Declarative Computation Model

Kernel language semantics
(Non-)Suspendable statements (VRH 2.4.3-2.4.4)

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Sequential declarative computation model

- The kernel language semantics
  - The environment: maps textual variable names (variable identifiers) into entities in the store
  - Abstract machine consists of an execution stack of semantic statements transforming the store
  - Interpretation (execution) of the kernel language elements (statements) by the use of an abstract machine
    - Non-suspendable statements
    - Suspendable statements

Kernel language syntax

The following defines the syntax of a statement, ⟨s⟩ denotes a statement

⟨s⟩ ::=	skip	*empty statement*
|	⟨x⟩ = ⟨y⟩	*variable-variable binding*
|	⟨x⟩ = ⟨v⟩	*variable-value binding*
|	$\langle s_1\rangle\ \langle s_2\rangle$	*sequential composition*
|	local ⟨x⟩ in $\langle s_1\rangle$ end	*declaration*
|	if ⟨x⟩ then $\langle s_1\rangle$ else $\langle s_2\rangle$ end	*conditional*
|	{ ⟨x⟩ $\langle y_1\rangle$ ... $\langle y_n\rangle$ }	*procedural application*
|	case ⟨x⟩ of ⟨pattern⟩ then $\langle s_1\rangle$ else $\langle s_2\rangle$ end	*pattern matching*
⟨v⟩ ::=	proc { \$ $\langle y_1\rangle$ ... $\langle y_n\rangle$ } $\langle s_1\rangle$ end | ...	*value expression*
⟨pattern⟩ ::=	...	

Computations (abstract machine)

- A computation defines how the execution state is transformed step by step from the initial state to the final state
- A *single assignment store* σ is a set of store variables, a variable may be unbound, bound to a partial value, or bound to a group of other variables
- An *environment E* is mapping from variable identifiers to variables or values in σ, e.g. $\{X \rightarrow x_1, Y \rightarrow x_2\}$
- A *semantic statement* is a pair
  (⟨s⟩ , *E*) where ⟨s⟩ is a statement
- *ST* is a stack of <u>semantic statements</u>

Computations (abstract machine)

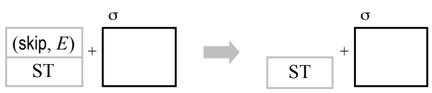
- A computation defines how the execution state is transformed step by step from the initial state to the final state
- The *execution state* is a pair
  (*ST* , σ)
  - where *ST* is a *stack of semantic statements* and σ is a *single assignment store*
- A *computation* is a sequence of execution states
  $(ST_0 , \sigma_0) \rightarrow (ST_1 , \sigma_1) \rightarrow (ST_2 , \sigma_2) \rightarrow \ldots$

Semantics

- To execute a program (i.e., a statement) ⟨*s*⟩ the initial execution state is
  ([(⟨*s*⟩ , ∅)] , ∅)
- *ST* has a single semantic statement (⟨*s*⟩ , ∅)
- The environment *E* is empty, and the store σ is empty
- [...] denotes the stack
- At each step the first element of *ST* is popped and execution proceeds according to the form of the element
- The final execution state (if any) is a state in which *ST* is empty

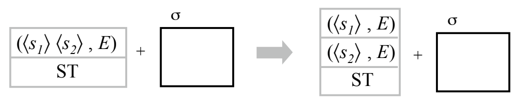
skip

- The semantic statement is
  (skip, E)
- Continue to next execution step



Sequential composition

- The semantic statement is
  $(\langle s_1 \rangle\ \langle s_2 \rangle\ , E)$
- Push $(\langle s_2 \rangle\ , E)$ and then push $(\langle s_1 \rangle\ , E)$ on ST
- Continue to next execution step

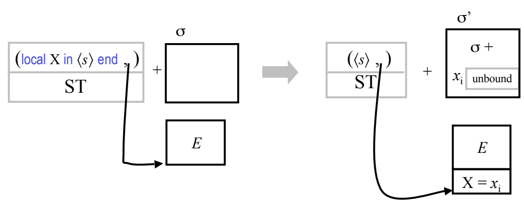


Variable declaration

- The semantic statement is
  (local $\langle x \rangle$ in $\langle s \rangle$ end, E)
- Create a new store variable x in the Store
- Let E' be $E+\{\langle x \rangle \rightarrow x\}$, i.e. E' is the same as E but the identifier $\langle x \rangle$ is mapped to x.
- Push $(\langle s \rangle\ , E')$ on ST
- Continue to next execution step

