Review from Lecture 19

- Operators as non-member functions, as member functions, and as friend functions.
- A hash table is a table implementation with constant time access. Like a map, we can store key-value pair associations in the hash table. But it’s even faster to do find, insert, and erase with a hash table! However, hash tables don’t store the data in sorted order.
- A hash table is implemented with an array at the top level. Each key is mapped to a slot in the array by a hash function, a simple function of one argument (the key) which returns an index (a bucket or slot in the array).
- CallerID Performance: Vectors vs. Binary Search Trees vs. Hash Tables

Today’s Lecture

- Collision resolution: separate chaining vs open addressing
- Using a hash table to implement a set.
  - Function objects
  - Iterators, find, insert, and erase

Reading: Ford&Topp, Chapter 12 through Section 12.5.

20.1 What makes a Good Hash Function?

- Goals: (1) fast computation, and (2) a random, uniform distribution of keys throughout the table, despite the actual distribution of keys that are to be stored.
- For example, using: \( f(k) = \text{abs}(k) \% N \) as our hash function satisfies the first requirement, but may not satisfy the second.
- Another example of a dangerous hash function on string keys is to add or multiply the ascii values of each char:

  ```cpp
  unsigned int hash(string const& k, unsigned int N) {
    unsigned int value = 0;
    for (unsigned int i=0; i<k.size(); ++i)
      value += k[i]; // conversion to int is automatic
    return k % N;
  }
  ```

  The problem is that different permutations of the same string result in the same hash table location.
- This can be improved through multiplications that involve the position and value of the key:

  ```cpp
  unsigned int hash(string const& k, unsigned int N) {
    unsigned int value = 0;
    for (unsigned int i=0; i<k.size(); ++i)
      value = value*8 + k[i]; // conversion to int is automatic
    return k % N;
  }
  ```

  The 2nd method is better, but can be improved further. The theory of good hash functions is quite involved.

20.2 How do we Resolve Collisions?

When two keys map to the same table location, we have a collision. Two classes of approaches to collision resolution are commonly used:

- In separate chaining, each table location stores a list or vector of the keys (and values) that are hashed to that location.
- In open addressing, when a table location already stores a key (and its associated value, if any), a different table location is sought in order to store the new value.
20.3 Collision Resolution: Separate Chaining

- Each table location stores a list of keys (and values) hashed to it. Thus, the hashing function really just selects the list to check.

- This works well when the number of items stored in each list is small, e.g. an average of 1. Other data structures, such as binary search trees, may be used in place of the list, but these have even greater overhead considering the number of items stored.

20.4 Collision Resolution: Open Addressing

- Here are three different open addressing variations to handle a collision during an insert operation:
  - **Linear probing:** If \( i \) is the hash location then the following sequence of table locations is tested (“probed”):
    \[
    (i+1)\%N, (i+2)\%N, (i+3)\%N, \ldots
    \]
    until an empty location is found.
  - **Quadratic probing:** If \( i \) is the hash location then the following sequence of table locations is tested:
    \[
    (i+1)\%N, (i+2\times2)\%N, (i+3\times3)\%N, (i+4\times4)\%N, \ldots
    \]
    More generally, the \( j \)th “probe” of the table is \( (i + c_1 j + c_2 j^2) \mod N \) where \( c_1 \) and \( c_2 \) are constants.
  - **Secondary hashing:** when a collision occurs a second hash function is applied to compute a new table location. This is repeated until an empty location is found.

- For each of these approaches, the find operation follows the same sequence of locations as the insert operation. The key value is determined to be absent from the table only when an empty location is found.

- The erase function must mark a location as “formerly occupied”. If a location is marked empty, instead, find may fail. Formerly-occupied locations may (and should) be reused, but only after the find operation has been run to completion.

- Problems with open addressing:
  - Slows dramatically when the table is nearly full (e.g. about 80% or higher). This is particularly problematic for linear probing.
  - Fails completely when the table is full.
  - Cost of computing new hash values.

20.5 A Set As a Hash Table

- The class is templated over both the key type and the hash function type.

\[
\text{template < typename KeyType, typename HashFunc >}
\]
\[
\text{class ds_hashset {}
\]
\[
\text{...}
\]

- We use separate chaining for collision resolution. Hence the main data structure inside the class is:

\[
\text{std::vector< std::list<KeyType> > m_table;}
\]

- We will use automatic resizing to handle the possibility that our table is too full. This resize is analogous to the automatic reallocation that occurs inside the vector push_back function.

20.6 Function Objects

- Our hash function is implemented as an object with a function call operator.

