CSCI-1200 Data Structures — Fall 2025
Lecture 12 – Operators & Friends

Review from Lectures 11

- Limitations of singly-linked lists
- Doubly-linked list `push_back` `erase`, `insert`, `copy_list`
- Our own version of the STL `list<T>` class, named `dslist`
- Implementing `list<T>::iterator`
- ... `friend` and `typename` keywords

Today's Lecture

- Operators as non-member functions
- Operators as member functions
- Operators as friend functions
- Also... return by copy, return by reference, pass-by-reference, pass-by-pointer

12.1 Complex Numbers — A Brief Review

- Complex numbers take the form $z = a + bi$, where $i = \sqrt{-1}$ and a and b are real.
- a is called the real part, b is called the imaginary part.
- If $w = c + di$, then
  - $w + z = (a + c) + (b + d)i$,
  - $w - z = (a - c) + (b - d)i$, and
  - $w \times z = (ac - bd) + (ad + bc)i$
- The magnitude of a complex number is $\sqrt{a^2 + b^2}$.

12.2 `Complex` Class declaration (`complex.h`)

```
class Complex {
public:
  Complex(double x=0, double y=0) : real_(x), imag_(y) {}
  // Note: The compiler-written copy constructor & assignment operator
  //   would do the correct thing for this class (so normally we would
  //   recommend not implementing them yourself).  But we implement them
  //   here for discussion purposes...
  Complex(const Complex& old) : real_(old.real_), imag_(old.imag_) {}
  Complex& operator= (const Complex& rhs);
  double Real() const { return real_; }
  void SetReal(double x) { real_ = x; }
  double Imaginary() const { return imag_; }
  void SetImaginary(double y) { imag_ = y; }
  double Magnitude() const { return sqrt(real_*real_ + imag_*imag_); }
  Complex operator+ (Complex const& rhs) const;
  Complex operator- () const; // unary operator- negates a complex number
  friend std::istream& operator>> (std::istream& istr, Complex& c);
private:
  double real_, imag_;
};

Complex operator- (Complex const& left, Complex const& right); // non-member function
std::ostream& operator<< (std::ostream& ostr, Complex const& c);  // non-member function
```

12.3 Implementation of `Complex` Class (`complex.cpp`)

```
// Assignment operator
Complex& Complex::operator= (Complex const& rhs) {
  real_ = rhs.real_;
  imag_ = rhs.imag_;
  return *this;
}

// Addition operator as member function. (alt: could implement as non-member function)
Complex Complex::operator+ (Complex const& rhs) const {
  double re = real_ + rhs.real_;
  double im = imag_ + rhs.imag_;
  return Complex(re, im);
}

// Subtraction operator as non-member function. (alt: could implement as a member function)
Complex operator- (Complex const& lhs, Complex const& rhs) {
  return Complex(lhs.Real()-rhs.Real(), lhs.Imaginary()-rhs.Imaginary());
}

// Unary negation operator.  Note that there are no arguments.
Complex Complex::operator- () const {
  return Complex(-real_, -imag_);
}

// Input stream operator as a friend function
istream& operator>> (istream & istr, Complex & c) {
  istr >> c.real_ >> c.imag_;
  return istr;
}

// Output stream operator as an ordinary non-member function
ostream& operator<< (ostream & ostr, Complex const& c) {
  if (c.Imaginary() < 0)  ostr << c.Real() << " - " << -c.Imaginary() << " i ";
  else                    ostr << c.Real() << " + " <<  c.Imaginary() << " i ";
  return ostr;
}
```

12.4 Operators as Non-Member Functions and as Member Functions

- We have already written our own operators, especially `operator<`, to sort objects stored in STL containers and to create our own keys for maps.
- We can write them as non-member functions (e.g., `operator-`). When implemented as a non-member function, the expression: `z - w` is translated by the compiler into the function call: `operator- (z, w)`
- We can also write them as member functions (e.g., `operator+`). When implemented as a member function, the expression: `z + w` is translated into: `z.operator+ (w)`

  This shows that `operator+` is a member function of `z`, since `z` appears on the left-hand side of the operator. Observe that the function has **only one** argument!

