

# Declarative Programming Techniques

Accumulators (CTM 3.4.3)

Difference Lists (CTM 3.4.4)

Carlos Varela

RPI

Adapted with permission from:

Seif Haridi

KTH

Peter Van Roy

UCL

December 1, 2015

# Accumulators

- *Accumulator programming* is a way to handle state in declarative programs. It is a programming technique that uses arguments to carry state, transform the state, and pass it to the next procedure.
- Assume that the state  $S$  consists of a number of components to be transformed individually:

$$S = (X, Y, Z, \dots)$$

- For each predicate  $P$ , each state component is made into a pair, the first component is the *input* state and the second component is the output state after  $P$  has terminated
- $S$  is represented as  
 $(X_{in}, X_{out}, Y_{in}, Y_{out}, Z_{in}, Z_{out}, \dots)$

# A Trivial Example in Prolog

```
increment(N0,N) :-  
    N is N0 + 1.
```

```
square(N0,N) :-  
    N is N0 * N0.
```

```
inc_square(N0,N) :-  
    increment(N0,N1),  
    square(N1,N).
```

**increment** takes N0 as the input and produces N as the output by adding 1 to N0.

**square** takes N0 as the input and produces N as the output by multiplying N0 to itself.

**inc\_square** takes N0 as the input and produces N as the output by using an intermediate variable N1 to carry N0+1 (the output of **increment**) and passing it as input to **square**. The pairs N0-N1 and N1-N are called *accumulators*.

# A Trivial Example in Oz

```
proc {Increment N0 N}  
  N = N0 + 1  
end
```

```
proc {Square N0 N}  
  N = N0 * N0  
end
```

```
proc {IncSquare N0 N}  
  N1 in  
    {Increment N0 N1}  
    {Square N1 N}  
end
```

**Increment** takes N0 as the input and produces N as the output by adding 1 to N0.

**Square** takes N0 as the input and produces N as the output by multiplying N0 to itself.

**IncSquare** takes N0 as the input and produces N as the output by using an intermediate variable N1 to carry N0+1 (the output of **Increment**) and passing it as input to **Square**. The pairs N0-N1 and N1-N are called *accumulators*.

# Accumulators

- Assume that the state  $S$  consists of a number of components to be transformed individually:

$$S = (X, Y, Z)$$

- Assume  $P_1$  to  $P_n$  are procedures in Oz

accumulator  
└─┘

```
proc {P X0 X Y0 Y Z0 Z}  
      :  
      {P1 X0 X1 Y0 Y1 Z0 Z1}  
      {P2 X1 X2 Y1 Y2 Z1 Z2}  
      :  
      {Pn Xn-1 X Yn-1 Y Zn-1 Z}  
end
```

The same  
concept  
applies to  
predicates in  
Prolog

- The procedural syntax is easier to use if there is more than one accumulator

# MergeSort Example

- Consider a variant of MergeSort with accumulator
- `proc {MergeSort1 N S0 S Xs}`
  - N is an integer,
  - S0 is an input list to be sorted
  - S is the remainder of S0 after the first N elements are sorted
  - Xs is the sorted first N elements of S0
- The pair (S0, S) is an accumulator
- The definition is in a procedural syntax in Oz because it has two outputs S and Xs

## Example (2)

```
fun {MergeSort Xs}  
  Ys in  
    {MergeSort1 {Length Xs} Xs _ Ys}  
  Ys  
end
```

```
proc {MergeSort1 N S0 S Xs}  
  if N==0 then S = S0 Xs = nil  
  elseif N ==1 then X in X|S = S0 Xs=[X]  
  else %% N > 1  
    local S1 Xs1 Xs2 NL NR in  
      NL = N div 2  
      NR = N - NL  
      {MergeSort1 NL S0 S1 Xs1}  
      {MergeSort1 NR S1 S Xs2}  
      Xs = {Merge Xs1 Xs2}  
    end  
  end  
end
```

