Declarative Computation Model

Kernel language semantics

Basic concepts, the abstract machine (CTM 2.4.1-2.4.2)

Carlos Varela

RPI

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Adapted with permission from:

Seif Haridi

KTH

Peter Van Roy

UCL

Sequential declarative computation model

- The single assignment store
  - declarative (dataflow) variables
  - partial values (variables and values are also called *entities*)
- The kernel language syntax
- The kernel language semantics
  - The environment: maps textual variable names (variable identifiers) into entities in the store
  - Interpretation (execution) of the kernel language elements (statements) by the use of an abstract machine
  - Abstract machine consists of an execution stack of statements transforming the store

Kernel language syntax

The following defines the syntax of a statement, ⟨s⟩ denotes a statement

⟨s⟩	::=	skip	*empty statement*
	|	⟨x⟩ = ⟨y⟩	*variable-variable binding*
	|	⟨x⟩ = ⟨v⟩	*variable-value binding*
	|	⟨s_1⟩ ⟨s_2⟩	*sequential composition*
	|	local ⟨x⟩ in ⟨s_1⟩ end	*declaration*
	|	if ⟨x⟩ then ⟨s_1⟩ else ⟨s_2⟩ end	*conditional*
	|	{ ⟨x⟩ ⟨y_1⟩ … ⟨y_n⟩ }	*procedural application*
	|	case ⟨x⟩ of ⟨pattern⟩ then ⟨s_1⟩ else ⟨s_2⟩ end	*pattern matching*
⟨v⟩	::=	proc { $ ⟨y_1⟩ … ⟨y_n⟩ } ⟨s_1⟩ end | ...	*value expression*
⟨pattern⟩	::=	...	

Examples

-
  ```
  local X in X = 1 end
  ```

-
  ```
  local X Y T Z in
        X = 5
        Y = 10
        T = (X>=Y)
        if T then Z = X else Z = Y end
        {Browse Z}
  end
  ```

-
  ```
  local S T in
        S = proc {$ X Y} Y = X*X end
        {S 5 T}
        {Browse T}
   end
  ```

Procedure abstraction

- Any statement can be abstracted to a procedure by selecting a number of the ’free’ variable identifiers and enclosing the statement into a procedure with the identifiers as parameters
- `if X >= Y then Z = X else Z = Y end`
- Abstracting over all variables

```
proc {Max X Y Z}
      if X >= Y then Z = X else Z = Y end
end
```

- Abstracting over X and Z

```
proc {LowerBound X Z}
      if X >= Y then Z = X else Z = Y end
end
```

Computations (abstract machine)

- A computation defines how the execution state is transformed step by step from the initial state to the final state
- A *single assignment store* σ is a set of store variables, a variable may be unbound, bound to a partial value, or bound to a group of other variables
- An *environment* E is mapping from variable identifiers to variables or values in σ, e.g. $\{X \rightarrow x_1, Y \rightarrow x_2\}$
- A *semantic statement* is a pair

  $(\langle \mathsf{s} \rangle , E)$ where $\langle \mathsf{s} \rangle$ is a statement
- *ST* is a stack of <u>semantic statements</u>

Computations (abstract machine)

- A computation defines how the execution state is transformed step by step from the initial state to the final state
- The *execution state* is a pair

  (ST , σ)
- *ST* is a stack of <u>semantic statements</u>
- A *computation* is a sequence of execution states
  $(ST_0 , \sigma_0) \rightarrow (ST_1 , \sigma_1) \rightarrow (ST_2 , \sigma_2) \rightarrow \ldots$

Semantics

- To execute a program (i.e., a statement) $\langle s \rangle$ the initial execution state is

