Declarative Computation Model

Single assignment store (VRH 2.2)
Kernel language syntax (VRH 2.3)

Carlos Varela
RPI
October 6, 2009

Adapted with permission from:
Seif Haridi
KTH
Peter Van Roy
UCL

On Academic Dishonesty

Academic dishonesty policies apply to this course, including:
– Academic Fraud
– Collaboration
– Copying
– Cribbing
– Fabrication
– Plagiarism
– Sabotage
– Substitution

If in doubt, ask the instructor, or see Rensselaer Handbook of Student Rights and Responsibilities at:
http://www.rpi.edu/dept/doso/handbook.html

Students found in violation of academic dishonesty policies may receive a failing grade for this course.

Sequential declarative computation model

• The single assignment store
  – declarative (dataflow) variables
  – partial values (variables and values are also called entities)
• The kernel language syntax
• The kernel language semantics
  – The environment: maps textual variable names (variable identifiers) into entities in the store
  – Interpretation (execution) of the kernel language elements (statements) by the use of an abstract machine
  – Abstract machine consists of an execution stack of statements transforming the store

Single assignment store

• A single assignment store is a store (set) of variables
• Initially the variables are unbound, i.e. do not have a defined value
• Example: a store with three variables, x₁, x₂, and x₃

Single assignment store (2)

• Variables in the store may be bound to values
• Example: assume we allow as values, integers and lists of integers

Single assignment store (3)

• Variables in the store may be bound to values
• Assume we allow as values, integers and lists of integers
• Example: x₁ is bound to the integer 314, x₂ is bound to the list [1 2 3], and x₃ is still unbound
Declarative (single-assignment) variables

- A declarative variable starts out as being unbound when created
- It can be bound to exactly one value
- Once bound it stays bound through the computation, and is indistinguishable from its value

Value store

- A store where all variables are bound to values is called a value store
- Example: a value store where \( x_1 \) is bound to integer 314, \( x_2 \) to the list \([1, 2, 3]\), and \( x_3 \) to the record (labeled tree) person(name: "George", age: 25)
- Functional programming computes functions on values, needs only a value store
- This notion of value store is enough for functional programming (ML, Haskell, Scheme)

Operations on the store (1)

Single assignment

\( \langle x \rangle = \langle v \rangle \)
- \( x_1 = 314 \)
- \( x_2 = [1, 2, 3] \)
- This assumes that \( \langle x \rangle \) is unbound

Single-assignment (2)

\( \langle x \rangle = \langle v \rangle \)
- \( x_1 = 314 \)
- \( x_2 = [1, 2, 3] \)
- The single assignment operation \( \langle = \rangle \) constructs the \( \langle v \rangle \) in the store and binds the variable \( \langle x \rangle \) to this value
- If the variable is already bound, the operation will test the compatibility of the two values
- If the test fails an error is raised

Variable identifiers

- Variable identifiers refers to store entities (variables or values)
- The environment maps variable identifiers to variables
- \( \text{declare } X \)
- \( \text{local } X \) in ...
- "X" is a (variable) identifier
- This corresponds to "environment" \( \{ X \rightarrow x_1 \} \)
Variable-value binding revisited (1)

- \( X = [1 \ 2 \ 3] \)
- Once bound the variable is indistinguishable from its value

\[
\begin{array}{c}
\text{X} \\
\leftarrow \ x_1 \ x_2 \ x_3
\end{array}
\]

\[
\begin{array}{c}
\text{X} \\
\leftarrow \ 1 \ \ 2 \ \ 3 \ \ \text{nil}
\end{array}
\]

Variable-value binding revisited (2)

- \( X = [1 \ 2 \ 3] \)
- Once bound the variable is indistinguishable from its value
- The operation of traversing variable cells to get the value is known as dereferencing and is invisible to the programmer

\[
\begin{array}{c}
\text{X} \\
\leftarrow \ x_1 \ x_2 \ x_3
\end{array}
\]

Partial Values

- A partial value is a data structure that may contain unbound variables
- The store contains the partial value: `person(name: "George" age: x_2)`
- `\text{X} = \text{person(name: "George" age: Y)}`
- The identifier `Y` refers to `x_2`

Partial Values (2)

Partial Values may be complete

- `\text{declare} \ Y \ X`
- `\text{X} = \text{person(name: "George" age: Y)}`
- `Y = 25`

Variable to variable binding

\( \langle x_1 \rangle = \langle x_2 \rangle \)

