Review from Lecture 22 & Lab 12

- “the single most important data structure known to mankind”
- Hash Tables, Hash Functions, and Collision Resolution
- Performance of: Hash Tables vs. Binary Search Trees
- Collision resolution: separate chaining
- Using a hash table to implement a set/map
  - Iterators, find, insert, and erase

Today’s Lecture

- Using STL’s `for_each`
- Something weird & cool in C++... Function Objects, a.k.a. *Functors*
- Hash Tables, part II
  - STL’s `unordered_set` (and `unordered_map`)
  - Hash functions as functors/function objects (or non-type template parameters, or function pointers)
  - Collision resolution: separate chaining vs. open addressing
- Bonus Data Structures (continuing from Lecture 21)
  - General Overview
  - Merging heaps are the motivation for *leftist heaps* (see Lecture 21)
  - Unrolled Linked List (see Lecture 21)
  - Skip List (see Lecture 21)
  - Quad Tree
  - Trie / Prefix Tree
  - Suffix Tree

23.1 Using STL’s `for_each`

- First, here’s a tiny helper function:
  ```cpp
  void float_print (float f) {
    std::cout << f << std::endl;
  }
  ```

- Let’s make an STL vector of floats:
  ```cpp
  std::vector<float> my_data;
  my_data.push_back(3.14);
  my_data.push_back(1.41);
  my_data.push_back(6.02);
  my_data.push_back(2.71);
  ```

- Now we can write a loop to print out all the data in our vector:
  ```cpp
  std::vector<float>::iterator itr;
  for (itr = my_data.begin(); itr != my_data.end(); itr++) {
    float_print(*itr);
  }
  ```
• Alternatively we can use it with STL’s \texttt{for\_each} function to visit and print each element:

\begin{verbatim}
std::for_each(my_data.begin(), my_data.end(), float_print);
\end{verbatim}

Wow! That’s alot less to type. Can I stop using regular \texttt{for} and \texttt{while} loops altogether?

• We can actually also do the same thing without creating & explicitly naming the \texttt{float\_print} function. We create an \textit{anonymous function} using \texttt{lambda}:

\begin{verbatim}
std::for_each(my_data.begin(), my_data.end(), [](float f){ std::cout << f << std::endl; });
\end{verbatim}

Lambda is new to the C++ language (part of C++11). But lambda is a core piece of many classic, older programming languages including Lisp and Scheme. Python lambdas and Perl anonymous subroutines are similar. (In fact lambda dates back to the 1930’s, before the first computers were built!) You’ll learn more about lambda more in later courses like CSCI 4430 Programming Languages!

23.2 Function Objects, a.k.a. \textit{Functors}

• In addition to the basic mathematical operators $+, -, \ast, /, <, >$, another operator we can overload for our C++ classes is the \textit{function call operator}.

Why do we want to do this? This allows instances or objects of our class, to be used like functions. It’s weird but powerful.

• Here’s the basic syntax. Any specific number of arguments can be used.

\begin{verbatim}
class my\_class\_name {
  public:
    // ... normal class stuff ...
    my\_return\_type operator() ( /* my list of args */ );
};
\end{verbatim}

23.3 Why are Functors Useful?

• One example is the default 3rd argument for \texttt{std::sort}. We know that by default STL’s sort routines will use the less than comparison function for the type stored inside the container. How exactly do they do that?

• First let’s define another tiny helper function:

\begin{verbatim}
bool float\_less(float x, float y) {
  return x < y;
}
\end{verbatim}

• Remember how we can sort the \texttt{my\_data} vector defined above using our own homemade comparison function for sorting:

\begin{verbatim}
std::sort(my\_data.begin(),my\_data.end(),float\_less);
\end{verbatim}

If we don’t specify a 3rd argument:

\begin{verbatim}
std::sort(my\_data.begin(),my\_data.end());
\end{verbatim}

This is what STL does by default:

\begin{verbatim}
std::sort(my\_data.begin(),my\_data.end(),std::less\langle float\rangle());
\end{verbatim}

• What is \texttt{std::less}? It’s a templated class. Above we have called the default constructor to make an instance of that class. Then, that instance/object can be used like it’s a function. Weird!