  There are several important properties of the implementation of an operator as a member function:

  - It is within the scope of class `Complex`, so private member variables can be accessed directly.
  - The member variables of `z`, whose member function is actually called, are referenced by directly by name.
  - The member variables of `w` are accessed through the parameter `rhs`.
  - The member function is `const`, which means that `z` will not (and can not) be changed by the function. Also, since `w` will not be changed since the argument is also marked `const`.
- Both `operator+` and `operator-` return `Complex` objects, so both must call `Complex` constructors to create these objects. Calling constructors for `Complex` objects inside functions, especially member functions that work on `Complex` objects, seems somewhat counter-intuitive at first, but it is common practice!

12.5 Exercises

- Write `operator*` for `Complex` numbers as a member function of the `Complex` class. Show the additions to `Complex.h` and `Complex.cpp`.

- Write `operator*` for `Complex` numbers as an ordinary non-member function instead of as a member function of the `Complex` class. Show the additions to `Complex.h` and `Complex.cpp`.

12.6 Assignment Operators

- The assignment operator: `z1 = z2;` becomes a function call: `z1.operator=(z2);`

  And cascaded assignments like: `z1 = z2 = z3;` are really: `z1 = (z2 = z3);`
  which becomes: `z1.operator= (z2.operator= (z3));`

  Studying these helps to explain how to write the assignment operator, which is usually a member function.

- The argument (the right side of the operator) is passed by constant reference. Its values are used to change the contents of the left side of the operator, which is the object whose member function is called. A reference to this object is returned, allowing a subsequent call to `operator=` (`z1`'s `operator=` in the example above).

  The identifier `this` is reserved as a pointer inside class scope to the object whose member function is called. Therefore, `*this` is a reference to this object.

- The fact that `operator=` returns a reference allows us to write code of the form: `(z1 = z2).real();`

12.7 Exercise

- Write an `operator+=` as a member function of the `Complex` class. To do so, you must combine what you learned about `operator=` and `operator+`. In particular, the new operator must return a reference, `*this`.

12.8 Returning Objects vs. Returning References to Objects

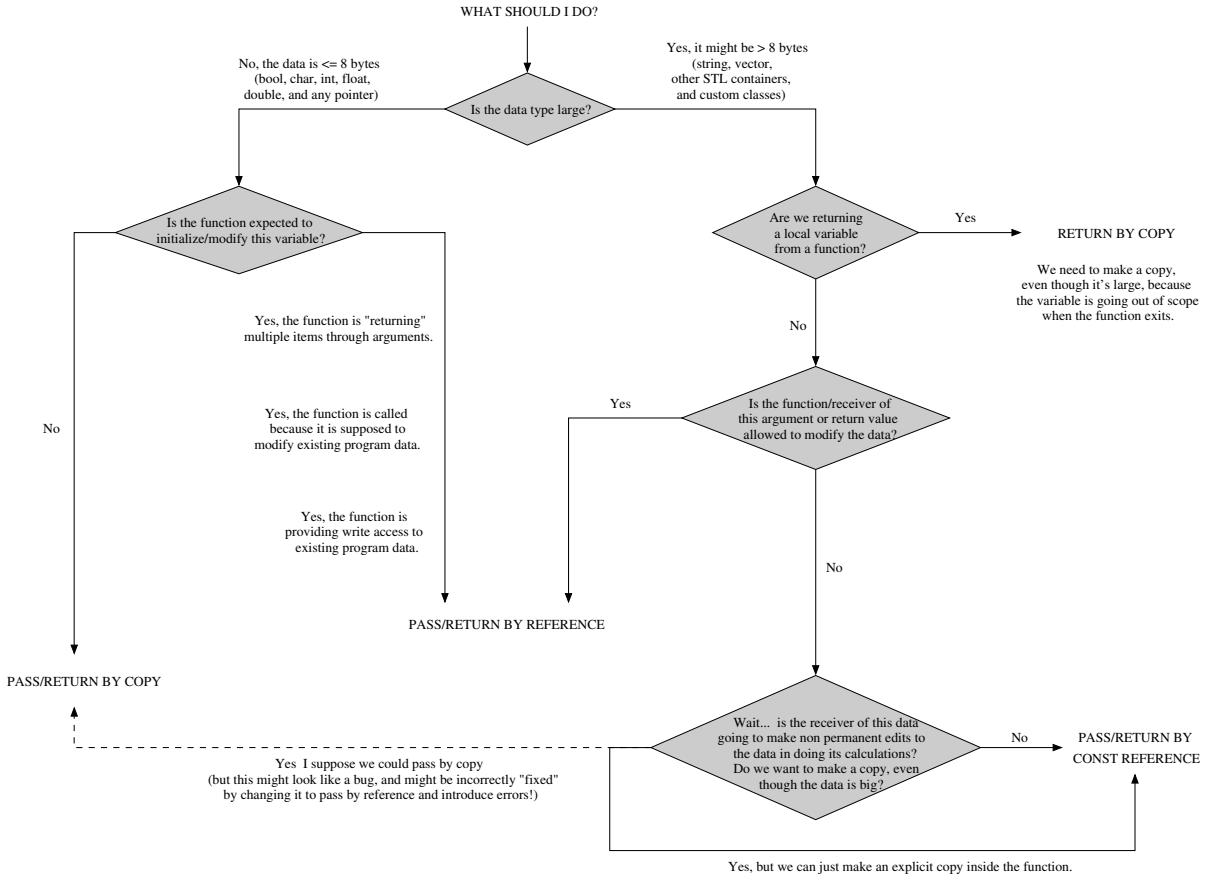
- In the `operator+` and `operator-` functions we create new `Complex` objects and simply return the new object. The return types of these operators are both `Complex`.

