CSCI-1200 Data Structures — Fall 2019
Lecture 12 — Problem Solving Techniques

Announcements: Test 2 Information

- Test 2 will be held Thursday, Oct 17th, 2019 from 6-7:50pm
  - Students will be randomly assigned to a test room, section, row, and seat. If you did not already do so for Test 1... indicate your right- or left-handedness on Submitty so you get an appropriate seat assignment. Test 2 seat assignments will be posted early next week.
  - Make-ups will only be given except for pre-approved absence or emergency or illness, and a written excuse from the Dean of Students or the Student Experience office or the RPI Health Center will be required.
  - If you have a letter from Disability Services for Students and you have not already emailed it to ds_instructors@cs.rpi.edu, please do so ASAP. Shianne Hulbert will be in contact with you about your accommodations for the test.

- Coverage: Lectures 1-12, Labs 1-7, and Homeworks 1-5.
  - Practice problems from previous tests are available on the course website. Sample solutions to the practice problems will be posted on Wednesday morning. The best way to prepare is to completely work through and write out your solution to each problem, before looking at the answers.

- OPTIONAL: Prepare a 2 page, black & white, 8.5x11”, portrait orientation .pdf of notes you would like to have during the test. This may be digitally prepared or handwritten and scanned or photographed. The file may be no bigger than 2MB. You will upload this file to Submitty (to the assignment “Test 2 OPTIONAL Notes Page”) before Wednesday Oct 16th @11:59pm. We will print this and attach it to your test. No other notes may be used during the test.

- Bring to the exam room:
  - Your Rensselaer photo ID card.
  - Pencil(s) & eraser (pens are ok, but not recommended). The exam will involve handwriting code on paper (and other short answer problem solving). Neat legible handwriting is appreciated. We will be somewhat forgiving to minor syntax errors – it will be graded by humans not computers ;)
  - Do not bring your own scratch paper. We will provide scratch paper.
  - Computers, cell-phones, smart watches, calculators, music players, etc. are not permitted. Please do not bring your laptop, books, backpack, etc. to the exam room – leave everything in your dorm room. Unless you are coming directly from another class or sports/club meeting.

Announcements: Test 2 Review Session

- UPE (the Computer Science honor society) will hold a Data Structures Test 2 Review Session on Monday Oct 14th, 2019 from 5-7pm (yes! on Columbus Day!) in the Folsom Library, lower level, ALAC tutoring space.

- Regular Data Structures office hours for Monday are cancelled due to the holiday.

Test Taking Skills

- Look at the point values for each problem, allocate time proportional to the problem points – Roughly 1 minute per point. (Don’t spend all of your time on one problem and neglect other big point problems).

- Look at the size of the answer box & the sample solution code line estimate for each problem. If your solution is going to take a lot more space than the box allows, we are probably looking for the solution to a simpler problem or a simpler solution to the problem.

- Going in to the test, you should know what big topics will be covered on the test. As you skim through the problems, see if you can match up those big topics to each question. Even if you are stumped about how to solve the whole problem, or some of the details of the problem, make sure you demonstrate your understanding of the big topic that is covered in that question.

- Re-read the problem statement carefully. Make sure you didn’t miss anything.

- Ask questions during the test if something is unclear.
Review from Lecture 11

- Finished Lecture 10 on List Implementation
- Rules for writing recursive functions:
  1. Handle the base case(s).
  2. Define the problem solution in terms of smaller instances of the problem. Use *wishful thinking*, i.e., if someone else solves the problem of \texttt{fact}(4) I can extend that solution to solve \texttt{fact}(5). This defines the necessary recursive calls. It is also the hardest part!
  3. Figure out what work needs to be done before making the recursive call(s).
  4. Figure out what work needs to be done after the recursive call(s) complete(s) to finish the computation. (What are you going to do with the result of the recursive call?)
  5. Assume the recursive calls work correctly, but make sure they are progressing toward the base case(s)!
- Standard Example Recursion: Binary Search – (Can easily be re-written iteratively)

Today’s Class

- Non-trivial Recursion: Merge sort – *Leftover from Lecture 11*
- Non-trivial Recursion: Non-linear maze search – *Leftover from Lecture 11*
- Today we will discuss how to design and implement algorithms using three steps or stages:
  1. Generating and Evaluating Ideas
  2. Mapping Ideas into Code
  3. Getting the Details Right