Variable declaration

- The semantic statement is
  (local X in $\langle s \rangle$ end, E)



Variable-variable equality

- The semantic statement is
  $(\langle x \rangle = \langle y \rangle, E)$
- Bind $E(\langle x \rangle)$ and $E(\langle y \rangle)$ in the store

Variable-value equality

- The semantic statement is
  $(\langle x \rangle = \langle v \rangle, E)$
- Where $\langle v \rangle$ is a record, a number, or a procedure
- Construct the value in the store and refer to it by the variable y.
- Bind $E(\langle x \rangle)$ and y in the store
- We have seen how to construct records and numbers, but what is a procedure value?

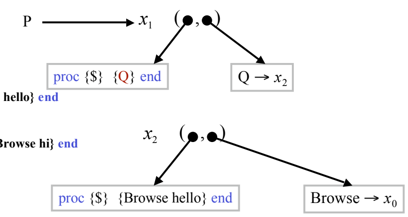
Procedure values

- Constructing a procedure value in the store is not simple because a procedure may have external references

```
local P Q in
   Q = proc {$} {Browse hello} end
   P = proc {$}  {Q} end
   local Q in
      Q = proc {$} {Browse hi} end
      {P}
   end
end
```


Procedure values (2)

```
local P Q in
   Q = proc {$} {Browse hello} end
   P = proc {$}  {Q} end
   local Q in
      Q = proc {$} {Browse hi} end
      {P}
   end
end
```



Procedure values (3)

- The semantic statement is
  $(\langle x \rangle$ = proc {\$ $\langle y_1 \rangle \ldots \langle y_n \rangle$} $\langle s \rangle$ end, $E)$
- $\langle y_1 \rangle \ldots \langle y_n \rangle$ are the (*formal*) parameters of the procedure
- Other free identifiers in $\langle s \rangle$ are called *external references* $\langle z_1 \rangle \ldots \langle z_k \rangle$
- These are defined by the environment E where the procedure is declared (lexical scoping)
- The contextual environment of the procedure CE is $E|_{\{\langle z_1 \rangle \ldots \langle z_k \rangle\}}$
- When the procedure is called CE is used to construct the environment for execution of $\langle s \rangle$

(proc {\$ $\langle y_1 \rangle \ldots \langle y_n \rangle$ }
  $\langle s \rangle$
end ,
CE)

Procedure values (4)

- Procedure values are pairs:
  (proc {\$ $\langle y_1 \rangle \ldots \langle y_n \rangle$} $\langle s \rangle$ end , CE)
- They are stored in the store just as any other value

(proc {\$ $\langle y_1 \rangle \ldots \langle y_n \rangle$ }
  $\langle s \rangle$
end ,
CE)

Procedure introduction

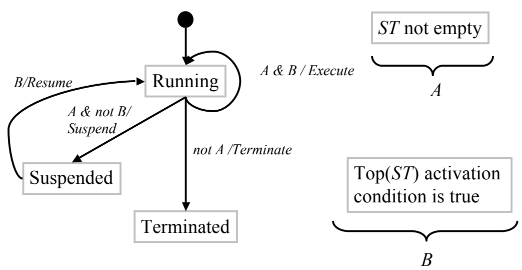
- The semantic statement is
  $(\langle x \rangle$ = proc {\$ $\langle y_1 \rangle \ldots \langle y_n \rangle$} $\langle s \rangle$ end, $E)$
- Create a contextual environment:
  $CE = E|_{\{\langle z_1 \rangle \ldots \langle z_k \rangle\}}$ where $\langle z_1 \rangle \ldots \langle z_k \rangle$ are external references in $\langle s \rangle$.
- Create a new procedure value of the form:
  (proc {\$ $\langle y_1 \rangle \ldots \langle y_n \rangle$} $\langle s \rangle$ end, CE) , refer to it by the variable x_P
- Bind the store variable $E(\langle x \rangle)$ to x_P
- Continue to next execution step