# MergeSort Example in Prolog

```
mergesort(Xs,Ys) :-  
    length(Xs,N),  
    mergesort1(N,Xs,_,Ys).
```

```
mergesort1(0,S,S,[]) :- !.  
mergesort1(1,[X|S],S,[X]) :- !.  
mergesort1(N,S0,S,Xs) :-  
    NL is N // 2,  
    NR is N - NL,  
    mergesort1(NL,S0,S1,Xs1),  
    mergesort1(NR,S1,S,Xs2),  
    merge(Xs1,Xs2,Xs).
```

# Multiple accumulators

- Consider a stack machine for evaluating arithmetic expressions
- Example:  $(1+4)-3$
- The machine executes the following instructions

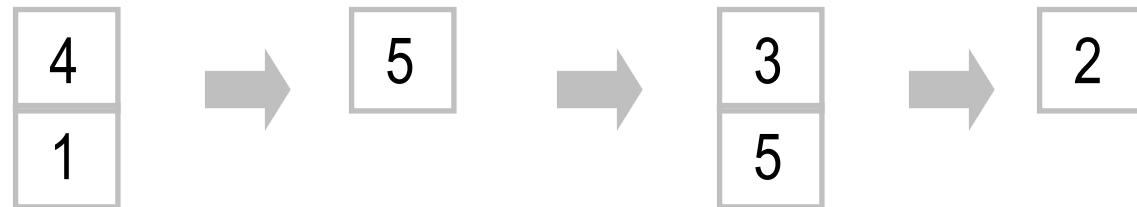
push(1)

push(4)

plus

push(3)

minus



# Multiple accumulators (2)

- Example:  $(1+4)-3$
- The arithmetic expressions are represented as trees:  
     $\text{minus}(\text{plus}(1\ 4)\ 3)$
- Write a procedure that takes arithmetic expressions represented as trees and output a list of stack machine instructions and counts the number of instructions

```
proc {ExprCode Expr Cin Cout Nin Nout}
```

- Cin: initial list of instructions
- Cout: final list of instructions
- Nin: initial count
- Nout: final count

# Multiple accumulators (3)

```
proc {ExprCode Expr C0 C N0 N}  
  case Expr  
  of plus(Expr1 Expr2) then C1 N1 in  
    C1 = plus|C0  
    N1 = N0 + 1  
    {SeqCode [Expr2 Expr1] C1 C N1 N}  
  [] minus(Expr1 Expr2) then C1 N1 in  
    C1 = minus|C0  
    N1 = N0 + 1  
    {SeqCode [Expr2 Expr1] C1 C N1 N}  
  [] I andthen {IsInt I} then  
    C = push(I)|C0  
    N = N0 + 1  
  end  
end
```

# Multiple accumulators (4)

```
proc {ExprCode Expr C0 C N0 N}  
  case Expr  
  of plus(Expr1 Expr2) then C1 N1 in  
    C1 = plus|C0  
    N1 = N0 + 1  
    {SeqCode [Expr2 Expr1] C1 C N1 N}  
  [] minus(Expr1 Expr2) then C1 N1 in  
    C1 = minus|C0  
    N1 = N0 + 1  
    {SeqCode [Expr2 Expr1] C1 C N1 N}  
  [] I andthen {IsInt I} then  
    C = push(I)|C0  
    N = N0 + 1  
  end  
end
```

```
proc {SeqCode Es C0 C N0 N}  
  case Es  
  of nil then C = C0 N = N0  
  [] E|Er then N1 C1 in  
    {ExprCode E C0 C1 N0 N1}  
    {SeqCode Er C1 C N1 N}  
  end  
end
```

## Shorter version (4)

```
proc {ExprCode Expr C0 C N0 N}  
  case Expr  
  of plus(Expr1 Expr2) then  
    {SeqCode [Expr2 Expr1] plus|C0 C N0 + 1 N}  
  [] minus(Expr1 Expr2) then  
    {SeqCode [Expr2 Expr1] minus|C0 C N0 + 1 N}  
  [] I andthen {IsInt I} then  
    C = push(I)|C0  
    N = N0 + 1  
  end  
end
```

```
proc {SeqCode Es C0 C N0 N}  
  case Es  
  of nil then C = C0 N = N0  
  [] E|Er then N1 C1 in  
    {ExprCode E C0 C1 N0 N1}  
    {SeqCode Er C1 C N1 N}  
  end  
end
```

