CSCI-1200 Data Structures — Fall 2022
Lecture 20 – Trees, Part IV

Review from Lecture 19 (& Lectures 17 & 18)

- General-Purpose Breadth-First, Tree Traversal Code

```
void breadth_first_traverse( Node* root ) {
  int level = 0;
  std::vector<Node*> current_level;
  std::vector<Node*> next_level;
  if (root == NULL) { return; }
  current_level.push_back(root);
  while ( current_level.size() != 0 ) {
    std::cout << "level " << level << ":";
    for (unsigned int i = 0; i < current_level.size(); i++) {
      if (current_level[i]->left != NULL)
        next_level.push_back(current_level[i]->left);
      if (current_level[i]->right != NULL)
        next_level.push_back(current_level[i]->right);
      std::cout << " " << current_level[i]->value;
    }
    current_level = next_level;
    level++;
    next_level.clear();
    std::cout << std::endl;
  }
}
```

- BST / `ds_set` iterator increment (`operator+`)
- Every node stores `Node` parent pointer *or*
  iterator stores a vector of `Node` pointers (the path from root Node).

Today's Lecture

- Some more practice exercises with trees & Big O Notation
- Limitations of our `ds_set` implementation, brief intro to red-black trees
- BONUS TOPIC: Template Specialization

20.1 Height and Height Calculation Algorithm

- The *height* of a node in a tree is the length of the longest path down the tree from that node to a leaf node. The height of a leaf is 1. We will think of the height of a null pointer as 0.
- The height of the tree is the height of the root node, and therefore if the tree is empty the height will be 0.

  Exercise: Write a simple recursive algorithm to calculate the height of a tree.

- What is the best/average/worst-case running time of this algorithm? What is the best/average/worst-case memory usage of this algorithm? Give a specific example tree that illustrates each case.

20.2 Shortest Paths to Leaf Node

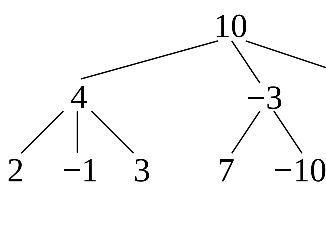
- Now let's write a function to instead calculate the *shortest* path to a NULL child pointer.

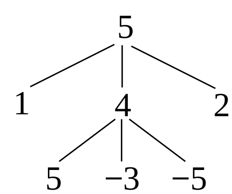
- What is the running time of this algorithm? Can we do better? *Hint: How does a breadth-first vs. depth-first algorithm for this problem compare?*

20.3 A Practice Test Tree Problem

A *trinary tree* is similar to a binary tree except that each node has at most 3 children. Write a *recursive* function named `EqualsChildrenSum` that takes one argument, a pointer to the root of a trinary tree, and returns true if the value at each non-leaf node is the sum of the values of all of its children and false otherwise. In the examples below, the tree on the left will return true and the tree on the right will return false.

```
class Node {
public:
  int value;
  Node* left;
  Node* middle;
  Node* right;
};
```





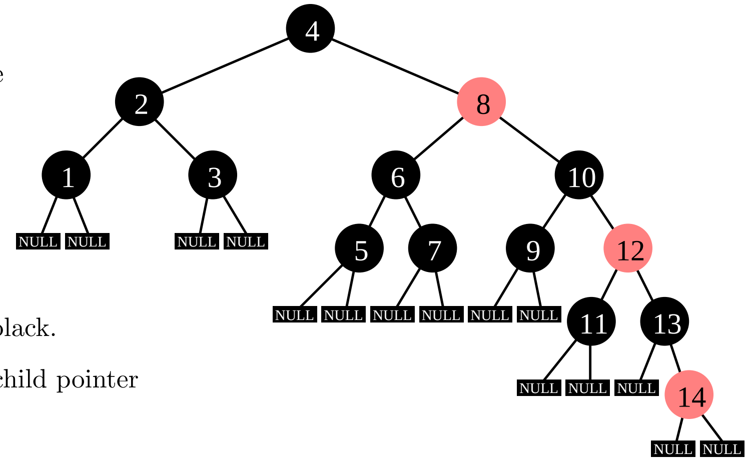
20.4 Limitations of Our BST Implementation

- The efficiency of the main insert, find and erase algorithms depends on the height of the tree.
- The best-case and average-case heights of a binary search tree storing n nodes are both $O(\log n)$. The worst-case, which often can happen in practice, is $O(n)$.
- Developing more sophisticated algorithms to avoid the worst-case behavior will be covered in Introduction to Algorithms. One elegant extension to binary search tree is described below...