Suspendable statements

- The remaining statements require $\langle x \rangle$ to be bound in order to execute
- The activation condition ($E(\langle x \rangle)$ is *determined*), is that $\langle x \rangle$ be bound to a number, a record or a procedure value

$\langle$s$\rangle$ ::= …	
| if $\langle$x$\rangle$ then $\langle$s$_1\rangle$ else $\langle$s$_2\rangle$ end	*conditional*
| { $\langle$x$\rangle$ $\langle$y$_1\rangle$ … $\langle$y$_n\rangle$ }	*procedural application*
| case $\langle$x$\rangle$ of $\langle$pattern$\rangle$ then $\langle$s$_1\rangle$ else $\langle$s$_2\rangle$ end	*pattern matching*

Life cycle of a thread



Conditional

- The semantic statement is
  (if ⟨x⟩ then $\langle s_1 \rangle$ else $\langle s_2 \rangle$ end , E)
- If the activation condition (E(⟨x⟩) is determined) is true:
  - If E(⟨x⟩) is not Boolean (true, false), raise an error
  - E(⟨x⟩) is true, push ($\langle s_1 \rangle$, E) on the stack
  - E(⟨x⟩) is false, push ($\langle s_2 \rangle$, E) on the stack
- If the activation condition (E(⟨x⟩) is determined) is false:
  - Suspend

Procedure application

- The semantic statement is
  ({ $\langle x \rangle \langle y_1 \rangle \ldots \langle y_n \rangle$ } , E)
- If the activation condition (E(⟨x⟩) is determined) is true:
  - If E(⟨x⟩) is not a procedure value, or it is a procedure with arity that is not equal to n, raise an error
  - If E(⟨x⟩) is (proc {\$ $\langle z_1 \rangle \ldots \langle z_n \rangle$} ⟨s⟩ end, CE) , push
    (⟨s⟩ , CE + {$\langle z_1 \rangle \rightarrow E(\langle y_1 \rangle) \ldots \langle z_n \rangle \rightarrow E(\langle y_n \rangle)$})
    on the stack
- If the activation condition (E(⟨x⟩) is determined) is false:
  - Suspend

Case statement

- The semantic statement is
  (case ⟨x⟩ of ⟨l⟩ ($\langle f_1 \rangle : \langle x_1 \rangle \ldots \langle f_n \rangle : \langle x_n \rangle$)
  then $\langle s_1 \rangle$
  else $\langle s_2 \rangle$ end , E)
- If the activation condition (E(⟨x⟩) is determined) is true:
  - If E(⟨x⟩) is a record, and the label of E(⟨x⟩) is ⟨l⟩ and its arity is [$\langle f_1 \rangle \ldots \langle f_n \rangle$]:
    push (local $\langle x_1 \rangle = \langle x \rangle . \langle f_1 \rangle \ldots \langle x_n \rangle = \langle x \rangle . \langle f_n \rangle$ in $\langle s_1 \rangle$ end, E)
    on the stack
  - Otherwise, push ($\langle s_2 \rangle$, E) on the stack
- If the activation condition (E(⟨x⟩) is determined) is false:
  - Suspend

Execution examples

$\langle s \rangle_1$ $\langle s \rangle_2$

local Max C in
  proc {Max X Y Z}
    $\langle s \rangle_3$ if X >= Y then Z=X else Z=Y end
  end
  {Max 3 5 C}
end

Execution examples (2)

$\langle s \rangle_1$ $\langle s \rangle_2$

local Max C in
  proc {Max X Y Z}
    $\langle s \rangle_3$ if X >= Y then Z=X else Z=Y end
  end
  $\langle s \rangle_4$ {Max 3 5 C}
end

- Initial state ([($\langle s \rangle_1$, ∅)], ∅)
- After local Max C in …
  ([($\langle s \rangle_2$, {Max → m, C → c})], {m, c})
- After Max binding
  ([($\langle s \rangle_4$, {Max → m, C → c})],
  {m = (proc{\$ X Y Z} $\langle s \rangle_3$ end , ∅) , c})