# Functional style (4)

```
fun {ExprCode Expr t(C0 N0) }  
  case Expr  
  of plus(Expr1 Expr2) then  
    {SeqCode [Expr2 Expr1] t(plus|C0 N0 + 1)}  
  [] minus(Expr1 Expr2) then  
    {SeqCode [Expr2 Expr1] t(minus|C0 N0 + 1)}  
  [] I andthen {IsInt I} then  
    t(push(I)|C0 N0 + 1)  
  end  
end
```

```
fun {SeqCode Es T}  
  case Es  
  of nil then T  
  [] E|Er then  
    T1 = {ExprCode E T} in  
    {SeqCode Er T1}  
  end  
end
```

# Difference lists in Oz

- A *difference list* is a pair of lists, each might have an unbound tail, with the invariant that one can get the second list by removing zero or more elements from the first list
- $X \# X$                       % Represent the empty list
- $\text{nil} \# \text{nil}$                 % idem
- $[a] \# [a]$                     % idem
- $(a|b|c|X) \# X$                 % Represents  $[a \ b \ c]$
- $[a \ b \ c \ d] \# [d]$             % idem
- $[a \ b \ c \ d|Y] \# [d|Y]$     % idem

# Difference lists in Prolog

- A *difference list* is a pair of lists, each might have an unbound tail, with the invariant that one can get the second list by removing zero or more elements from the first list
- $X, X$                       % Represent the empty list
- $[], []$                       % idem
- $[a], [a]$                     % idem
- $[a,b,c|X], X$                 % Represents  $[a,b,c]$
- $[a,b,c,d], [d]$               % idem
- $[a,b,c,d|Y], [d|Y]$         % idem

## Difference lists in Oz (2)

- When the second list is unbound, an append operation with another difference list takes constant time
- ```
fun {AppendD D1 D2}
    S1 # E1 = D1
    S2 # E2 = D2
in   E1 = S2
    S1 # E2
end
```
- ```
local X Y in {Browse {AppendD (1|2|3|X)#X (4|5|Y)#Y}} end
```
- Displays `(1|2|3|4|5|Y)#Y`

# Difference lists in Prolog (2)

- When the second list is unbound, an append operation with another difference list takes constant time

`append_dl(S1,E1, S2,E2, S1,E2) :- E1 = S2.`

- `?- append_dl([1,2,3|X],X, [4,5|Y],Y, S,E).`

Displays

`X = [4, 5|_G193]`

`Y = _G193`

`S = [1, 2, 3, 4, 5|_G193]`

`E = _G193 ;`

# A FIFO queue with difference lists (1)

- A *FIFO queue* is a sequence of elements with an insert and a delete operation.
  - Insert adds an element to the end and delete removes it from the beginning
- Queues can be implemented with lists. If  $L$  represents the queue content, then deleting  $X$  can remove the head of the list matching  $X|T$  but inserting  $X$  requires traversing the list  $\{\text{Append } L [X]\}$  (insert element at the end).
  - *Insert is inefficient*: it takes time proportional to the number of queue elements
- With difference lists we can implement a queue with *constant-time insert and delete operations*
  - The queue content is represented as  $q(N \ S \ E)$ , where  $N$  is the number of elements and  $S\#E$  is a difference list representing the elements

# A FIFO queue with difference lists (2)

```
fun {NewQueue} X in q(0 X X) end
```

```
fun {Insert Q X}  
  case Q of q(N S E) then E1 in E=X|E1 q(N+1 S E1) end  
end
```

```
fun {Delete Q X}  
  case Q of q(N S E) then S1 in X|S1=S q(N-1 S1 E) end  
end
```

```
fun {EmptyQueue Q} case Q of q(N S E) then N==0 end end
```

- Inserting 'b':
  - In: q(1 a|T T)
  - Out: q(2 a|b|U U)
- Deleting X:
  - In: q(2 a|b|U U)
  - Out: q(1 b|U U)  
and X=a
- Difference list allows operations at **both ends**
- N is needed to keep track of the number of queue elements

# Flatten

```
fun {Flatten Xs}  
case Xs  
of nil then nil  
[] X|Xr andthen {IsLeaf X} then  
  X|{Flatten Xr}  
[] X|Xr andthen {Not {IsLeaf X}} then  
  {Append {Flatten X} {Flatten Xr}}  
end  
end
```

Flatten takes a list of elements and sub-lists and returns a list with only the elements, e.g.:

$\{\text{Flatten } [1 \ [2] \ [[3]]]\} = [1 \ 2 \ 3]$

Let us replace lists by difference lists and see what happens.

