REPLICATION

$$(!P) \stackrel{def}{=} P \mid !P$$

$$\underbrace{\bar{a}\langle u,v\rangle.\,P}_{\text{send two values over channel } a \text{ to the same process}} \mid \underbrace{a(x,y).\,Q}_{\text{receive two values over channel } a} \mid a(x,y).\,R$$

$(m \cdot n)\underline{n}\}$

$m \cdot \underline{n} \cdot n\ \_$

$$\underline{\bar{a}u}.\bar{a}v.P \mid$$
$$\underline{a(x)}.a(y).Q \mid$$
$$a(x).a(y).R$$
$$\xrightarrow{\tau} \underline{\bar{a}v}.P \mid a(y).Q\{u/x\} \mid \underline{a(x)}.a(y).R$$
$$\xrightarrow{\tau} P \mid a(y).Q\{u/x\} \mid a(y).R\{v/x\}$$

$$(\nu w)(\underline{\bar{a}w}.\bar{w}u.\bar{w}v.P) \mid$$
$$\underline{a(z)}.z(x).z(y).Q \mid$$
$$a(z).z(x).z(y).R$$
$$\xrightarrow{\tau} (\nu w)(\underline{\bar{w}u}.\bar{w}v.P \mid \underline{w(x)}.w(y).Q) \mid$$
$$a(z).z(x).z(y).R$$
$$\xrightarrow{\tau} (\nu w)(\underline{\bar{w}v}.P \mid \underline{w(y)}.Q\{u/x\}) \mid \ldots$$
$$\xrightarrow{\tau} (\nu w)(P \mid Q\{u,v/x,y\}) \mid \ldots$$

POLYADIC π-CALCULUS

$a(x_1 \cdots x_n)$ Multiple input

$\bar{a}\langle x_1 \cdots x_n \rangle$ Multiple output.

we choose c, s.t $c \notin fn(P)$:

$$a(x_1 \cdots x_n).P$$

$$\equiv a(c).\ c(x_1).c(x_2).\ \cdots c(x_n).P$$

$$\bar{a}\langle x_1 \ldots x_n \rangle.P$$

$$\equiv (\nu c)\ \cancel{\bar{a}c.}\ \cancel{\bar{c}x_1.\bar{c}x_2 \ldots \bar{c}x_n\, P}$$

$$(\nu c)\,\bar{a}c.\bar{c}x_1 \ldots \bar{c}x_n.P$$

$$a(x_1 \cdots x_n).P \mid \bar{a}\langle y_1 \cdots y_n \rangle.Q$$

$$\xrightarrow{\tau} P\{y_1 \cdots y_n / x_1 \cdots x_n\} \mid Q$$

A REFERENCE CELL IN π-CALCULUS

$$\mathrm{Ref}(r,w,i) = (\nu \ell)\,(\,\bar{\ell}i \mid \mathrm{ReadServer}(\ell,r) \mid \mathrm{WriteServer}(\ell,w)\,)$$

$$\mathrm{ReadServer}(\ell,r) = !\,r(c).\,\ell(v).\,(\,\bar{c}v \mid \bar{\ell}v\,)$$

$$\mathrm{WriteServer}(\ell,w) = !\,w(c,v').\,\ell(v).\,(\,\bar{c} \mid \bar{\ell}v'\,)$$

Example using reference cell:

$$(\nu c)\,\bar{w}\langle c,v\rangle.\,c.\,(\nu d)\,\bar{r}d.\,d(e).\,Q$$

will receive the value v over the channel d assuming no other processes interacting with the reference cell.

TWO-WAY COMMUNICATION

$$(\nu x)\, \bar{r}x . x(y) . \bar{y} . Q \mid r(e) . \bar{e}z . z . P$$

OPERATIONAL SEMANTICS OF π-CALCULUS
LABELLED TRANSITION SYSTEM

$$P \xrightarrow[\tau]{\alpha} Q$$

$$a(x).P \xrightarrow{a(x)} P$$

$$\bar{a}x.P \xrightarrow{\bar{a}x} P$$

$$a(b).\bar{b}c.0 \xrightarrow{a(w)} \bar{w}c.0 \xrightarrow{\bar{w}c} 0$$

$$\tau.P \xrightarrow{\tau} P$$

α

$(\nu x)\,\bar{x}a.P$ semantically equivalent to 0

$$\underbrace{(\nu a)\,\bar{x}a.P}\ |\ x(b).Q \xrightarrow{\tau} (\nu a)(P \mid Q\{a/b\})$$