  Technically, we don't return the new object (which is stored only locally and will disappear once the scope of the function is exited). Instead we create a copy of the object and return the copy. This automatic copying happens outside of the scope of the function, so it is *safe* to access outside of the function. *Note: It's important that the copy constructor is correctly implemented!* Good compilers can minimize the amount of redundant copying without introducing semantic errors.

- When you change an existing object inside an operator and need to return that object, you must return a **reference** to that object. This is why the return types of `operator=` and `operator+=` are both `Complex&`. This avoids creation of a new object.

- A common error made by beginners (and some non-beginners!) is attempting to return a reference to a locally created object! This results in someone having a pointer to stale memory. The pointer may behave correctly for a short while... until the memory under the pointer is allocated and used by someone else.

12.9 REVIEW: When to use & (reference) vs. const & (const reference)



12.10 REVIEW: References and Return Values

- A reference is an *alias* for another variable. For example:

```
string a = "Tommy";
string b = a;    // a new string is created using the string copy constructor
string& c = a;   // c is an alias/reference to the string object a
b[1] = 'i';
cout << a << " " << b << " " << c << endl;    // outputs:  Tommy Timmy Tommy
c[1] = 'a';
cout << a << " " << b << " " << c << endl;    // outputs:  Tammy Timmy Tammy
```

  The reference variable `c` refers to the same string as variable `a`. Therefore, when we change `c`, we change `a`.

- Exactly the same thing occurs with reference parameters to functions and the return values of functions. Let's look at the `Student` class from Lecture 4 again:

```
class Student {
public:
  const string& first_name() const { return first_name_; }
  const string& last_name() const { return last_name_; }
private:
  string first_name_;
  string last_name_;
};
```

- In the main function we had a vector of students:

```
vector<Student> students;
```

  Based on our discussion of references above and looking at the class declaration, what if we wrote the following. Would the code then be changing the internal contents of the i-th Student object?

```
string & fname = students[i].first_name();
fname[1] = 'i'
```

- The answer is NO! The `Student` class member function `first_name` returns a **const** reference. The compiler will complain that the above code is attempting to assign a const reference to a non-const reference variable.
- If we instead wrote the following, then compiler would complain that you are trying to change a const object.

```
const string & fname = students[i].first_name();
fname[1] = 'i'
```

- Hence in both cases the Student class would be "safe" from attempts at external modification.
- However, the author of the `Student` class would get into trouble if the member function return type was only a reference, and not a const reference. Then external users could access and change the internal contents of an object! This is a bad idea in most cases.

