Higher-Order Programming:
Closures, procedural abstraction, genericity, instantiation, embedding. Control abstractions: iterate, map, reduce, fold, filter (CTM Sections 1.9, 3.6, 4.7)

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Higher-order programming

- Assume we want to write another Pascal function which instead of adding numbers, performs exclusive-or on them.
- It calculates for each number whether it is odd or even (parity).
- Either write a new function each time we need a new operation, or write one generic function that takes an operation (another function) as argument.
- The ability to pass functions as arguments, or return a function as a result is called *higher-order programming*.
- Higher-order programming is an aid to build generic abstractions.
Variations of Pascal

• Compute the parity Pascal triangle

\[
\text{fun } \{ \text{Xor X Y} \} \text{ if } X == Y \text{ then } 0 \text{ else } 1 \text{ end end}
\]

\[
\begin{array}{cccccc}
1 & & & & & 1 \\
1 & 1 & & & & 1 \\
1 & 2 & 1 & & & 1 \\
1 & 3 & 3 & 1 & & 1 \\
1 & 4 & 6 & 4 & 1 & 1 \\
\end{array}
\]
Higher-order programming

```plaintext
fun {GenericPascal Op N} 
  if N==1 then [1] 
  else L in L = {GenericPascal Op N-1} 
    {OpList Op {ShiftLeft L} {ShiftRight L}} 
  end 
end 

fun {OpList Op L1 L2} 
  case L1 of H1|T1 then 
    case L2 of H2|T2 then 
    {Op H1 H2}|{OpList Op T1 T2} 
  end 
  else nil end 
end 

fun {Add N1 N2} N1+N2 end 
fun {Xor N1 N2} 
  if N1==N2 then 0 else 1 end 
end 

fun {Pascal N} {GenericPascal Add N} end 
fun {ParityPascal N} 
  {GenericPascal Xor N} 
end 
```

Add and Xor functions are passed as arguments.
The Iterate control abstraction

```
fun \{Iterate S IsDone Transform\}
  if \{IsDone S\} then S
  else S1 in
    S1 = \{Transform S\}
    \{Iterate S1 IsDone Transform\}
  end
end
```

```
fun \{Iterate S_i\}
  if \{IsDone S_i\} then S_i
  else S_{i+1} in
    S_{i+1} = \{Transform S_i\}
    \{Iterate S_{i+1}\}
  end
end
```
Sqrt using the control abstraction

fun {Sqrt X}
  {Iterate
    1.0
    fun {$ G} {Abs X - G*G}/X < 0.000001 end
    fun {$ G} (G + X/G)/2.0 end
  }
end

IsDone and Transform anonymous functions are passed as arguments.
Sqrt in Haskell

let sqrt x = head (dropWhile (not . goodEnough) sqrtGuesses)
where
    goodEnough guess = (abs (x – guess*guess))/x < 0.00001
    improve guess = (guess + x/guess)/2.0
    sqrtGuesses = 1:(map improve sqrtGuesses)

This sqrt example uses infinite lists enabled by lazy evaluation, and the map control abstraction.
Functions are procedures in Oz

fun {Map Xs F}
    case Xs
    of nil then nil
    [] X|Xr then {F X}|{Map Xr F}
    end
end

proc {Map Xs F Ys}
    case Xs
    of nil then Ys = nil
    [] X|Xr then Y Yr in
        Ys = Y|Yr
        {F X Y}
        {Map Xr F Yr}
    end
end
Map in Haskell

\[
\text{map}' :: (a \to b) \to [a] \to [b]
\]

\[
\text{map}' \_ [] = []
\]

\[
\text{map}' f (h:t) = f h : \text{map}' f t
\]

_ means that the argument is not used (read “don’t care”). map’ is to distinguish it from the Prelude’s map function.
Higher-order programming

• Higher-order programming = the set of programming techniques that are possible with procedure values (lexically-scoped closures)

• Basic operations
  – Procedural abstraction: creating procedure values with lexical scoping
  – Genericity: procedure values as arguments
  – Instantiation: procedure values as return values
  – Embedding: procedure values in data structures

• Higher-order programming is the foundation of component-based programming and object-oriented programming
Procedural abstraction

• Procedural abstraction is the ability to convert any statement into a procedure value
  – A procedure value is usually called a closure, or more precisely, a lexically-scoped closure
  – A procedure value is a pair: it combines the procedure code with the environment where the procedure was created (the contextual environment)

• Basic scheme:
  – Consider any statement <s>
  – Convert it into a procedure value: \( P = \text{proc}\{\}$ <s> end \)
  – Executing \( \{P\} \) has exactly the same effect as executing <s>
Procedural abstraction

fun {AndThen B1 B2}
    if B1 then B2 else false
    end
end
Procedural abstraction

fun {AndThen B1 B2}
  if {B1} then {B2} else false
  end
end
end
Procedure abstraction

- Any statement can be abstracted to a procedure by selecting a number of the ’free’ variable identifiers and enclosing the statement into a procedure with the identifiers as parameters
  - if $X \geq Y$ then $Z = X$ else $Z = Y$ end
- Abstracting over all variables
  
