Announcements: Test 3 Information

- Test 3 will be held on Tuesday, November 16th, 2010 from 2-3:50pm in West Hall Auditorium. No make-ups will be given except for emergency situations, and even then a written excuse from the Dean of Students office will be required.
- Closed-book and closed-notes except for 1 sheet of notes on 8.5x11 inch paper (front & back) that may be handwritten or printed.

Review from Lecture 18

- Longest & shortest paths: discussion depth-first vs. breadth-first traversal
- Increment and decrement operations on iterators
- Red-black trees

Today’s Lecture

- Operators as non-member functions, as member functions, and as friend functions.
- Queues and Stacks, What’s a Priority Queue?
- A Priority Queue as a Heap, percolate_up and percolate_down
- Ford & Topp Sections 8.1, 8.5-8.6.

19.1 Complex Numbers — A Brief Review

- Complex numbers take the form $z = a + bi$, where $i = \sqrt{-1}$ and $a$ and $b$ are real. $a$ is called the real part, $b$ is called the imaginary part.
- If $w = c + di$, then
  - $w + z = (a + c) + (b + d)i$,
  - $w - z = (a - c) + (b - d)i$, and
  - $w \times z = (ac - bd) + (ad + bc)i$
- The magnitude of a complex number is $\sqrt{a^2 + b^2}$.

19.2 Complex Class declaration (complex.h)

```cpp
class Complex {
public:
  Complex(double x=0, double y=0) : real_(x), imag_(y) {} // default constructor
  Complex(Complex const& old) : real_(old.real_), imag_(old.imag_) {} // copy constructor
  Complex& operator= (Complex const& rhs); // Assignment operator
  double Real() const { return real_; }
  void SetReal(double x) { real_ = x; }
  double Imaginary() const { return imag_; }
  void SetImaginary(double y) { imag_ = y; }
  double Magnitude() const { return sqrt(real_*real_ + imag_*imag_); }
  Complex operator+ (Complex const& rhs) const;
  Complex operator- () const; // unary operator- negates a complex number
  friend istream& operator>> (istream& istr, Complex& c);
private:
  double real_, imag_;}

Complex operator- (Complex const& left, Complex const& right); // non-member function
ostream& operator<< (ostream& ostr, Complex const& c); // non-member function
```
19.3 Implementation of Complex Class (complex.cpp)

// Assignment operator
Complex& Complex::operator= (Complex const& rhs) {
    real_ = rhs.real_;  
    imag_ = rhs.imag_;  
    return *this;
}

// Addition operator as a member function.
Complex Complex::operator+ (Complex const& rhs) const {
    double re = real_ + rhs.real_;  
    double im = imag_ + rhs.imag_;  
    return Complex(re, im); 
}

// Subtraction operator as a non-member function.
Complex operator- (Complex const& lhs, Complex const& rhs) {
    return Complex(lhs.Real()-rhs.Real(), lhs.Imaginary()-rhs.Imaginary()); 
}

// Unary negation operator. Note that there are no arguments.
Complex Complex::operator- () const {
    return Complex(-real_, -imag_); 
}

// Input stream operator as a friend function
istream& operator>> (istream & istr, Complex & c) {
    istr >> c.real_ >> c.imag_;  
    return istr;
}

// Output stream operator as an ordinary non-member function
ostream& operator<< (ostream & ostr, Complex const& c) {
    if (c.Imaginary() < 0) ostr << c.Real() << " - " << -c.Imaginary() << " i ";  
    else ostr << c.Real() << " + " << c.Imaginary() << " i ";  
    return ostr;
}

19.4 Operators as Non-Member Functions and as Member Functions

- We have already written our own operators, especially operator<, to sort objects stored in STL containers and to create our own keys for maps.

- We can write them as non-member functions (e.g., operator-). When implemented as a non-member function, the expression: \( z - w \) is translated by the compiler into the function call: \( \text{operator-}(z, w) \)

- We can also write them as member functions (e.g., operator+). When implemented as a member function, the expression: \( z + w \) is translated into: \( z.\text{operator+}(w) \)

This shows that operator+ is a member function of \( z \), since \( z \) appears on the left-hand side of the operator. Observe that the function has only one argument!

There are several important properties of the implementation of an operator as a member function:
- It is within the scope of class Complex, so private member variables can be accessed directly.
- The member variables of \( z \), whose member function is actually called, are referenced by directly by name.
- The member variables of \( w \) are accessed through the parameter rhs.
- The member function is const, which means that \( z \) will not (and can not) be changed by the function. Also, since \( w \) will not be changed since the argument is also marked const.

