In this assignment you will build an extension of the basic linked list data structure called a _multi-linked list_. Our particular variation for this homework has elements of singly-linked, doubly-linked, and circularly linked lists. This style of data structure can be advantageous is a variety of applications; for example, in storing and organizing a musical song database that can be iterated/viewed/played in chronological (insertion) order, in alphabetical (sorted) order, and in random order (a.k.a. "shuffle" mode). You’re welcome to read more about multi-linked lists on Wikipedia (but you’re not allowed to search for, use, or study in detail any code or pseudocode that might be available online!). **Please read the entire handout before beginning your implementation.**

To complete the assignment you will modify the helper classes (Node and list_iterator) as well as the main class, dslist, which will be renamed “MultiLL”. Below is a picture of the relationships between the classes (compare this to the picture of the dslist class from Lecture 11):

That’s a whole lot of pointers!!! Each Node object has been expanded to include several new pointers to its fellow nodes. In addition to the basic doubly-linked list pointers (renamed chrono_next and chrono_prev and drawn in red), each node contains pointers for a second doubly-linked structure in which the nodes are linked in alphabetical order (through the blue sorted_next and sorted_prev pointers). Finally, a
third order of the nodes is stored with a *singly-linked, circular* chain through the `random_next` pointers (in green). Note that the random ordering is re-created (*re-randomized*) each time a random iterator is requested/attached to the list.

These three orderings are maintained by the top level `MultiLL` object, and are accessed through appropriately named private `head` and `tail` pointers. Furthermore, because there are three different orderings, the iterator objects that attach to the `MultiLL` also include a `type` variable that is used when incrementing or decrementing the iterator to specify which of the three orderings should be followed. This type can be implemented as a custom `enum` type or with boolean values or an integer (your choice). In the diagram above, we show three iterators of different types attached to the list, as they would be initialized with the corresponding `begin_chronological()`, `begin_sorted()`, and `begin_random()` function calls. The dashed lines illustrate where these iterators will move if they are pushed forward with the `++` increment operator (called separately on each iterator).

Nodes can be inserted into the `MultiLL` using the `add` function. We specify that the new node is *always* inserted at the tail end of the chronological ordering, and the node is *automatically* inserted into the sorted list to maintain the consistent alphabetical ordering. The other common methods of adding elements to a linked list (e.g., `push_front`, `push_back`, and `insert`) are *not available* in our `MultiLL`.

Just like regular linked lists, the `MultiLL` supports speedy `erase` operations of any `Node` from the beginning, middle, or end of the list. Of course, this involves a lot of pointer rearrangement! The `erase` operation is passed an iterator (of one of the three types), and after carefully removing the `Node` that the iterator points to, it returns an iterator of the same type, that points to the `Node` that is “next” in the order specified by that type.

**Randomness**

Each time the `begin_random()` member function is called to attach a new iterator to the `MLList`, the singly-linked circular chain of `random_next` pointers is re-randomized. The new ordering should be a proper random distribution; that is, each element in the list is equally likely to be chosen as the new head for the random ordering. And each remaining element is equally likely to be chosen as the “next” node after the head, etc. Finally, the last node in random order points back to the head node. All elements should appear in the circular list. Note that because this is a circular list, incrementing the random iterator can be done infinitely. The random order is generated once for the initial loop and the nodes simply repeat this same order on all subsequent loops.

Randomness is simulated by a computer using a *pseudo-random number generator*. The C library contains a few built-in options for generating random numbers, but we suggest you use the provided “Mersenne Twister” code (see also [http://en.wikipedia.org/wiki/Mersenne_twister](http://en.wikipedia.org/wiki/Mersenne_twister)). To use, simply create a new `MTRand` object. Note that by default the `MTRand` object is “seeded” with the computer’s internal time (or other similar source), so that subsequent runs will be different. To extract a random number between 0 and `n` (inclusive), simply call the member function `RandInt(int)` on your `MTRand` object with the argument `n`. See the Mersenne Twister code for additional details and features. Note: You do not need to understand how this random number generator works to use it.

**Your Tasks**

Your new data structure should be a templated class named `MultiLL` and its public interface should support the following member functions: `size`, `empty`, `clear`, `add`, `erase`, `begin_chronological`, `begin_sorted`, `begin_random`, `end_chronological`, and `end_sorted`. The `begin_chronological`, `begin_sorted`, `end_chronological`, and `end_sorted` functions must run in *constant time*. Note that the running time of the `begin_random` function is not specified. The chronological and sorted iterators should support bi-directional movement (i.e., increment `++` and decrement `--` operations) and the random type iterator
should support the increment operator. All of these iterator operations should run in constant time. Note: You should implement all standard class constructors, the assignment operator, and the destructor and these functions should properly allocate and deallocate the necessary dynamic memory. As with Homework 3, we will enable the homework server to give you the output from Valgrind for debugging memory usage and we will use Valgrind for grading as well. You are encouraged to use Valgrind or Dr. Memory on your local machine. Please ask the instructor or TAs for help if you have trouble installing or using these tools.

To get started, we highly recommend that you study and heavily borrow from the dslist class that we discussed in Lecture 11. The code is available on the course website. We provide a main.cpp file that will exercise the required features of your class, but it does not include tests for each “corner” case. You should add your own test cases to the bottom of the main.cpp file. Be sure to test every member function each of the constructors, the assignment operator, and the destructor, and exercise the templated class with types other than std::string. Discuss in your README.txt file your strategy for thoroughly debugging your assignment.

In the implementation of the MList class you may not use the STL list or the STL vector classes (or any other advanced STL container). You may only use the STL list and/or vector classes to aid in testing and debugging, in the main.cpp file.

Order Notation and Iterator Invalidation Evaluation
You must follow the memory structure and performance requirements outlined in this handout for the MList class. Overall, you should pay attention to and strive for efficiency when implementing the data structure. However, for the basic assignment you are specifically encouraged to use a simple \(O(n^2)\) sorting algorithm such as insertion sort. What is the order notation of each of the member functions in your class? Include your answers and discussion of this analysis in your README.txt.

The interaction of active MList iterators with the add and erase functions, and the simultaneous use of multiple MList iterators on the same MList object, may cause inconsistencies that are non-trivial, inefficient, and/or impossible to resolve. Think carefully about potential problems that might arise. In your README.txt file discuss any restrictions you chose to place on the use of your class; specifically, if the user should be aware of the invalidation of active iterators during the use of specific member functions. Be sure to appropriately justify the need for these restrictions.

Extra Credit
Your order notation evaluation should have uncovered one function that will benefit from an improved sorting algorithm (something better than insertion sort). Which function? Hint: It’s not add! Caution, your overall class must still meet the performance requirements outlined above. For one point of extra credit, identify this function. For several more points of extra credit re-implement this function to achieve a significant improvement in performance, correctly identifying the order notation of both your original and revised implementations. Discuss in your README.txt.

Additional Information
Do all of your work in a new folder named hw5 inside of your homeworks directory. Use good coding style when you design and implement your program. Be sure to make up new test cases to fully test your program and don’t forget to comment your code! Use the template README.txt to list your collaborators and any notes you want the grader to read. When you are finished please zip up your hw5 folder exactly as instructed for the previous assignments and submit it through the course webpage.