- The basic form of the function call operator is shown below. Any specific number of arguments can be used.

\[
\text{class name {}
\]
\[
\text{public:}
\]
\[
\text{// ...}
\]
\[
\text{return_type operator() ( *** args *** );}
\]
Here is an example of a templated function object implementing the less-than comparison operation. This is the default 3rd argument to `std::sort`.

```cpp
template <typename T>
class less_than {
    public:
        bool operator() (const T& x, const T& y) { return x<y; }
};
```

Constructors of functions objects can be used to specify parameters for use in comparisons. For example

```cpp
class between_values {
    private:
        int x, y;
    public:
        between_values(int in_x, int in_y) : x(in_x), y(in_y) {}
        bool operator() (int z) { return x <= z && z <= y; }
    }
```

This can be used in combination with `find_if`. For example if `v` is a vector of integers, then

```cpp
between_values low_range(-99, 99);
if (std::find_if(v.begin(), v.end(), low_range) != v.end())
    std::cout << "A value between -99 and 99 is in the vector.\n";
```

### 20.7 Our Hash Function Object

```cpp
class hash_string_obj {
    public:
        unsigned int operator() (std::string const& key) const {
            // This implementation comes from
            // http://www.partow.net/programming/hashfunctions/
            unsigned int hash = 1315423911;
            for(unsigned int i = 0; i < key.length(); i++)
                hash ^= ((hash << 5) + key[i] + (hash >> 2));
            return hash;
        }
};
```

The type `hash_string_obj` is one of the template parameters to the declaration of a `ds_hashset`. E.g.,

```cpp
ds_hashset<std::string, hash_string_obj> my_hashset;
```

### 20.8 Hash Set Iterators

- Iterators move through the hash table in the order of the storage locations rather than the ordering imposed by (say) an `operator<`. The order depends on the hash function and the table size.
  - Hence the increment operators must move to the next entry in the current list or, if the end of the current list is reached, to the first entry in the next non-empty list.

- The declaration is nested inside the `ds_hashset` declaration in order to avoid explicitly templating the iterator over the hash function type.

- The iterator must store a pointer to the hash table it is associated with.
  - This reflects a subtle point about types: even though the `iterator` class is declared inside the `ds_hashset`, this does not mean an iterator automatically knows about any particular `ds_hashset`.

- The iterator must also store:
  - The index of the current list in the hash table.
  - An iterator referencing the current location in the current list.

- Because of the way the classes are nested, the `iterator` class object must declare the `ds_hashset` class as a friend, but the reverse is unnecessary.
20.9 Implementing begin() and end()

- **begin()**: Skips over empty lists to find the first key in the table. It must tie the iterator being created to the particular ds_hashset object it is applied to. This is done by passing the this pointer to the iterator constructor.

- **end()**: Also associates the iterator with the specific table, assigns an index of -1 (indicating it is not a normal valid index), and thus does not assign the particular list iterator.

**Exercise**: Implement the begin() function.

20.10 Iterator Increment, Decrement, & Comparison Operators

- The increment operators must find the next key, either in the current list, or in the next non-empty list.

- The decrement operator must check if the iterator in the list is at the beginning and if so it must proceed to find the previous non-empty list and then find the last entry in that list. This might sound expensive, but remember that the lists should be very short.

- The comparison operators must accommodate the fact that when (at least) one of the iterators is the end, the internal list iterator will not have a useful value.

20.11 Insert & Find

- Computes the hash function value and then the index location.

- If the key is already in the list that is at the index location, then no changes are made to the set, but an iterator is created referencing the location of the key, a pair is returned with this iterator and false.

- If the key is not in the list at the index location, then the key should be inserted in the list (at the front is fine), and an iterator is created referencing the location of the newly-inserted key a pair is returned with this iterator and true.

**Exercise**: Implement the insert() function, ignoring for now the resize operation.

- Find is similar to insert, computing the hash function and index, followed by a std::find operation.

20.12 Erase

- Two versions are implemented, one based on a key value and one based on an iterator. These are based on finding the appropriate iterator location in the appropriate list, and applying the list erase function.

20.13 Resize

- Must copy the contents of the current vector into a scratch vector, resize the current vector, and then re-insert each key into the resized vector.

**Exercise**: Write resize()

20.14 Hash Table Iterator Invalidation

- Any insert operation invalidates all ds_hashset iterators because the insert operation could cause a resize of the table.

- The erase function only invalidates an iterator that references the current object.

20.15 Summary

- The algorithmic steps are not particularly difficult.

- The use of C++ is more sophisticated than recent examples.

- The standard library is exploited to simplify the implementation.

- This imposes some hidden costs in the implementation, the most substantial being in resize where
  - lists are copied and then reallocated, and
  - hash function values are recomputed.
#ifndef ds_hashset_h_
#define ds_hashset_h_

// The set class as a hash table instead of a binary search tree.  The
// primary external difference between ds_set and ds_hashset is that
// the iterators do not step through the hashset in any meaningful
// order.  It is just the order imposed by the hash function.
#include <iostream>
#include <list>
#include <string>
#include <vector>