  ```
  proc {Max X Y Z}
      if $X \geq Y$ then $Z = X$ else $Z = Y$ end
  end
  ```
- Abstracting over $X$ and $Z$
  
  ```
  proc {LowerBound X Z}
      if $X \geq Y$ then $Z = X$ else $Z = Y$ end
  end
  ```
Lexical scope

local P Q in
  proc {P …} {Q …} end
  proc {Q …} {Browse hello} end
local Q in
  proc {Q …} {Browse hi} end
  {P …}
end
end
Procedure values

• Constructing a procedure value in the store is not simple because a procedure may have external references

```plaintext
local P Q in
  P = proc {$ …} {Q …} end
  Q = proc {$ …} {Browse hello} end
local Q in
  Q = proc {$ …} {Browse hi} end
  {P …}
end
end
```
local P Q in
    P = proc {$ ...} {Q ...} end
    Q = proc {$ ...} {Browse hello} end
local Q in
    Q = proc {$ ...} {Browse hi} end end
end

proc {$ ...} {Q ...} end
Q → x_2

P → x_1

proc {$ ...} {Browse hello} end
Browse → x_0

x_0 (, , )

x_2 (, , )

x_1 (, )
Procedure values (3)

- The semantic statement is \(\text{proc } \langle x \rangle \langle y_1 \rangle \ldots \langle y_n \rangle \langle s \rangle \text{ end, } E\)
- \(\langle y_1 \rangle \ldots \langle y_n \rangle\) are the (formal) parameters of the procedure
- Other free identifiers of \(\langle s \rangle\) are called external references \(\langle z_1 \rangle \ldots \langle z_k \rangle\)
- These are defined by the environment \(E\) where the procedure is declared (lexical scoping)
- The contextual environment of the procedure \(CE\) is \(E \mid \{\langle z_1 \rangle \ldots \langle z_k \rangle\}\)
- When the procedure is called \(CE\) is used to construct the environment of \(\langle s \rangle\)
Procedure values (4)

- Procedure values are pairs:
  \[ (\text{proc} \{\langle y_1 \rangle \ldots \langle y_n \rangle \langle s \rangle \text{ end }, CE) \]

- They are stored in the store just as any other value
A common limitation

• Most popular imperative languages (C, Pascal) do not have procedure values
• They have only half of the pair: variables can reference procedure code, but there is no contextual environment
• This means that control abstractions cannot be programmed in these languages
  – They provide a predefined set of control abstractions (for, while loops, if statement)
• Generic operations are still possible
  – They can often get by with just the procedure code. The contextual environment is often empty.
• The limitation is due to the way memory is managed in these languages
  – Part of the store is put on the stack and deallocated when the stack is deallocated
  – This is supposed to make memory management simpler for the programmer on systems that have no garbage collection
  – It means that contextual environments cannot be created, since they would be full of dangling pointers
• Object-oriented programming languages can use objects to encode procedure values by making external references (contextual environment) instance variables.
Genericity

- Replace specific entities (zero 0 and addition +) by function arguments
- The same routine can do the sum, the product, the logical or, etc.

```plaintext
fun {SumList L}
  case L
  of nil then 0
      [] X|L2 then X+{SumList L2}
  end
end

fun {FoldR L F U}
  case L
  of nil then U
      [] X|L2 then {F X {FoldR L2 F U}}
  end
end
```
Genericity in Haskell

- Replace specific entities (zero 0 and addition +) by function arguments
- The same routine can do the sum, the product, the logical or, etc.

```
sumlist :: (Num a) => [a] -> a
sumlist [] = 0
sumlist (h:t) = h+sumlist t
```

```
foldr' :: (a->b->b) -> b -> [a] -> b
foldr' _ u [] = u
foldr' f u (h:t) = f h (foldr' f u t)
```
Instantiation

- Instantiation is when a procedure returns a procedure value as its result
- Calling \{FoldFactory fun \{A B\} A+B end 0\} returns a function that behaves identically to SumList, which is an « instance » of a folding function

fun \{FoldFactory F U\}
  fun \{FoldR L\}
    case L
      of nil then U
      [] X|L2 then \{F X \{FoldR L2\}\}
    end
  end
in
FoldR
end
Currying

- Currying is a technique that can simplify programs that heavily use higher-order programming.
- The idea: function of n arguments $\Rightarrow$ n nested functions of one argument.
- Advantage: The intermediate functions can be useful in themselves.

```plaintext
fun {Max X Y}
  if X>=Y then X else Y end
end

fun {Max X}
  fun {$ Y}
    if X>=Y then X else Y end
  end
end
```
Embedding

- Embedding is when procedure values are put in data structures
- Embedding has many uses:
  - Modules: a module is a record that groups together a set of related operations
  - Software components: a software component is a generic function that takes a set of modules as its arguments and returns a new module. It can be seen as specifying a module in terms of the modules it needs.
  - Delayed evaluation (also called explicit lazy evaluation): build just a small part of a data structure, with functions at the extremities that can be called to build more. The consumer can control explicitly how much of the data structure is built.
Control Abstractions

declare
proc {For I J P}
    if I >= J then skip
    else {P I} {For I+1 J P}
end
end

