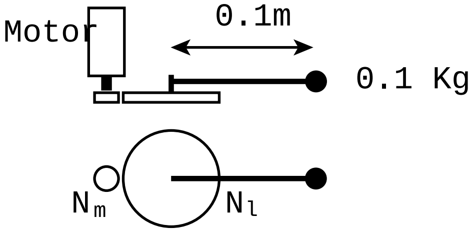
CSCI 4961/6961 Advanced robotics, Spring 2000
Assignment 3: out February 9, due February 16

1. In this problem, we will use a motor (along with some gears) to move a simple link (a mass at the end of a massless rod) as shown below.



To do the controls analysis, we will use the simplified model of a motor as derived in class and use the parameters from a real motor whose spec sheet appears below.

Recall that the (simplified) electrical circuit of the motor is:

$$V = IR + K_e \omega \tag{1}$$

where K_e is the Back-EMF constant, ω is $\dot{\theta}_m$ (and θ_m is the angle of the motor shaft), V is the voltage applied to the motor, I is the current through the motor, and R is the armature resistance. The torque the motor produces is given by:

$$T = K_t I \tag{2}$$

where K_t is the torque constant.

To keep this problem simpler, we will ignore friction in the motor as well as the rotational inertia of the gears. The mechanical system is then described by:

$$T = I_m \ddot{\theta}_m + n T_l \tag{3}$$

where I_m is the armature inertia, $n = \frac{N_m}{N_l}$ is the gear ratio, and T_l is the torque "delivered" to the load. Assume that $n = \frac{1}{30}$. The acceleration of the link (load) is given by:

$$T_l = I_l \ddot{\theta}_l \tag{4}$$

where $I_l = mr^2$ is the moment of inertia of the link and $\theta_l = n\theta_m$ its orientation.

Assume we have a measurement of the position of the link and will use the error from the reference position to do proportional control. Our feedback law is:

$$V = K_p(\theta_r - \theta_l) \tag{5}$$

Note that some of the parameters on the spec sheet are in odd (English) units. In order to avoid confusion, I would suggest converting everything to metric. The following conversion factors will be helpful:

1 radian/second = 9.55 rpm (rotations per minute)
1 ounce inch = 7.09×10^{-3} Newton meter
1 ounce inch second^2 = 7.09×10^{-3} kilogram meter^2

The following steps should take you through this problem. You may want to look at the feedback control notes that will be on the web page (also hardcopies outside my office door) on Thursday February 10.

Please note that you should *present* a solution that demonstrates your understanding of the material, not just scribble down some unintelligible equations that magically produce the answer.

(a) Find the differential equation for the open loop system which relates the motor input voltage (V) to the angle of the link (θ_l). This will be a differential equation in θ_l which you will find from combining Equations 1–4. Do NOT substitute numbers for the constants yet!

(b) Find the differential equation for the closed loop system. This is the open loop differential equation from part (a) combined with Equation 5 which describes how the input voltage will be determined from the error (difference between the desired link angle θ_r and the actual link angle θ_l). Do NOT substitute numbers for the constants yet!

(c) What must the gain K_p be in order for the system to be critically damped? You can put the equation from part (b) into standard form and figure out the natural frequency of the system and then the damping ratio. I suggest substituting numbers for the constants only at the very end. (Don't forget to convert to the proper units!)

(d) Suppose the system is at rest with $\theta_r = \theta_l = 0$, and then we change the commanded velocity θ_r to 30 degrees. How long does it take for the link to move to 27 degrees (i.e. 90% of the way to the final position at 30 degrees).