# Flatten with difference lists (1)

- Flatten of nil is  $X\#X$
- Flatten of a leaf  $X|Xr$  is  $(X|Y1)\#Y$ 
  - flatten of  $Xr$  is  $Y1\#Y$
- Flatten of  $X|Xr$  is  $Y1\#Y$  where
  - flatten of  $X$  is  $Y1\#Y2$
  - flatten of  $Xr$  is  $Y3\#Y$
  - equate  $Y2$  and  $Y3$

Here is what it looks like  
as text

# Flatten with difference lists (2)

```
proc {FlattenD Xs Ds}  
  case Xs  
  of nil then Y in Ds = Y#Y  
    [] X|Xr andthen {IsLeaf X} then Y1 Y in  
      {FlattenD Xr Y1#Y2}  
      Ds = (X|Y1)#Y  
    [] X|Xr andthen {IsList X} then Y0 Y1 Y2 in  
      Ds = Y0#Y2  
      {FlattenD X Y0#Y1}  
      {FlattenD Xr Y1#Y2}  
    end  
  end  
end  
fun {Flatten Xs} Y in {FlattenD Xs Y#nil} Y end
```

Here is the new program. It is much more efficient than the first version.

# Reverse

- Here is our recursive reverse:

```
fun {Reverse Xs}  
  case Xs  
  of nil then nil  
  [] X|Xr then {Append {Reverse Xr} [X]}  
  end  
end
```

- Rewrite this with difference lists:
  - Reverse of nil is  $X\#X$
  - Reverse of  $X|Xs$  is  $Y1\#Y$ , where
    - reverse of  $Xs$  is  $Y1\#Y2$ , and
    - equate  $Y2$  and  $X|Y$

# Reverse with difference lists (1)

- The naive version takes time proportional to the **square** of the input length
- Using difference lists in the naive version makes it **linear time**
- We use two arguments Y1 and Y instead of Y1#Y
- With a minor change we can make it **iterative** as well

```
fun {ReverseD Xs}  
  proc {ReverseD Xs Y1 Y}  
    case Xs  
    of nil then Y1=Y  
    [] X|Xr then Y2 in  
      {ReverseD Xr Y1 Y2}  
      Y2 = X|Y  
    end  
  end  
  R in  
  {ReverseD Xs R nil}  
  R  
end
```

# Reverse with difference lists (2)

```
fun {ReverseD Xs}  
  proc {ReverseD Xs Y1 Y}  
    case Xs  
    of nil then Y1=Y  
    [] X|Xr then  
      {ReverseD Xr Y1 X|Y}  
    end  
  end  
  R in  
    {ReverseD Xs R nil}  
  R  
end
```

# Difference lists: Summary

- Difference lists are a way to represent lists in the declarative model such that **one append operation can be done in constant time**
  - A function that builds a big list by concatenating together lots of little lists can usually be written efficiently with difference lists
  - The function can be written naively, using difference lists and append, and will be efficient when the append is expanded out
- Difference lists are declarative, yet have **some of the power of destructive assignment**
  - Because of the single-assignment property of dataflow variables
- Difference lists originated from **Prolog** and are used to implement, e.g., definite clause grammar rules for natural language parsing.

# Exercises

91. Rewrite the Oz multiple accumulators example in Prolog.
92. Rewrite the Oz FIFO queue with difference lists in Prolog.
93. Draw the search trees for Prolog queries:
  - `append([1,2],[3],L) .`
  - `append(X,Y,[1,2,3]) .`
  - `append_dl([1,2|X],X,[3|Y],Y,S,E) .`
94. CTM Exercise 3.10.11 (page 232)
95. CTM Exercise 3.10.14 (page 232)
96. CTM Exercise 3.10.15 (page 232)