CSCI–4190 Introduction to Robotic Algorithms, Spring 2003
Assignment 3: out March 20, due April 7

SONAR mapping

For this assignment, you will implement both the certainty grid and the Dempster-Shafer theory approaches to SONAR mapping and compare the results. The support code will provide representation and display for grid-based maps and a simulator to produce SONAR data.

From a programming standpoint, the two SONAR mapping algorithms are very similar — they differ primarily in the method of combining information from a new SONAR reading with the current map. One challenge in implementing the basic algorithms is to determine which cells are in the interior or on the arc of a cone.

Another aspect of this assignment is to evaluate the resulting maps. You will need to devise a method to compare the SONAR map with the world.

Program interface

Here is a rough idea of the interface your program should provide. See the web page for the exact specification.

- The user should, at any time, be able to switch to any of 4 different views:
  - the empty map
  - the full map
  - the combined map
  - the "working" map (see below for details)
- The SONAR data will consist of a number of scans (readings for each sensor in the SONAR ring) from different configurations. The user should be able to "step" through the SONAR data, incorporating a single reading from a scan. There will also be a command for incorporating an entire scan, 10 scans, and the remaining scans.
- The user can "break down" the process of incorporating a single reading from a scan and see the results in the "working" map. For example, the user should be able to see the SONAR cone drawn over the grid, which cells are determined to be in the interior of the cone, which are on the arc, etc. This should be useful for your debugging and testing as well as for us to see that your program is working properly.
- The user will be able to save the sonar map as an image. You may want to write a separate program to compare the generated SONAR map with the actual world.

The support code will have a "skeleton" for most of this functionality.

Written report

You will turn in a written report on this project. Here are some of the things that it should cover. Again, see the web page for a definitive list of topics to cover:

- Describe the details of your implementation: anything that was not straightforward. One thing in particular is how you determined which cells are in the interior or on the arc of the SONAR cone.
- Describe your results: qualitatively, how good are the maps? Describe the method you used to compare the maps with the actual world? How good are the maps according to this measure? Which SONAR mapping method is better?

SONAR mapping methods

Certainty grids

The model of the SONAR sensor is:

$$
\begin{aligned}
Emp_k(r,\theta) &= \left[1 - \left(\frac{r - R_{min}}{R - \epsilon - R_{min}}\right)^2\right]\left[1 - \left(\frac{\theta}{\beta}\right)^2\right] \\
Occ_k(r,\theta) &= \left[1 - \left(\frac{r - R}{\epsilon}\right)^2\right]\left[1 - \left(\frac{\theta}{\beta}\right)^2\right]
\end{aligned}
$$

where (r, θ) are the polar coordinates of the cell, R is the range returned by the SONAR sensor, β is the angular half width of the SONAR cone, ϵ is the half width of the "occupied" distribution, and R_{min} is the minimum possible SONAR range reading.

SONAR readings are incorporated into the current map by the following process:

1. Reset the map to reflect not knowing whether the cell is empty or occupied:

$$
\begin{aligned}
Emp(X,Y) &= 0 \\
Occ(X,Y) &= 0
\end{aligned}
$$

2. For each sonar reading k:

   (a) Superimpose empty areas:

   - Enhance — combine the "empty" certainty from the reading with the current "empty" certainty in the grid.

$$
Emp(X,Y) = Emp(X,Y) + Emp_k(X,Y) - Emp(X,Y) \times Emp_k(X,Y) \quad (1)
$$

   (b) Superimpose occupied areas:

   - Cancel — adjust the "occupied" certainties of the SONAR reading based upon the current "empty" certainties.

$$
Occ_k(X,Y) = Occ_k(X,Y) \times (1 - Emp(X,Y)) \quad (2)
$$

   - Normalize — the sum of all occupied certainties should sum to one since we assume that a single occupied cell causes the SONAR reading

$$
Occ_k(X,Y) = \frac{Occ_k(X,Y)}{\sum Occ_k(X,Y)} \quad (3)
$$

   - Enhance — combine the "occupied" certainty from the reading with the current "occupied" certainty in the grid.

$$
Occ(X,Y) = Occ(X,Y) + Occ_k(X,Y) - Occ(X,Y) \times Occ_k(X,Y) \quad (4)
$$

3. Threshold

$$
Map(X,Y) = \begin{cases} Occ(X,Y) & \text{if } Occ(X,Y) \geq Emp(X,Y) \\ -Emp(X,Y) & \text{otherwise} \end{cases} \quad (5)
$$

Dempster-Shafer based mapping

The model of the SONAR is:

- For cells on the arc of the sonar cone:
  - The probability of being full is based on the assumtions that a single occupied cell is responsible for the reading and that we consider each cell equally likely to be full.

    $$P_F(i,j) = \frac{1}{n}$$

    where n is the number of cells on the arc.
  - There is no evidence that these cells are empty, so

    $$P_E(i,j) = 0$$

- For cells within the cone:
  - The probability of being empty is assumed to be equal over all the cells.

    $$P_E(i,j) = \rho$$

    [Although it is not quite clear from Pagac et al. [**?**], it appears that they take $\rho = \frac{1}{n}$ where n is the number of cells on the arc.]
  - There is no evidence that these cells are full, so

    $$P_F(i,j) = 0$$

SONAR readings are incorporated into the current map by the following equations:

- for the new empty probability of a cell:

$$m_m \oplus m_s(E) = \frac{m_m(E)m_s(E) + m_m(E)m_s(\{E,F\}) + m_m(\{E,F\})m_s(E)}{1 - m_m(E)m_s(F) - m_m(F)m_s(E)}$$

- for the new full probability of a cell:

$$m_m \oplus m_s(F) = \frac{m_m(F)m_s(F) + m_m(F)m_s(\{E,F\}) + m_m(\{E,F\})m_s(F)}{1 - m_m(E)m_s(F) - m_m(F)m_s(E)}$$