20.5 Red-Black Trees

In addition to the binary search tree properties, the following red-black tree properties are maintained throughout all modifications to the data structure:

1. Each node is either red or black.
2. The NULL child pointers are black.
3. Both children of every red node are black.
   Thus, the parent of a red node must also be black.
4. All paths from a particular node to a NULL child pointer contain the same number of black nodes.



What tree does our `ds_set` implementation produce if we insert the numbers 1-14 *in order*?
The tree at the right is the result using a red-black tree. Notice how the tree is still quite balanced.
Visit these links for an animation of the sequential insertion and re-balancing:

https://www.cs.usfca.edu/~galles/visualization/RedBlack.html
http://www.youtube.com/watch?v=vDHFF4wjWYU&noredirect=1

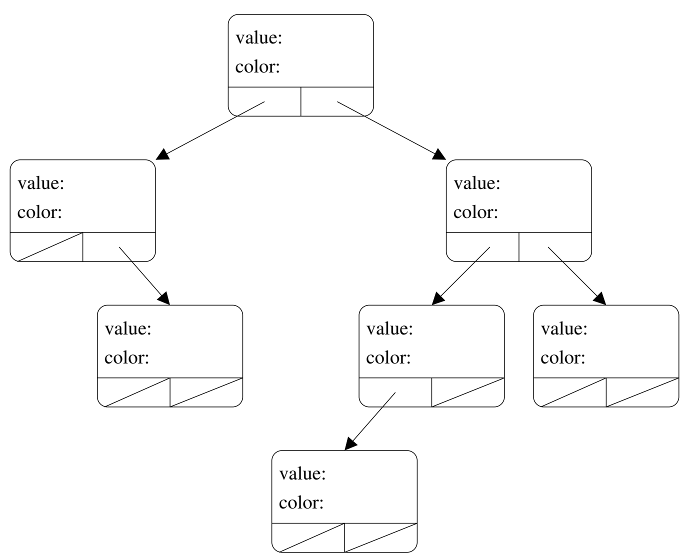
- What is the best/average/worst case height of a red-black tree with n nodes?

- What is the best/average/worst case shortest-path from root to leaf node in a red-black tree with n nodes?

20.6 Exercise [/6]

Fill in the tree on the right with the integers 1-7 to make a binary search tree. Also, color each node "red" or "black" so that the tree also fulfills the requirements of a Red-Black tree.

Draw two other red-black binary search trees with the values 1-7.



Note: Red-Black Trees are just one algorithm for *self-balancing binary search tree*.
Others include: AVL trees, Splay Trees, (& more!).

20.7 BONUS TOPIC: Template Specialization Example

Writing templated functions is elegant and powerful, but sometimes we do not want to handle all types in exactly the same way. Sometimes we want to write different versions of the function depending on the type:

- Let's study and discussion the following code:

```
// We'll use this templated function (unless we find a specialized
// implementation for our type)
template <class T>
void print_vec (const std::vector<T> &v) {
  std::cout << "count=" << v.size() << "  data=";
  for (unsigned int i = 0; i < v.size(); i++) {
    std::cout << " " << v[i]; }
  std::cout << std::endl;
}

// This will match doubles (but not floats)
void print_vec (const std::vector<double> &v) {
  std::cout << "count=" << v.size() << "  data=";
  for (unsigned int i = 0; i < v.size(); i++) {
    std::cout << std::setprecision(1) << std::fixed << " " << v[i]; }
  // unset the formatting
  std::cout << std::defaultfloat << std::endl;
}

int main() {
  // note: this syntax for initialization of vector contents is available with C++11
  std::vector<int> int_v = { 1, 2, 3, 4, 5 };
  std::vector<double> double_v = { 1, 2, 3, 4, 5 };
  std::vector<float> float_v = { 1, 2, 3, 4, 5 };
  std::vector<std::string> string_v = { "1", "2", "3", "4", "5" };
  print_vec(int_v);
  print_vec(double_v);
  print_vec(float_v);
  print_vec(string_v);
}

// This would match strings...  but because it's placed after the
// usage in main it's not used!?!?!
void print_vec (const std::vector<std::string> &v) {
  std::cout << "count=" << v.size() << "  data=";
  for (unsigned int i = 0; i < v.size(); i++) {
    std::cout << " \"" << v[i] << "\""; }
  std::cout << std::endl;
}
```

- If we commented out the specialized implementations of `print_vec` for the double and string types:

```
count=5  data= 1 2 3 4 5
count=5  data= 1 2 3 4 5
count=5  data= 1 2 3 4 5
count=5  data= 1 2 3 4 5
```

- If we run the original code:

```
count=5  data= 1 2 3 4 5
count=5  data= 1.0 2.0 3.0 4.0 5.0
count=5  data= 1 2 3 4 5
count=5  data= 1 2 3 4 5
```