12.1 Generating and Evaluating Ideas

- Most importantly, play with examples! Can you develop a strategy for solving the problem? You should try any strategy on several examples. Is it possible to map this strategy into an algorithm and then code?
- Try solving a simpler version of the problem first and either learn from the exercise or generalize the result.
- Does this problem look like another problem you know how to solve?
- If someone gave you a partial solution, could you extend this to a complete solution?
- What if you split the problem in half and solved each half (recursively) separately?
- Does sorting the data help?
- Can you split the problem into different cases, and handle each case separately?
- Can you discover something fundamental about the problem that makes it easier to solve or makes you able to solve it more efficiently?
- Once you have an idea that you think will work, you should evaluate it: will it indeed work? are there other ways to approach it that might be better / faster? if it doesn’t work, why not?

12.2 Mapping Ideas Into Code

- How are you going to represent the data? What is most efficient and what is easiest?
- Can you use classes to organize the data? What data should be stored and manipulated as a unit? What information needs to be stored for each object? What operations (beyond simple accessors) might be helpful?
- How can you divide the problem into units of logic that will become functions? Can you reuse any code you’re previously written? Will any of the logic you write now be re-usable?
- Are you going to use recursion or iteration? What information do you need to maintain during the loops or recursive calls and how is it being “carried along”?
- How effective is your solution? Is your solution general? How is the performance? (What is the order notation of the number of operations)? Can you now think of better ideas or approaches?
- Make notes for yourself about the logic of your code as you write it. These will become your *invariants*; that is, what should be true at the beginning and end of each iteration / recursive call.
12.3 Getting the Details Right

- Is everything being initialized correctly, including boolean flag variables, accumulation variables, max / min variables?
- Is the logic of your conditionals correct? Check several times and test examples by hand.
- Do you have the bounds on the loops correct? Should you end at $n$, $n - 1$ or $n - 2$?
- Tidy up your “notes” to formalize the invariants. Study the code to make sure that your code does in fact have it right. When possible use assertions to test your invariants. (Remember, sometimes checking the invariant is impossible or too costly to be practical.)
- Does it work on the corner cases; e.g., when the answer is on the start or end of the data, when there are repeated values in the data, or when the data set is very small or very large?

12.4 Exercises: Practice using these Techniques on Simple Problems

- A perfect number is a number that is the sum of its factors. The first perfect number is 6. Let’s write a program that finds all perfect numbers less than some input number $n$.

```cpp
int main() {
    std::cout << "Enter a number: ";
    int n;
    std::cin >> n;
}
```

- Given a sequence of $n$ floating point numbers, find the two that are closest in value.

```cpp
int main() {
    float f;
    while (std::cin >> f) {
    }
}
```

- Now let’s write code to remove duplicates from a sequence of numbers:

```cpp
int main() {
    int x;
    while (std::cin >> x) {
    }
}
12.5 Example: Merge Sort

- In Lecture 11, we saw the basic framework for the merge sort algorithm and we finished the implementation of the merge helper function. How did we Map Ideas Into Code?

- What invariants can we write down within the `merge_sort` and `merge` functions? Which invariants can we test using assertions? Which ones are too expensive (i.e., will affect the overall performance of the algorithm)?

// We split the vector in half, recursively sort each half, and merge the two sorted halves into a single sorted interval.

```cpp
template <class T>
void mergesort(int low, int high, vector<T>& values, vector<T>& scratch) {
    if (low >= high) return;
    int mid = (low + high) / 2;

    mergesort(low, mid, values, scratch);
    mergesort(mid+1, high, values, scratch);

    merge(low, mid, high, values, scratch);
}
```

// Non-recursive function to merge two sorted intervals (low..mid & mid+1..high) of a vector, using "scratch" as temporary copying space.

```cpp
template <class T>
void merge(int low, int mid, int high, vector<T>& values, vector<T>& scratch) {
    int i=low, j=mid+1, k=low;
    // while there's still something left in one of the sorted subintervals...
    while (i <= mid && j <= high) {
        // look at the top values, grab the smaller one, store it in the scratch vector
        if (values[i] < values[j]) {
            scratch[k] = values[i]; ++i;
        } else {
            scratch[k] = values[j]; ++j;
        }
        ++k;
    }
    // Copy the remainder of the interval that hasn't been exhausted
    for ( ; i<=mid; ++i, ++k ) scratch[k] = values[i]; // low interval
    for ( ; j<=high; ++j, ++k ) scratch[k] = values[j]; // high interval

    // Copy from scratch back to values
    for ( i=low; i<=high; ++i ) values[i] = scratch[i];
}
```
12.6 Example: Nonlinear Word Search

- What did we need to think about to Get the Details Right when we finished the implementation of the nonlinear word search program? What did we worry about when writing the first draft code (a.k.a. pseudo-code)? When debugging, what test cases should we be sure to try? Let’s try to break the code and write down all the “corner cases” we need to test.