  $([(\langle s \rangle , \varnothing)] \ , \varnothing)$
- *ST* has a single semantic statement $(\langle s \rangle , \varnothing)$
- The environment *E* is empty, and the store σ is empty
- [...] denotes the stack
- At each step the first element of *ST* is popped and execution proceeds according to the form of the element
- The final execution state (if any) is a state in which *ST* is empty

skip

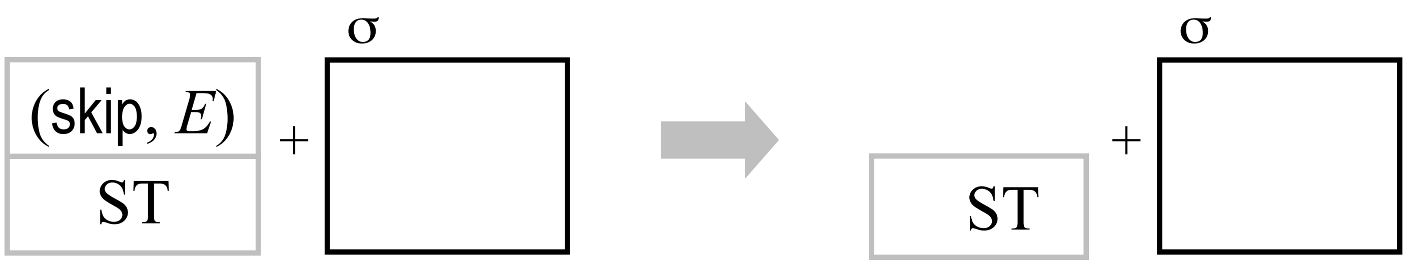
- The semantic statement is
  (skip, E)
- Continue to next execution step

skip

- The semantic statement is

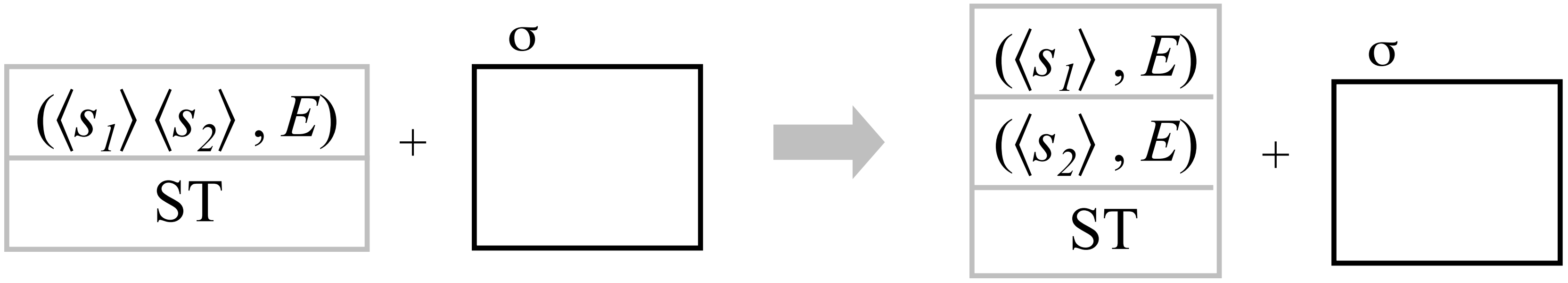
  (skip, *E*)

- Continue to next execution step



Sequential composition

- The semantic statement is
  $(\langle s_1 \rangle \, \langle s_2 \rangle \, , E)$
- Push $(\langle s_2 \rangle \, , E)$ and then push $(\langle s_1 \rangle \, , E)$ on ST
- Continue to next execution step



$(\langle s_1 \rangle \, \langle s_2 \rangle \, , E)$
ST
+
σ
$(\langle s_1 \rangle \, , E)$
$(\langle s_2 \rangle \, , E)$
ST
+
σ

Calculating with environments

- E is mapping from identifiers to entities (both store variables and values) in the store
- The notation $E(\langle y \rangle)$
  retrieves the entity x associated with the identifier $\langle y \rangle$ from the store
- The notation $E + \{\langle y \rangle_1 \rightarrow x_1, \langle y \rangle_2 \rightarrow x_2, \ldots , \langle y \rangle_n \rightarrow x_n \}$
  - denotes a new environment E' constructed from E by adding the mappings
    $\{\langle y \rangle_1 \rightarrow x_1, \langle y \rangle_2 \rightarrow x_2, \ldots , \langle y \rangle_n \rightarrow x_n\}$
  - $E'(\langle z \rangle)$ is x_k if $\langle z \rangle$ is equal to $\langle y \rangle_k$, otherwise $E'(\langle z \rangle)$ is equal to $E(\langle z \rangle)$
- The notation $E|_{\{\langle y \rangle_1, \langle y \rangle_2, \ldots , \langle y \rangle_n\}}$ denotes the projection of E onto the set $\{\langle y \rangle_1, \langle y \rangle_2, \ldots ,\langle y \rangle_n\}$, i.e., E restricted to the members of the set