// The ds_hashset is templated over both the type of key and the type
// of the hash function, a function object.
// template < typename KeyType, typename HashFunc >
class ds_hashset {
private:
    typedef typename std::list<KeyType>::iterator hash_list_itr;
public:
    // =================================================================
    // THE ITERATOR CLASS
    // Defined as a nested class and thus is not separately templated.
    class iterator {
        friend class ds_hashset; // allows access to private variables
    private:
        // ITERATOR REPRESENTATION
        ds_hashset* m_hs;
        int m_index;  // current index in the hash table
        hash_list_itr m_list_itr; // current iterator at the current index
    private:
        // private constructors for use by the ds_hashset only
        iterator(ds_hashset* hs, int index, hash_list_itr loc)
            : m_hs(hs), m_index(index), m_list_itr(loc) {}
        iterator() : m_hs(0), m_index(-1) {}    iterator(iterator const& itr)
            : m_hs(itr.m_hs), m_index(itr.m_index), m_list_itr(itr.m_list_itr) {}
        iterator& operator=(iterator const& old) {
            m_hs = old.m_hs;
            m_index = old.m_index;
            m_list_itr = old.m_list_itr;
            return *this;
        }
        // The dereference operator need only worry about the current
        // list iterator, and does not need to check the current index.
        const KeyTypes operator*() const { return m_list_itr; }
    // The comparison operators must account for the list iterators
    // being unassigned at the end.
        friend bool operator==(const iterator& lft, const iterator& rgt)
            { return lft.m_hs == rgt.m_hs && lft.m_index == rgt.m_index &&
                (lft.m_index == -1 || lft.m_list_itr == rgt.m_list_itr); }
        friend bool operator!=(const iterator& lft, const iterator& rgt)
            { return lft.m_hs != rgt.m_hs || lft.m_index != rgt.m_index ||
                (lft.m_index == -1 && lft.m_list_itr != rgt.m_list_itr); }
        // increment and decrement
        iterator operator++() { return *this; }
        iterator operator+(int) { iterator temp(*this);
            this->next();
            return temp; }
        iterator & operator--() { return *this; }
        iterator operator--(int) { iterator temp(*this);
            this->prev();
            return temp; }
    private:
        // Find the next entry in the table
        void next() {
            // If we are at the end of this list
            if (m_list_itr == m_hs->m_table[m_index].end()) {
                // Find the next non-empty list in the table
                for (++m_index;
                    m_index < int(m_hs->m_table.size()) && m_hs->m_table[m_index].empty();
                    ++m_index) {}    // If one is found, assign the m_list_itr to the start
                if (m_index != int(m_hs->m_table.size()))
                    m_list_itr = m_hs->m_table[m_index].begin();
                // Otherwise, we are at the end
                else
                    m_index = -1;
            }    // end of ITERATOR CLASS
        }
        // Find the previous entry in the table
        void prev() {
            // If we aren’t at the start of the current list, just decrement
            // the list iterator
            if (m_list_itr != m_hs->m_table[m_index].begin())
                m_list_itr =
            else {
                // Otherwise, back down the table until the previous
                // non-empty list in the table is found
                for (; m_list_itr != m_hs->m_table[m_index].end(); ++p,
                    ++m_list_itr) {
                }
            }
        }
    // =================================================================
}
#endif  // ifndef  ds_hashset_h_
private:
  // HASH SET REPRESENTATION
  std::vector<std::list<KeyType> > m_table; // actual table
  HashFunc m_hash; // hash function
  unsigned int m_size; // number of keys

public:
  // HASH SET IMPLEMENTATION

  // Constructor for the table accepts the size of the table. Default
  // constructor for the hash function object is implicitly used.
  ds_hashset(unsigned int init_size = 10) : m_table(init_size), m_size(0) {}  

  // Copy constructor just uses the member function copy constructors.
  ds_hashset(const ds_hashset<KeyType, HashFunc>& old) : m_table(old.m_table), m_size(old.m_size) {}  

  ~ds_hashset() {}  

  ds_hashset& operator=(const ds_hashset<KeyType, HashFunc>& old) {  
    if (&old != this)  
      *this = old;  
  }  

  unsigned int size() const {  
    return m_size;  
  }  

  // Insert the key if it is not already there.
  std::pair< iterator, bool > insert(KeyType const & key) {  
    const float LOAD_FRACTION_FOR_RESIZE = 1.25;  
    if (m_size >= LOAD_FRACTION_FOR_RESIZE * m_table.size())  
      this->resize_table(2*m_table.size()+1);  
    // implemented in class  
    }  

  // Erase the key
  int erase(const KeyType& key) {  
    // Find the key and use the erase iterator function.
    iterator p = find(key);  
    if (p == end())  
      return 0;  
    else {  
      erase(p);  
      return 1;  
    }  

    // Erase at the iterator
    void erase(iterator p) {  
      m_table[ p.m_index ].erase(p.m_list_itr);  
    }  

    // Find the first entry in the table and create an associated iterator
    iterator begin() {  
      // implemented in class  
    }  

    // Create an end iterator.
    iterator end() {  
      iterator p(this);  
      p.m_index = -1;  
      return p;  
    }  

    // A public print utility.
    void print(std::ostream & ostr) {  
      for (unsigned int i=0; i<m_table.size(); ++i)  
        ostr << i << " : " ;  
      for (hash_list_itr p = m_table[i].begin(); p != m_table[i].end(); ++p)  
        ostr << *p ;  
      ostr << std::endl;  
    }  

  }  

private:
  // resize the table with the same values but a
  void resize_table(unsigned int new_size) {  
    // implemented in class  
  }