- If we swap the order of the main function and the string version of `print_vec`:

```
count=5  data= 1 2 3 4 5
count=5  data= 1.0 2.0 3.0 4.0 5.0
count=5  data= 1 2 3 4 5
count=5  data= "1" "2" "3" "4" "5"
```

```cpp
// -------------------------------------------------------------------
// DS_SET CLASS -- WITH NESTED TREE NODE & TREE ITERATOR CLASSES (ALTERNATE STYLE)
template <class T>
class ds_set {
public:
  // -------------------------------------------------------------------
  // TREE NODE CLASS
  class TreeNode {
  public:
    TreeNode() : left(NULL), right(NULL), parent(NULL) {}
    TreeNode(const T& init) : value(init), left(NULL), right(NULL), parent(NULL) {}
    T value;
    TreeNode* left;
    TreeNode* right;
    // one way to allow implementation of iterator increment & decrement
    TreeNode* parent;
  };
  // -------------------------------------------------------------------
  // TREE NODE ITERATOR CLASS
  class iterator {
  public:
    iterator() : ptr_(NULL), set_(NULL) {}
    iterator(TreeNode* p, const ds_set * s) : ptr_(p), set_(s) {}
    // operator* gives constant access to the value at the pointer
    const T& operator*() const { return ptr_->value; }
    // comparions operators are straightforward
    bool operator== (const iterator& rgt) { return ptr_ == rgt.ptr_; }
    bool operator!= (const iterator& rgt) { return ptr_ != rgt.ptr_; }
    // increment & decrement operators
    iterator & operator++() {
       /* discussed & implemented in Lecture 19 */




    }
    iterator operator++(int) { iterator temp(*this); ++(*this); return temp; }
    iterator& operator--() {  /* implementation omitted */ }
    iterator operator--(int) { iterator temp(*this); --(*this); return temp; }
  private:
    // representation
    TreeNode* ptr_;
    const ds_set* set_;
  };
  // -------------------------------------------------------------------
  // CONSTRUCTORS, ASSIGNMENT OPERATOR, DESTRUCTOR
  ds_set() : root_(NULL), size_(0) {}
  ds_set(const ds_set<T>& old) : size_(old.size_) { root_=this->copy_tree(old.root_,NULL); }
  ~ds_set() { this->destroy_tree(root_); root_ = NULL; }
  ds_set& operator=(const ds_set<T>& old) {
    if (&old != this) {
      this->destroy_tree(root_);
      root_ = this->copy_tree(old.root_,NULL);
      size_ = old.size_;
    }
    return *this;
  }
  int size() const { return size_; }
  bool operator==(const ds_set<T>& old) const { return (old.root_ == this->root_); }
  // FIND, INSERT & ERASE
  iterator find(const T& key_value) { return find(key_value, root_); }
  std::pair<iterator,bool> insert(T const& key_value) {return insert(key_value,root_,NULL);}
  int erase(T const& key_value) { return erase(key_value, root_); }
  // ITERATORS
  iterator begin() const {
    if (!root_) return iterator(NULL,this);
    TreeNode* p = root_;
    while (p->left) p = p->left;
    return iterator(p,this);
  }
  iterator end() const { return iterator(NULL,this); }
private:
  // REPRESENTATION
  TreeNode* root_;
  int size_;
  // PRIVATE HELPER FUNCTIONS
  TreeNode*  copy_tree(TreeNode* old_root, TreeNode* the_parent) {
    if (old_root == NULL) return NULL;
    TreeNode *answer = new TreeNode();
    answer->value = old_root->value;
    answer->left = copy_tree(old_root->left,answer);
    answer->right = copy_tree(old_root->right,answer);
    answer->parent = the_parent;
    return answer;
  }
  void destroy_tree(TreeNode* p) {
    if (p) { destroy_tree(p->left); destroy_tree(p->right); delete p; }}
  iterator find(const T& key_value, TreeNode* p) {
    if (!p) return end();
    if      (p->value > key_value) return find(key_value, p->left);
    else if (p->value < key_value) return find(key_value, p->right);
    else                           return iterator(p,this);
  }
  std::pair<iterator,bool> insert(const T& key_value, TreeNode*& p, TreeNode* the_parent) {
    if (!p) {
      p = new TreeNode(key_value);
      p->parent = the_parent;
      this->size_++;
      return std::pair<iterator,bool>(iterator(p, this), true);
    }
    else if (key_value < p->value)
      return insert(key_value, p->left, p);
    else if (key_value > p->value)
      return insert(key_value, p->right, p);
    else
      return std::pair<iterator,bool>(iterator(p,this), false);
  }
  int erase(T const& key_value, TreeNode* &p) {
    /* implemented in Lecture 20 */











  }
};
```