Calculating with environments (2)

- E = {X → 1, Y → [2 3], Z → x_i}
- E' = E + {X → 2}
- E'(X) = 2,
  E(X) = 1
- $E|_{\{X,Y\}}$ restricts E to the 'domain' {X,Y},
  i.e., it is equal to {X → 1, Y → [2 3]}

Calculating with environments (3)

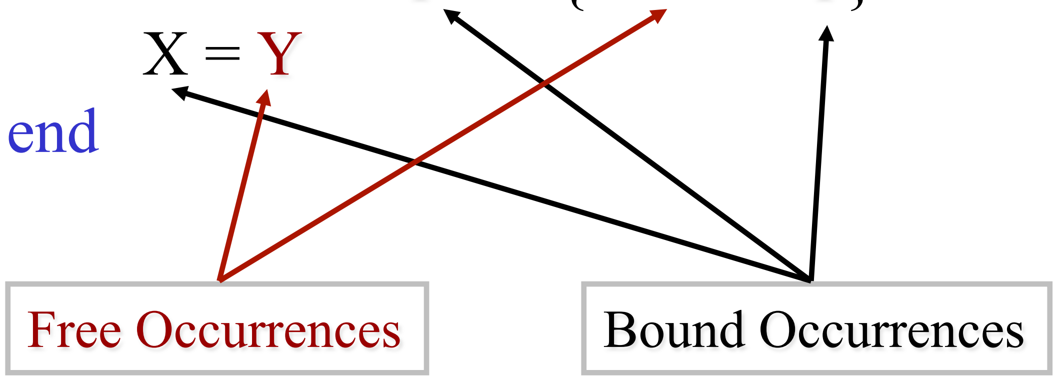
```
local X in
      X = 1                   (E)
      local X in
            X = 2             (E')
            {Browse X}
      end                     (E)
      {Browse X}
end
```

Lexical scoping

- Free and bound identifier occurrences
- An identifier occurrence is *bound* with respect to a statement ⟨s⟩ if it is in the scope of a declaration inside ⟨s⟩
- A variable identifier is declared either by a 'local' statement, as a parameter of a procedure, or implicitly declared by a case statement
- An identifier occurrence is *free* otherwise
- In a running program every identifier is bound (i.e., declared)

Lexical scoping (2)

-
  ```
  proc {P X}
        local Y in Y = 1 {Browse Y} end
        X = Y
  end
  ```



Lexical scoping (3)

```
• local Arg1 Arg2 in
        Arg1 = 111*111
        Arg2 = 999*999
        Res = Arg1*Arg2
  end
```

Free Occurrences

Bound Occurrences

This is not a runnable program!

Lexical scoping (4)

```
• local Res in
      local Arg1 Arg2 in
            Arg1 = 111*111
            Arg2 = 999*999
            Res = Arg1*Arg2
      end
      {Browse Res}
  end
```

Lexical scoping (5)

```
local P Q in
   proc {P}  {Q} end
   proc {Q} {Browse hello} end
   local Q in
         proc {Q}  {Browse hi} end
         {P}
   end
end
```

Exercises

42. Translate the following function to the kernel language:

```
fun {AddList L1 L2}
  case L1 of H1|T1 then
    case L2 of H2|T2 then
           H1+H2|{AddList T1 T2}
    end
  else nil end
end
```

43. Translate the following function call to the kernel language:

```
{Browse {Max 5 7}}
```

Exercises

44. Explain the difference between static scoping and dynamic scoping. Give an example program that produces different results with static and dynamic scoping.
45. Think of a reason why static scoping may be preferable to dynamic scoping. Think of a reason why dynamic scoping may be preferable to static scoping.