12.11 BONUS: How are references and pointers related?

- In C++, reference parameters allow the function to modify data passed as arguments to the function and/or "return" data through parameters (in addition to the usual function return value).
- In C, *we cannot use pass-by-reference function parameters*. But we can do nearly the same thing by passing in a pointer.

```
// function argument passed by reference - only available in C++
void double_me(int& x) {
  x *= 2;
}

// function argument passed by pointer - available in C and C++
void double_me(int* y) {
  *y *= 2;
}

int main() {
  int foo = 7;
  std::cout << "initial: foo is " << foo << std::endl;
  double_me(foo);
  std::cout << "intermediate: foo is " << foo << std::endl;
  double_me(&foo);
  std::cout << "final: foo is " << foo << std::endl;
}
```

And here is the program output:

```
initial: foo is 7
intermediate: foo is 14
final: foo is 28
```

- When we write a function in C++, when should we choose pass-by-reference and when should we choose pass-by-pointer?
  - The syntax of pass-by-reference is simpler, and easier-to-read.
    - You don't have to "take the address of" your variable when you call the function.
    - You don't have to de-reference the variable when you want to use it or edit it within the function.
  - The syntax of pass-by-pointer is appropriate if your code might be ported to C or re-used in a C program. C programmers are very familiar with pass-by-pointer arguments. Many C library functions use pass-by-pointer arguments.
  - A function parameter pass-by-reference *is not allowed* to be `NULL` or `nullptr`. The compiler will help enforce that your argument is never NULL. This may be appropriate/useful assumption for your program.
  - A function parameter pass-by-pointer *is allowed* to be `NULL` or `nullptr`. This may be appropriate/useful for your program.

12.12 Friend Classes vs. Friend Functions

- We're now going to shift gears slightly and discuss `friend` classes and functions. This will lead to the third method of writing an operator. Friendship is often used for closely related (interdependent) classes, *but should be used sparingly.*
- In the example below, the `Foo` class has designated the `Bar` to be a **friend**. This must be done in the `public` area of the declaration of `Foo`.

```
class Foo {
public:
  friend class Bar;
  ...
};
```

  This allows member functions in class `Bar` to access *all of* the private member functions and variables of a `Foo` object as though they were public (but not vice versa). Note that `Foo` is giving friendship (access to its private contents) rather than `Bar` claiming it. What could go wrong if we allowed friendships to be claimed?

- Alternatively, within the definition of the class, we can designate specific functions to be "`friend`"s, which grants these functions access similar to that of a member function. The most common example of this is operators, and especially stream operators.

12.13 Stream Operators as Friend Functions

- The operators `>>` and `<<` are defined for the `Complex` class. These are binary operators.
  The compiler translates: `cout << z3` into: `operator<< (cout, z3)`
  Consecutive calls to the `<<` operator, such as: `cout << "z3 = " << z3 << endl;`
  are translated into: `((cout << "z3 = ") << z3) << endl;`
  Each application of the operator returns an `ostream` object so that the next application can occur.
- If we wanted to make one of these stream operators a regular member function, it would have to be a member function of the `ostream` class because this is the first argument (left operand). *We cannot make it a member function of the* `Complex` *class.* This is why stream operators are never member functions.
- Stream operators are either ordinary non-member functions (if the operators can do their work through the public class interface) or friend functions (if they need non public access).

12.14 Summary of Operator Overloading in C++

- Unary operators that can be overloaded: `+ - * & ~ ! ++ -- -> ->*`
- Binary operators that can be overloaded: `+ - * / % ^ & | << >> += -= *= /= %= ^= &= |= <<= >>= < <= > >= == != && || , [] () new new[] delete delete[]`
- There are only a few operators that can not be overloaded: `. .* ?: ::`
- We can't create new operators and we can't change the number of arguments (except for the function call operator, which has a variable number of arguments).
- There are three different ways to overload an operator. When there is a choice, we recommend trying to write operators in this order:
  - Non-member function
  - Member function
  - Friend function
- The most important rule for clean class design involving operators is to **NEVER change the intuitive meaning of an operator**. The whole point of operators is lost if you do. One (bad) example would be defining the increment operator on a `Complex` number.

12.15 Extra Practice

- Implement the following operators for the `Complex` class (or explain why they cannot or should not be implemented). Think about whether they should be non-member, member, or friend.

  `operator*` `operator==` `operator!=` `operator<`