- It is to perform the bind operation between variables
- Example:
  - `X = Y`
  - `X = [1 \ 2 \ 3]`
- The operations equates (merges) the two variables

Variable to variable binding (2)

\( \langle x_1 \rangle = \langle x_2 \rangle \)

- It is to perform a single assignment between variables
- Example:
  - `X = Y`
  - `X = [1 \ 2 \ 3]`
- The operations equates the two variables (forming an equivalence class)
C. Varela; Adapted w/permission from S. Haridi and P. Van Roy

Variable to variable binding (3)

\[ (x_1) \neq (x_2) \]
- It is to perform a single assignment between variables
- Example:
  - \( X = [1 \ 2 \ 3] \)
  - All variables (X and Y) are bound to \([1 \ 2 \ 3]\)

The Store
X
[1 2 3]
Y
[1 2 3]

C. Varela; Adapted w/permission from S. Haridi and P. Van Roy

Summary
Variables and partial values
- Declarative variable:
  - is an entity that resides in a single-assignment store, that is initially unbound, and can be bound to exactly one (partial) value
  - it can be bound to several (partial) values as long as they are compatible with each other
- Partial value:
  - is a data-structure that may contain unbound variables
  - when one of the variables is bound, it is replaced by the (partial) value it is bound to
  - a complete value, or value for short is a data structure that does not contain any unbound variables

Declaration and use of variables
- Assume that variables can be declared (introduced) and used separately
- What happens if we try to use a variable before it is bound?
  1. Use whatever value happens to be in the memory cell occupied by the variable (C, C++)
  2. The variable is initialized to a default value (Java), use the default
  3. An error is signaled (Prolog). Makes sense if there is a single activity running (pure sequential programs)
  4. An attempt to use the variable will wait (suspends) until another activity binds the variable (Oz/Mozart)

Kernel language syntax
The following defines the syntax of a statement, \( s \) denotes a statement

\[
(s) \triangleright= \begin{cases} 
\text{skip} & \text{empty statement} \\
(x) = (y) & \text{variable-variable binding} \\
(x) = (v) & \text{variable-value binding} \\
 lokal((x) in (s_1) end) & \text{sequential composition} \\
\{ \text{if} (x) \text{then} (s_1) \text{else} (s_2) \text{end} \} & \text{declaration} \\
\{ \text{case} (x) \text{of} (\text{pattern}) \text{then} (s_1) \text{else} (s_2) \text{end} \} & \text{conditional} \\
\{ \text{case} (y) \text{of} (\text{pattern}) \text{then} (s_1) \text{else} (s_2) \text{end} \} & \text{procedural application} \\
(value) \triangleright= \ldots & \text{pattern matching} \\
(pattern) \triangleright= \ldots & \text{value expression}
\end{cases}
\]

C. Varela; Adapted w/permission from S. Haridi and P. Van Roy

Variable identifiers
- \( (x), (y), (z) \) stand for variables
- In the concrete kernel language variables begin with uppercase letter followed by a (possibly empty) sequence of alphanumeric characters or underscore
- Any sequence of printable characters within back-quote
- Examples:
  - X
  - Y
  - Hello_World
  - "hello this is a $5 bill" (back-quote)
Values and types

- A data type is a set of values and a set of associated operations.
- Example: Int is the data type "Integer", i.e. set of all integer values.
- 1 is of type Int.
- Int has a set of operations including +,-,*,div, etc.
- The model comes with a set of basic types.
- Programs can define other types, e.g., abstract data types ADT.

Data types

- Value
  - Number
  - Record
  - Procedure
- Procedure
  - Int
  - Float
  - Literal
  - Tuple
  - List
  - Char
  - Atom
  - Boolean
  - True
  - False
  - String

Data types (2)

Value
- Number
- Record
- Procedure
- Tuple
- Literal
- Atom
- Boolean
- True
- False
- String

Value expressions

(v) ::= (procedure) | (record) | (number)
(procedure) ::= proc 'k' ('y_1' ... 'y_n') 's' end
(record), (pattern) ::= (literal)
  | (literal) (feature_1) : (x_1) ... (feature_n) : (x_n)
(literal) ::= (atom) | (bool)
(feature) ::= (int) | (atom) | (bool)
(bool) ::= true | false
(number) ::= (int) | (float)

Numbers

- Integers
  - 314, 0
  - -10 (minus 10)
- Floats
  - 1.0, 3.4, 2.0e2, 2.0E2 (2×10^2)