π-CALCULUS OPERATIONAL SEMANTICS

STRUCT $$\frac{P' \equiv P, \quad P \xrightarrow{\alpha} Q, \quad Q \equiv Q'}{P' \xrightarrow{\alpha} Q'}$$

PREFIX $$\frac{}{\alpha.P \xrightarrow{\alpha} P}$$

SUM $$\frac{P \xrightarrow{\alpha} P'}{P+Q \xrightarrow{\alpha} P'}$$

MATCH $$\frac{P \xrightarrow{\alpha} P'}{\text{if } x=x \text{ then } P \xrightarrow{\alpha} P'}$$

MISMATCH $$\frac{P \xrightarrow{\alpha} P', \quad x \neq y}{\text{if } x \neq y \text{ then } P \xrightarrow{\alpha} P'}$$

PAR $$\frac{P \xrightarrow{\alpha} P', \quad bn(\alpha) \cap fn(Q) = \emptyset}{P|Q \xrightarrow{\alpha} P'|Q}$$

COM $$\frac{P \xrightarrow{a(x)} P', \quad Q \xrightarrow{\bar{a}u} Q'}{P|Q \xrightarrow{\tau} P'\{u/x\}|Q'}$$

RES $$\frac{P \xrightarrow{\alpha} P', \quad x \notin \alpha}{(\nu x)P \xrightarrow{\alpha} (\nu x)P'}$$

OPEN $$\frac{P \xrightarrow{\bar{a}x} P', \quad a \neq x}{(\nu x)P \xrightarrow{\bar{a}\nu x} P'}$$

$$fn(\bar{a}\nu x) = fn(\nu x\, \underbrace{\bar{a}x}) = \{a\}$$

$$bn(\bar{a}\nu x) = bn(\nu x\, \bar{a}x) = \{x\}$$

EXAMPLE OF STRUCT RULE:

$$\frac{P \equiv P' \;;\; P \xrightarrow{\alpha} Q \;;\; Q \equiv Q'}{P' \xrightarrow{\alpha} Q'}\ \text{STRUCT}$$

$$\underbrace{\bar{a}x.P \mid 0}_{P'} \equiv \underbrace{\bar{a}x.P}_{P} \qquad \underbrace{P}_{Q} \equiv \underbrace{P \mid 0}_{Q'}$$

$$\dfrac{\dfrac{}{\underbrace{\bar{a}x.P}_{P} \xrightarrow{\bar{a}x} \underbrace{P}_{Q}}\ \text{PREFIX}}{\underbrace{\bar{a}x.P \mid 0}_{P'} \xrightarrow{\bar{a}x} \underbrace{P \mid 0}_{Q'}}\ \text{STRUCT}$$

DUAL RULES

SUM$_2$ $$\frac{Q \xrightarrow{\alpha} Q'}{P + Q \xrightarrow{\alpha} Q'}$$

Can be derived from SUM and STRUCT

Combining RES and STRUCT

$$
\text{STRUCT}\ \dfrac{\text{RES}\ \dfrac{a(x).P \mid \bar{a}u.Q \xrightarrow{\tau} P\{u/x\} \mid Q}{(\nu u)(a(x).P \mid \bar{a}u.Q) \xrightarrow{\tau} (\nu u)(P\{u/x\} \mid Q)}}{a(x).P \mid (\nu u)\bar{a}u.Q \xrightarrow{\tau} (\nu u)(P\{u/x\} \mid Q)}
$$

Combining PAR and COM

$$\text{PREFIX}\ \frac{}{a(x).P \xrightarrow{a(x)} P}$$

$$\text{PAR}\ \frac{}{(a(x).P)\,|\,Q \xrightarrow{a(x)} P\,|\,Q} \qquad , \qquad \text{PREFIX}\ \frac{}{\bar{a}u.R \xrightarrow{\bar{a}u} R}$$

$$\text{COM}\ \frac{}{((a(x).P)\,|\,Q)\,|\,\bar{a}u.R \xrightarrow{\tau} (P\,|\,Q)\{u/x\}\,|\,R}$$