• How does it do that? \texttt{std::less} is a teeny tiny class that just contains the overloaded function call operator.

\begin{verbatim}
template <class T>
class less {
  public:
    bool operator() (const T& x, const T& y) const { return x < y; }
};
\end{verbatim}

You can use this instance/object/functor as a function that expects exactly two arguments of type \texttt{T} (in this example \texttt{float}) that returns a bool. That’s exactly what we need for \texttt{std::sort}! This ultimately does the same thing as our tiny helper homemade compare function (but for any type \texttt{T})!
23.4 Another more Complicated Functor Example

• Constructors of function objects can be used to specify internal data for the functor that can then be used during computation of the function call operator! For example:

```
class between_values {
  private:
    float low, high;
  public:
    between_values(float l, float h) : low(l), high(h) {}
    bool operator() (float val) { return low <= val && val <= high; }
};
```

• The range between low & high is specified when a functor/an instance of this class is created. We might have multiple different instances of the `between_values` functor, each with their own range. Later, when the functor is used, the query value will be passed in as an argument. The function call operator accepts that single argument val and compares against the internal data low & high.

• This can be used in combination with STL’s `find_if` construct. For example:

```
between_values two_and_four(2,4);
if (std::find_if(my_data.begin(), my_data.end(), two_and_four) != my_data.end()) {
    std::cout << "Found a value greater than 2 & less than 4!" << std::endl;
}
```

• Alternatively, we could create the functor without giving it a variable name. And in the use below we also capture the return value to print out the first item in the vector inside this range. Note that it does not print all values in the range.

```
std::vector<float>::iterator itr;
itr = std::find_if(my_data.begin(), my_data.end(), between_values(2,4));
if (itr != my_data.end()) {
    std::cout << "my_data contains " << *itr 
    << ", a value greater than 2 & less than 4!" << std::endl;
}
```

“Weird Things we can do in C++” Finished – Now back to Hash Tables!

23.5 Hash Table in STL?

• The Standard Template Library standard and implementation of hash table have been slowly evolving over many years. Unfortunately, the names “hashset” and “hashmap” were spoiled by developers anticipating the STL standard, so to avoid breaking or having name clashes with code using these early implementations...

• STL’s agreed-upon standard for hash tables: `unordered_set` and `unordered_map`

• Depending on your OS/compiler, you may need to add the `-std=c++11` flag to the compile line (or other configuration tweaks) to access these more recent pieces of STL. (And this will certainly continue to evolve in future years!)

• For many types STL has a good default hash function, so in those cases you do not need to provide your own hash function. But sometimes we do want to write our own...
23.6 Writing our own Hash Functions or Hash Functors

- Often the programmer/designer for the program using a hash function has the best understanding of the distribution of data to be stored in the hash function. Thus, they are in the best position to define a custom hash function (if needed) for the data & application.

- Here’s an example of a (generically) good hash function for STL strings:

  Note: This implementation comes from http://www.partow.net/programming/hashfunctions/

  ```cpp
  unsigned int MyHashFunction(std::string const& key) {
    unsigned int hash = 1315423911;
    for(unsigned int i = 0; i < key.length(); i++)
      hash ^= ((hash << 5) + key[i] + (hash >> 2));
    return hash;
  }
  ```

- Alternately, this same string hash code can be written as a functor – which is just a class wrapper around a function, and the function is implemented as the overloaded function call operator for the class.

  ```cpp
  class MyHashFunctor {
  public:
    unsigned int operator() (std::string const& key) const {
      unsigned int hash = 1315423911;
      for(unsigned int i = 0; i < key.length(); i++)
        hash ^= ((hash << 5) + key[i] + (hash >> 2));
      return hash;
    }
  };
  ```

- Once our new type containing the hash function is defined, we can create instances of our hash set object containing std::string by specifying the type MyHashFunctor as the second template parameter to the declaration of a ds_hashset. E.g.,

  ```cpp
ds_hashset<std::string, MyHashFunctor> my_hashset;
  ```

23.7 Using STL’s Associative Hash Table (Unordered Map)

- Using the default std::string hash function.
  - With no specified initial table size.
    ```cpp
    std::unordered_map<std::string,Foo> m;
    ```
  - Optionally specifying initial (minimum) table size.
    ```cpp
    std::unordered_map<std::string,Foo> m(1000);
    ```