Atoms and booleans

- A sequence starting with a lower-case character followed by characters or digits, ...
  - person, peter
  - 'Seif Haridi'
- Booleans:
  - true
  - false
Records

• Compound representation (data-structures)
  - ⟨lj (x1 · · · xn)⟩ (tuple)
  - {l} is a literal

• Examples
  - person(age:X1 name:X2)
  - person(1:X1 2:X2)
  - '| (1:H 2:T)
  - nil
  - person

Syntactic sugar (tuples)

• Tuples
  - ⟨lj (x1 · · · xn)⟩ (tuple)
  - This is equivalent to the record
    ⟨lj (x1 : · · · n: xn)⟩

• Example:
  - person('George' 25)
  - This is the record
    person(1:'George' 2:25)

Syntactic sugar (lists)

• Lists
  - ⟨lx1 | lx2⟩ (a cons with the infix operator '|')
  - This is equivalent to the tuple
    ⟨lx1 | ⟨lx2⟩⟩

• Example:
  - H | T
  - This is the tuple
    ⟨H T⟩

Syntactic sugar (list)

• Lists
  - ⟨lx1 | lx2 | lx3⟩
  - '}' associates to the right
    ⟨lx1 | ⟨lx2 | lx3⟩⟩

• Example:
  - 1 | 2 | 3 | nil
  - This is the tuple
    ⟨1 (2 (3 nil))⟩

Strings

• A string is a list of character codes enclosed with double quotes
  - Ex: "E=mc^2"
  - Means the same as [69 61 109 99 94 50]
Procedure declarations

- According to the kernel language
  \( (x) \, \text{proc} \{ (y_1) \ldots (y_n) \} \, (s) \, \text{end} \)
  is a legal statement
- It binds \( (x) \) to a procedure value
- This statement actually declares (introduces) a procedure
- Another syntactic variant which is more familiar is
  \( \text{proc} \{ (x) \, (y_1) \ldots (y_n) \} \, (s) \, \text{end} \)
- This introduces (declares) the procedure \( (x) \)

Operations of basic types

- Arithmetics
  - Floating point numbers: +, -, *, and /
  - Integers: +, -, div (integer division, i.e. truncate fractional part), mod (the remainder after a division, e.g. 10 mod 3 = 1)
- Record operations
  - Arity, Label, and ".
  - \( X = \text{person(name="George", age=25)} \)
  - \{Arity X\} = [age name]
  - \{Label X\} = person, X.age = 25
- Comparisons
  - Boolean comparisons, including ==, \( \neq \) (equality)
  - Numeric comparisons, =<, <, >, >=, compares integers, floats, and atoms

Value expressions

\( (v) ::= \) (procedure) | (record) | (number) | (basicExpr)

(basicExpr) ::= ... | (numberExpr) ... 

(numberExpr) ::= (x)_1 \, \ast \, (x)_2 \, ... 

....

Syntactic sugar (multiple variables)

- Multiple variable introduction
  \( \text{local} \, X \, Y \, \text{in} \, (\text{statement}) \, \text{end} \)
- is transformed to
  \( \text{local} \, X \, \text{in} \, \text{local} \, Y \, \text{in} \, (\text{statement}) \, \text{end} \)
- where \( T \) is a fresh (‘new’) variable identifier

Syntactic sugar (basic expressions)

- Basic expression nesting
  
  if \( \text{basicExpr} \) then \( \text{statement}_1 \) else \( \text{statement}_2 \) end
-
  is transformed to
  
  local \( T \) in
  \( T = \text{basicExpr} \)
  if \( T \) then \( \text{statement}_1 \) else \( \text{statement}_2 \) end
  
  where \( T \) is a fresh (‘new’) variable identifier

Syntactic sugar (variables)

- Variable initialization
  \( \text{local} \, X = \text{(value)} \, \text{in} \, (\text{statement}) \, \text{end} \)
- is transformed to
  
  local \( X \) in
  \( X = \text{(value)} \, (\text{statement}) \, \text{end} \)
Exercises

42. Using Oz, perform a few basic operations on numbers, records, and booleans (see Appendix B1-B3).

43. Explain the behavior of the `declare` statement in the interactive environment. Give an example of an interactive Oz session where "declare" and "declare ... in" produce different results. Explain why.

44. VRH Exercise 2.9.1

45. Describe what an anonymous procedure is, and write one in Oz. When are anonymous procedures useful?