- Both operator+ and operator- return Complex objects, so both must call Complex constructors to create these objects. Calling constructors for Complex objects inside functions, especially member functions that work on Complex objects, seems somewhat counter-intuitive at first, but it is common practice!
19.5 Assignment Operators

- The assignment operator: \( z_1 = z_2; \) becomes a function call: \( z_1.\text{operator\=}(z_2); \)
  
  And cascaded assignments like: \( z_1 = z_2 = z_3; \) are really: \( z_1 = (z_2 = z_3); \)
  
  which becomes: \( z_1.\text{operator\=} (z_2.\text{operator\=} (z_3)); \)
  
  Studying these helps to explain how to write the assignment operator, which is usually a member function.

- The argument (the right side of the operator) is passed by constant reference. Its values are used to change
  the contents of the left side of the operator, which is the object whose member function is called. A reference
  to this object is returned, allowing a subsequent call to \( \text{operator\=} \) (\( z_1 \)’s \( \text{operator\=} \) in the example above).
  The identifier \textit{this} is reserved as a pointer inside class scope to the object whose member function is called.
  Therefore, \texttt{*this} is a reference to this object.

- The fact that \texttt{operator=} returns a reference allows us to write code of the form: \( (z_1 = z_2).\text{real}(); \)

19.6 Exercise

Write an \texttt{operator+=} as a member function of the \texttt{Complex} class. To do so, you must combine what you learned
about \texttt{operator=} and \texttt{operator+}. In particular, the new operator must return a reference, \texttt{*this}.

19.7 Returning Objects vs. Returning References to Objects

- In the \texttt{operator\+} and \texttt{operator\-} functions we create new \texttt{Complex} objects and simply return the new object.
  
  The return types of these operators are both \texttt{Complex}.

  Technically, we don’t return the new object (which is stored only locally and will disappear once the scope of
  the function is exited). Instead we create a copy of the object and return the copy. This automatic copying
  happens outside of the scope of the function, so it is \textit{safe} to access outside of the function. \textit{Note: It’s important
  that the copy constructor is correctly implemented!} Good compilers can minimize the amount of redundant
  copying without introducing semantic errors.

- When you change an existing object inside an operator and need to return that object, you must return a
  \texttt{reference} to that object. This is why the return types of \texttt{operator=} and \texttt{operator+=} are both \texttt{Complex&}.
  
  This avoids creation of a new object.

- A common error made by beginners (and some non-beginners!) is attempting to return a reference to a locally
  created object! This results in someone having a pointer to stale memory. The pointer may behave correctly
  for a short while... until the memory under the pointer is allocated and used by someone else.

19.8 Friend Classes vs. Friend Functions

- In the example below, the \texttt{Foo} class has designated the \texttt{Bar} to be a \texttt{friend}. This must be done in the \texttt{public}
  area of the declaration of \texttt{Foo}.

  ```
  class Foo {
  public:
    friend class Bar;
  ...
  }
  ```

  This allows member functions in class \texttt{Bar} to access \textit{all} of the private member functions and variables of a
  \texttt{Foo} object as though they were public (but not vice versa). Note that \texttt{Foo} is giving friendship (access to its private
  contents) rather than \texttt{Bar} claiming it. What could go wrong if we allowed friendships to be claimed?

- Alternatively, within the definition of the class, we can designate specific functions to be “friend”s, which
  grants these functions access similar to that of a member function. The most common example of this is
  operators, and especially stream operators.
19.9 Stream Operators as Friend Functions
- The operators >> and << are defined for the Complex class. These are binary operators.
  
  The compiler translates: `cout << z3` into: `operator<<(cout, z3)`
  
  Consecutive calls to the << operator, such as: `cout << "z3 = " << z3 << endl;`
  are translated into: `((cout << "z3 = ") << z3) << endl;`
  
  Each application of the operator returns an ostream object so that the next application can occur.
- If we wanted to make one of these stream operators a regular member function, it would have to be a member function of the ostream class because this is the first argument (left operand). *We cannot make it a member function of the Complex class.* This is why stream operators are never member functions.
- Stream operators are either ordinary non-member functions (if the operators can do their work through the public class interface) or friend functions (if they need non public access).

19.10 Summary of Operator Overloading in C++
- Unary operators that can be overloaded: `+ - * & ~ ! ++ -- -> -*`
- Binary operators that can be overloaded: `+ - * / % ^ & | << >> += -= *= /= %= ^= &= |= <<= >>= <<= >>= operator* operator== operator!= operator<`         
- There are only a few operators that can not be overloaded: `. .* ?: ::`
- We can’t create new operators and we can’t change the number of arguments (except for the function call operator, which has a variable number of arguments).
- There are three different ways to overload an operator. When there is a choice, we recommend trying to write operators in this order:
  - Non-member function
  - Member function
  - Friend function
  
  The most important rule for clean class design involving operators is to **NEVER change the intuitive meaning of an operator.** The whole point of operators is lost if you do. One (bad) example would be defining the increment operator on a Complex number.