- Using a home-made std::string hash function. Note: We are required to specify the initial table size.
  - Manually specifying the hash function type.
    ```cpp
    std::unordered_map<std::string,Foo,std::function<unsigned int(std::string)>> m(1000, MyHashFunction);
    ```
  - Using the decltype specifier to get the “declared type of an entity”.
    ```cpp
    std::unordered_map<std::string,Foo,decltype(&MyHashFunction)> m(1000, MyHashFunction);
    ```

- Using a home-made std::string hash functor or function object.
  - With no specified initial table size.
    ```cpp
    std::unordered_map<std::string,Foo,MyHashFunctor> m;
    ```
  - Optionally specifying initial (minimum) table size.
    ```cpp
    std::unordered_map<std::string,Foo,MyHashFunctor> m(1000);
    ```

- Note: In the above examples we’re creating a association between two types (STL strings and custom Foo object). If you’d like to just create a set (no associated 2nd type), simply switch from unordered_map to unordered_set and remove the Foo from the template type in the examples above.
23.8 How do we Resolve Collisions? METHOD 1: Separate Chaining

NOTE: We used this method in the last lecture & in lab!

- Each table location stores a linked list of keys (and values) hashed to that location. Thus, the hashing function really just selects which list to search or modify.
- This works well when the number of items stored in each list is small, e.g., an average of 1. Other data structures, such as binary search trees, may be used in place of the list, but these have even greater overhead considering the (hopefully, very small) number of items stored per bin.

23.9 How do we Resolve Collisions? METHOD 2: Open Addressing

- Let’s eliminate the individual memory allocations and pointer indirection / dereferencing that are necessary for separate chaining. This will improve memory / data access performance.
- We will directly store the data (key/key-value pair) in the the top level vector, and store at most one item per index/location.
- When the chosen table index/location already stores a key (or key-value pair), we will seek a different table location to store the new value (or pair).
- Here are three different open addressing variations to handle a collision during an insert operation:
  - Linear probing: If i is the chosen hash location then the following sequence of table locations is tested (“probed”) until an empty location is found:
    \[(i+1)\%N, (i+2)\%N, (i+3)\%N, \ldots\]
  - Quadratic probing: If i is the hash location then the following sequence of table locations is tested:
    \[(i+1)\%N, (i+2\times2)\%N, (i+3\times3)\%N, (i+4\times4)\%N, \ldots\]
    More generally, the \(j^{th}\) “probe” of the table is \((i + c_1j + c_2j^2) \mod N\) where \(c_1\) and \(c_2\) are constants.
  - Secondary hashing: When a collision occurs a second hash function is applied to compute a new table location. If that location is also full, we go to a third hash function, etc. This is repeated until an empty location is found.
    We can generate a sequence/family of hash functions by swapping in a fixed random-like sequence of big (prime?) constants values into the same general function structure.

- For each of these approaches, the find operation follows the same sequence of locations as the insert operation. The key value is determined to be absent from the table only when an empty location is found.
- When using open addressing to resolve collisions, the erase function must mark a location as “formerly occupied”. If a location is instead marked empty, find may fail to return elements that are actually in the table. Formerly-occupied locations may (and should) be reused, but only after the find search operation has been run to completion to determine the item is definitely not in the table.
- Advantages of open addressing:
  - No linked lists! No pointers! It’s faster! (Indirect memory accesses are slow!)
- Problems with open addressing:
  - Fails completely when the table is full. So we MUST resize before it gets full!
  - Slows dramatically when the table is nearly full (e.g. about 80% or higher).
    This is particularly problematic for linear probing.
  - Memory cache performance can be poor when we are jumping around unpredictably in the top level array.
  - Cost of computing new hash values (linear < quadratic < secondary hashing).
  - Careful testing and parameter tuning is necessary to achieve optimal memory/speed performance.
23.10 The Basic Data Structures

This term we’ve covered a number of core data structures. These structures have fundamentally different memory layouts. These data structures are classic, and are not unique to C++.

- array / vector
- linked list
- binary search tree
- binary heap / priority queue
- hash table

23.11 A Few Variants of the Basic Data Structures

Many variants and advanced extensions and hybrid versions of these data structures are possible. Different applications with different requirements and patterns of data and data sizes and computer hardware will benefit from or leverage different aspects of these variants.