Execution examples (3)

$$\langle s\rangle_1 \begin{cases} \text{local Max C in} \\ \quad \langle s\rangle_2 \begin{cases} \text{proc \{Max X Y Z\}} \\ \quad \langle s\rangle_3 \ \text{if X >= Y then Z=X else Z=Y end} \\ \text{end} \\ \langle s\rangle_4 \ \text{\{Max 3 5 C\}} \end{cases} \\ \text{end} \end{cases}$$

- After Max binding
  $([(\langle s\rangle_4, \{\text{Max} \rightarrow m, \text{C} \rightarrow c\})],$
  $\{m = (\text{proc}\{\$ \text{ X Y Z}\} \langle s\rangle_3 \text{end}, \varnothing), c\})$
- After procedure call
  $([(\langle s\rangle_3, \{\text{X} \rightarrow t_1, \text{Y} \rightarrow t_2, \text{Z} \rightarrow c\})],$
  $\{m = (\text{proc}\{\$ \text{ X Y Z}\} \langle s\rangle_3 \text{end}, \varnothing), t_1=3, t_2=5, c\})$

Execution examples (4)

$$\langle s\rangle_1 \begin{cases} \text{local Max C in} \\ \quad \langle s\rangle_2 \begin{cases} \text{proc \{Max X Y Z\}} \\ \quad \langle s\rangle_3 \ \text{if X >= Y then Z=X else Z=Y end} \\ \text{end} \\ \langle s\rangle_4 \ \text{\{Max 3 5 C\}} \end{cases} \\ \text{end} \end{cases}$$

- After procedure call
  $([(\langle s\rangle_3, \{\text{X} \rightarrow t_1, \text{Y} \rightarrow t_2, \text{Z} \rightarrow c\})],$
  $\{m = (\text{proc}\{\$ \text{ X Y Z}\} \langle s\rangle_3 \text{end}, \varnothing), t_1=3, t_2=5, c\})$
- After T = (X>=Y)
  $([(\langle s\rangle_3, \{\text{X} \rightarrow t_1, \text{Y} \rightarrow t_2, \text{Z} \rightarrow c, \text{T} \rightarrow t\})],$
  $\{m = (\text{proc}\{\$ \text{ X Y Z}\} \langle s\rangle_3 \text{end}, \varnothing), t_1=3, t_2=5, c, t=\text{false}\})$
- $([(\text{Z=Y}, \{\text{X} \rightarrow t_1, \text{Y} \rightarrow t_2, \text{Z} \rightarrow c, \text{T} \rightarrow t\})],$
  $\{m = (\text{proc}\{\$ \text{ X Y Z}\} \langle s\rangle_3 \text{end}, \varnothing), t_1=3, t_2=5, c, t=\text{false}\})$

Execution examples (5)

$$\langle s\rangle_1 \begin{cases} \text{local Max C in} \\ \quad \langle s\rangle_2 \begin{cases} \text{proc \{Max X Y Z\}} \\ \quad \langle s\rangle_3 \ \text{if X >= Y then Z=X else Z=Y end} \\ \text{end} \\ \langle s\rangle_4 \ \text{\{Max 3 5 C\}} \end{cases} \\ \text{end} \end{cases}$$

- $([(\text{Z=Y}, \{\text{X} \rightarrow t_1, \text{Y} \rightarrow t_2, \text{Z} \rightarrow c, \text{T} \rightarrow t\})],$
  $\{m = (\text{proc}\{\$ \text{ X Y Z}\} \langle s\rangle_3 \text{end}, \varnothing), t_1=3, t_2=5, c, t=\text{false}\})$
- $([\ ],$
  $\{m = (\text{proc}\{\$ \text{ X Y Z}\} \langle s\rangle_3 \text{end}, \varnothing), t_1=3, t_2=5, c=5, t=\text{false}\})$

Exercises

50. Does dynamic binding require keeping an environment in a closure (procedure value)? Why or why not?
51. VRH Exercise 2.9.2 (page 107)
52. After translating the following function to the kernel language:

```
fun {AddList L1 L2}
   case L1 of H1|T1 then
      case L2 of H2|T2 then
            H1+H2|{AddList T1 T2}
      end
   else nil end
end
```

Use the operational semantics to execute the call

```
{AddList [1 2] [3 4]}
```