19.11 Extra Practice
- Implement the following operators for the Complex class (or explain why they cannot or should not be implemented). Think about whether they should be non-member, member, or friend.
  
  `operator* operator== operator!= operator<`

19.12 Additional STL Container Classes: Stacks and Queues
- We’ve studied STL vectors, lists, maps, and sets. These data structures provide a wide range of flexibility in terms of operations. One way to obtain computational efficiency is to consider a simplified set of operations or functionality. 2 examples are:
  
  **Stacks** allow access, insertion and deletion from only one end called the top
  - There is no access to values in the middle of a stack.
  - Stacks may be implemented efficiently in terms of vectors and lists, although vectors are preferable.
  - All stack operations are $O(1)$
  
  **Queues** allow insertion at one end, called the back and removal from the other end, called the front
  - There is no access to values in the middle of a queue.
  - Queues may be implemented efficiently in terms of a list. Using vectors for queues is also possible, but requires more work to get right.
  - All queue operations are $O(1)$
19.13 What’s a Priority Queue?

- Priority queues are used in prioritizing operations. Examples include jobs on a shop floor, packet routing in a network, scheduling in an operating system, or events in a simulation.

- Among the data structures we have studied, their interface is most similar to a queue, including the idea of a front or top and a tail or a back.

- Each item is stored in a priority queue using an associated “priority” and therefore, the top item is the one with the lowest value of the priority score. The tail or back is never accessed through the public interface to a priority queue.

- The main operations are insert or push, and pop (or delete_min).

19.14 Some Data Structure Options for Implementing a Priority Queue

- Vector or list, either sorted or unsorted
  - At least one of the operations, push or pop, will cost linear time, at least if we think of the container as a linear structure.

- Binary search trees
  - If we use the priority as a key, then we can use a combination of finding the minimum key and erase to implement pop. An ordinary binary-search-tree insert may be used to implement push.
  - This costs logarithmic time in the average case (and in the worst case as well if balancing is used).

- The latter is the better solution, but we would like to improve upon it — for example, it might be more natural if the minimum priority value were stored at the root.
  - We will achieve this using a binary heap, giving up the complete ordering imposed in the binary search tree.

19.15 Definition: Binary Heaps

- A binary heap is a complete binary tree such that at each internal node, p, the value stored is less than the value stored at either of p’s children.
  - A complete binary tree is one that is completely filled, except perhaps at the lowest level, and at the lowest level all leaf nodes are as far to the left as possible.

- Binary heaps will be drawn as binary trees, but implemented using vectors! (more on this next lecture)

- Alternatively, the heap could be organized such that the value stored at each internal node is greater than the values at its children.

19.16 Exercise: Drawing Binary Heaps

Draw two different binary heaps with these values: 52 13 48 7 32 40 18 25 4
19.17 Implementing Pop (a.k.a. Delete Min)

- The top (root) of the tree is removed.
- It is replaced by the value stored in the last leaf node.
  - This has echoes of the erase function in binary search trees.
  - We have not yet discussed how to find the last leaf.
- The last leaf node is removed.
- The (following) `percolate_down` function is then run to restore the heap property. This function is written here in terms of tree nodes with child pointers (and the priority stored as a value), but later it will be written in terms of vector subscripts.

```cpp
percolate_down(TreeNode<T> * p) {
    while (p->left) {
        TreeNode<T>* child;
        // Choose the child to compare against
        if (p->right && p->right->value < p->left->value)
            child = p->right;
        else
            child = p->left;
        if (child->value < p->value) {
            swap(child, p); // value and other non-pointer member vars
            p = child;
        }
        else
            break;
    }
}
```

19.18 Push / Insert

- To add a value to the heap, a new last leaf node in the tree is created and then the following `percolate_up` function is run. It assumes each node has a pointer to its parent.

```cpp
percolate_up(TreeNode<T> * p) {
    while (p->parent) {
        if (p->value < p->parent->value) {
            swap(p, parent); // value and other non-pointer member vars
            p = p->parent;
        }
        else
            break;
    }
}
```

19.19 Analysis

- Both `percolate_down` and `percolate_up` are $O(\log n)$ in the worst-case. Why?
- But, `percolate_up` (and as a result `push`) can be $O(1)$ in the average case. Why? (The full answer is beyond the scope of this course.)

19.20 Exercise

Suppose the following operations are applied to an initially empty binary heap of integers. Show the resulting heap after each `delete_min` operation. (Remember, the tree must be **complete**!)

```plaintext
push 5, push 3, push 8, push 10, push 1, push 6, pop,
push 14, push 2, push 4, push 7, pop,
pop,
pop
```