This term we’ve already discussed / implemented a number of data structure variants:

- single vs. doubly linked lists
  using more memory can improve convenience and running time for key operations
- dummy nodes or circular linked lists – can reduce need for special case / corner case code
- jagged array (Homework 3)
  avoids overhead of separate memory allocations, pointer arithmetic is faster than pointer dereferencing
- linked tube/grid/matrix (Test 2, Problem 1) – 2-dimensional linked list with efficient merge/split
- red-black tree – an algorithm to automatically balance a binary search tree
- bounding volume hierarchy (Homework 8) – good for organizing spatial data (2D, 3D, etc.), especially data that overlaps – medium/large items can still be stored in a single leaf
- stack and queue – restricted/reduced(!) set of operations on array/vector and list
- hash table: separate chaining vs open addressing – reduce memory and avoid pointer dereferencing
- priority queue with backpointers (Homework 10) – when you need to update data already in the structure

We’ll discuss just a few additional variants today. The list below is certainly not comprehensive!

- unrolled linked list
- skip list
- quad tree
- leftist heap (notes from Lecture 23)
- trie (a.k.a. prefix tree)
- suffix tree
23.12 Quad Tree - Overview

The *quad tree* data structure is a 2D generalization of the 1-dimensional binary search tree. The 3D version is called an *octree*, or in higher dimensions it is called a *k-d tree*. These structures are used to improve the performance of applications that use large spatial data sets including: ray tracing in computer graphics, collision detection for simulation and gaming, motion planning for robotics, nearest neighbor calculation, and image processing.

The diagrams below illustrate the incremental construction of a quad tree. We add the 21 *two-dimensional points* shown in the first image to the tree structure. We will add them in the alphabetical order of their letter *labels*. Each time a point is added we locate the rectangular region containing that point and subdivide that region into 4 smaller rectangles using the $x,y$ coordinates of that point as the vertical and horizontal dividing lines.

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Each node in the structure has 4 children. (Or 8 children if we’re making a 3-dimensional octree). Here’s a 'sideways' printing of the finished tree structure from the example above:

A (20,10)
B (10,5)
F (5,3)
G (15,2)
H (4,7)
I (14,8)
C (30,4)
J (25,1)
K (35,2)
L (26,7)
M (36,6)
D (11,15)
N (3,13)
O (16,12)
P (4,17)
Q (15,18)
E (31,16)
R (25,13)
S (37,14)
T (24,19)
U (36,18)

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23.13 Quad Tree - Discussion

- How does the order of point insertion affect the constructed tree?
  What if we inserted the point labeled 'B' first, and the point labeled 'A' second?

- Can we easily erase an item from the quad tree? What if it’s not a leaf node?

- Alternately (in fact, more typically), the quad tree / octree / k-d tree may simply split at the midpoint in each dimension. In this way the intermediate tree nodes don’t store a data point. All data points are stored only at the leaves of the tree.
23.14 Trie / Prefix Tree - Overview

- Next up, let’s look at an alternate to a hash table for storing key strings and an associated value type.
- In a trie or prefix tree, the key is defined not by storing the data at the node or leaf, but instead by the path of to get to that node. Each edge from the root node stores one character of the string. The node stores the value for the key (or NULL or a special value, e.g., '-1', if the path to that point is not a valid key in the structure).

![Trie Diagram]

- Lookup in the structure is fast, $O(m)$ where $m$ is the length (# of characters) in the string. A hash table has similar lookup (since we have to hash the string which generally involves looking at every letter). If $m << n$, we can say this is $O(1)$.

23.15 Trie / Prefix Tree - Discussion

- What is the worst case # of children for a single node? What are the member variables for the Node class?
- Unlike a hash table, we can iterate over the keys in a trie / prefix tree in sorted order.
  **Exercise:** Implement the trie sorted-order iterator (in code or pseudocode) and print the table on the right.

23.16 Suffix Tree - A Brief Introduction...

- Instead of only encoding the complete string when walking from root to leaf... let’s store every possibly substring of the input.
- This toy example stores 'banana', and all suffix substrings of 'banana'. Each leaf node stores the start position of the substring within the original string. The '$' character is a special terminal character.
- Suffix trees clearly require much more memory than other data structures to store the input string, but do so to gain performance on certain operations....
  Suffix trees help us efficiently find the longest common substring – in linear time. This is an important problem in genome sequencing and computational biology.
- Clever algorithms have been developed to efficiently construct suffix trees.

... and we’re certainly out of time for today. There are many more wonderful data structures to explore. This semester you have learned the tools to study new structures, compare and contrast operation efficiency and memory usage of different structures, and to develop your own data structures for specific applications.