{For 1 10 Browse}

for I in 1..10 do {Browse I} end
Control Abstractions

\[
\text{proc } \{\text{ForAll Xs P}\}
\quad \text{case Xs}
\quad \text{of nil then skip}
\quad \text{[] X|Xr then}
\quad \quad \{P X\} \quad \{\text{ForAll Xr P}\}
\quad \text{end}
\quad \text{end}
\]

\[
\{\text{ForAll [a b c d]} \}
\quad \text{proc}\{\$ I\} \quad \{\text{System.showInfo "the item is: "} \# I\} \quad \text{end}\}
\]

\[
\text{for I in [a b c d] do}
\quad \{\text{System.showInfo "the item is: "} \# I\}
\quad \text{end}
\]
Control Abstractions

fun \{FoldL Xs F U\}
   case Xs
      of nil then U
      [] X|Xr then \{FoldL Xr F \{F X U\}\}
   end
end

Assume a list \[x1\] \[x2\] \[x3\] ....\]
\[S0 \rightarrow S1 \rightarrow S2\]
\[U \rightarrow \{F x1 U\} \rightarrow \{F x2 \{F x1 U\}\} \rightarrow ....\]
Control Abstractions

fun {FoldL Xs F U}
    case Xs
    of nil then U
    [] X|Xr then {FoldL Xr F {F X U}}
    end
end
end

What does this program do?
{Browse {FoldL [1 2 3]
    fun {$ X Y} X|Y end nil}
**FoldL in Haskell**

\[
\text{foldl}' :: (b \rightarrow a \rightarrow b) \rightarrow b \rightarrow [a] \rightarrow b \\
\text{foldl'} _ u [] = u \\
\text{foldl'} f u (h:t) = \text{foldl'} f (f u h) t
\]

Notice the unit \( u \) is of type \( b \), and the function \( f \) is of type \( b \rightarrow a \rightarrow b \).
List-based techniques

fun {Map Xs F}
   case Xs
   of nil then nil
   [] X|Xr then
       {F X}|{Map Xr F}
   end
end

fun {Filter Xs P}
   case Xs
   of nil then nil
   [] X|Xr andthen {P X} then
       X|{Filter Xr P}
   [] X|Xr then {Filter Xr P}
   end
end
Filter in Haskell

\[
\text{filter'} :: (a -> \text{Bool}) \rightarrow [a] \rightarrow [a]
\]

\[
\text{filter'} \_ \ [] = []
\]

\[
\text{filter'} \ p \ (h \ : \ t) = \begin{cases} 
\text{if } p \ h \ \text{then} \ h : \text{filter'} \ p \ t \\
\text{else} \ \text{filter'} \ p \ t
\end{cases}
\]
Filter as FoldR application

\[
\text{fun} \ {\text{Filter P L}} \\
\quad \{\text{FoldR fun} \ \{\$ H T\} \\
\quad \quad \text{if} \ \{P H\} \ \text{then} \\
\quad \quad \quad H|T \\
\quad \quad \text{else} \ T \ \text{end} \\
\quad \text{end} \ \text{nil L}\}
\]

\[
\text{filter"} :: (a-> \text{Bool}) \rightarrow [a] \rightarrow [a] \\
\text{filter"} \ p \ l = \text{foldr} \\
\quad (\ \text{\_ t ->} \ \text{if} \ p \ h \\
\quad \quad \text{then} \ h:t \\
\quad \quad \text{else} \ t) \ [] \ l
\]
Tree-based techniques

proc {DFS Tree}
  case Tree of tree(node:N sons:Sons ...) then
    {Browse N}
    for T in Sons do {DFS T} end
  end
end

Call {P T} at each node T

proc {VisitNodes Tree P}
  case Tree of tree(node:N sons:Sons ...) then
    {P N}
    for T in Sons do {VisitNodes T P} end
  end
end
Explicit lazy evaluation

• Supply-driven evaluation. (e.g. The list is completely calculated independent of whether the elements are needed or not.)
• Demand-driven execution. (e.g. The consumer of the list structure asks for new list elements when they are needed.)
• Technique: a programmed trigger.
• How to do it with higher-order programming? The consumer has a function that it calls when it needs a new list element. The function call returns a pair: the list element and a new function. The new function is the new trigger: calling it returns the next data item and another new function. And so forth.
Explicit lazy functions

fun lazy \{\text{From } N\}
    N | \{\text{From } N+1\}
end

fun \{\textbf{From } N\}
    fun \{\$\} N | \{\textbf{From } N+1\} end
end
Exercises

23. Define an IncList function to take a list of numbers and increment all its values, using the Map control abstraction. For example:

   \{\text{IncList} \ [3 \ 1 \ 7] \} \Rightarrow [4 \ 2 \ 8]

24. Create a higher-order MapReduce function that takes as input two functions corresponding to Map and Reduce respectively, and returns a function to perform the composition. Illustrate your MapReduce function with an example.

25. Write solutions for exercises 23 and 24 in both Oz and Haskell. Compare your solutions.