```cpp
bool search_from_loc(loc position, const vector<string>& board, const string& word, vector<loc>& path) {
    // start by adding this location to the path
    path.push_back(position);
    // BASE CASE: if the path length matches the word length, we’re done!
    if (path.size() == word.size()) return true;
    // search all the places you can get to in one step
    for (int i = position.row-1; i <= position.row+1; i++) {
        for (int j = position.col-1; j <= position.col+1; j++) {
            // don’t walk off the board though!
            if (i < 0 || i >= board.size()) continue;
            if (j < 0 || j >= board[0].size()) continue;
            // don’t consider locations already on our path
            if (on_path(loc(i,j),path)) continue;
            // if this letter matches, recurse!
            if (word[path.size()] == board[i][j]) {
                // if we find the remaining substring, we’re done!
                if (search_from_loc(loc(i,j),board,word,path))
                    return true;
            }
        }
    }
    // We have failed to find a path from this loc, remove it from the path
    path.pop_back();
    return false;
}
```

12.7 Exercise: Maximum Subsequence Sum

- Problem: Given is a sequence of \( n \) values, \( a_0, \ldots, a_{n-1} \), find the maximum value of \( \sum_{i=j}^{k} a_i \) over all possible subsequences \( j \ldots k \).

- For example, given the integers: 14, \(-4\), 6, \(-9\), \(-8\), 8, \(-3\), 16, \(-4\), 12, \(-7\), 4
  The maximum subsequence sum is: \( 8 + (-3) + 16 + (-4) + 12 = 29 \).

- Let’s write a first draft of the code, and then talk about how to make it more efficient.

```cpp
int main() {
    std::vector<int> v;
    int x;
    while (std::cin >> x) {
        v.push_back(x);
    }
    return 0;
}
```
12.8 Problem Solving Strategies

Here is an outline of the major steps to use in solving programming problems:

1. Before getting started: study the requirements, carefully!

2. Get started:
   (a) What major operations are needed and how do they relate to each other as the program flows?
   (b) What important data / information must be represented? How should it be represented? Consider and analyze several alternatives, thinking about the most important operations as you do so.
   (c) Develop a rough sketch of the solution, and write it down. There are advantages to working on paper first. Don’t start hacking right away!

3. Review: reread the requirements and examine your design. Are there major pitfalls in your design? Does everything make sense? Revise as needed.

4. Details, level 1:
   (a) What major classes are needed to represent the data / information? What standard library classes can be used entirely or in part? Evaluate these based on efficiency, flexibility and ease of programming.
   (b) Draft the main program, defining variables and writing function prototypes as needed.
   (c) Draft the class interfaces — the member function prototypes.
   These last two steps can be interchanged, depending on whether you feel the classes or the main program flow is the more crucial consideration.

5. Review: reread the requirements and examine your design. Does everything make sense? Revise as needed.

6. Details, level 2:
   (a) Write the details of the classes, including member functions.
   (b) Write the functions called by the main program. Revise the main program as needed.

7. Review: reread the requirements and examine your design. Does everything make sense? Revise as needed.

8. Testing:
   (a) Test your classes and member functions. Do this separately from the rest of your program, if practical. Try to test member functions as you write them.
   (b) Test your major program functions. Write separate “driver programs” for the functions if possible. Use the debugger and well-placed output statements and output functions (to print entire classes or data structures, for example).
   (c) Be sure to test on small examples and boundary conditions.
   The goal of testing is to incrementally figure out what works — line-by-line, class-by-class, function-by-function. When you have incrementally tested everything (and fixed mistakes), the program will work.

Notes

- For larger programs and programs requiring sophisticated classes / functions, these steps may need to be repeated several times over.
- Depending on the problem, some of these steps may be more important than others.
  - For some problems, the data / information representation may be complicated and require you to write several different classes. Once the construction of these classes is working properly, accessing information in the classes may be (relatively) trivial.
  - For other problems, the data / information representation may be straightforward, but what’s computed using them may be fairly complicated.
  - Many problems require combinations of both.