CSCI–4190 Introduction to Robotic Algorithms, Spring 2003
Exercise 1: out January 16, due January 23

# Robot kinematics

1. Suppose you command a differential drive robot to travel at some translational velocity $v$ and rotational velocity $\omega$. What should the left and right wheel velocities ($v_L$ and $v_R$) be? Assume that the distance between the wheels is $b$.

2. Suppose a differential drive robot is commanded to execute the following motions:

   - Straight 0.77 meters
   - Right turn for 36 degrees with a 0.73 meter turning radius
   - Straight 1.16 meters
   - Left turn for 60 degrees with a 1.01 meter turning radius

   Assume that the world coordinate frame has its origin at the starting position with the $y$ axis pointing forwards and the $x$ axis pointing to the robot's right. Assume the robot coordinate frame is coincident with the world frame at the starting point.

   (a) What is the path length of the two curved segments?

   (b) the robot's configuration (position and orientation) after each segment above.

3. Suppose you command a three roller-wheeled robot to travel at some translational velocity $\vec{v} = (v_x, v_y)$ and rotational velocity $\omega$. What are the wheel velocities $v_1$, $v_2$, and $v_3$ required?

   Assume that the three wheels are in counterclockwise order, and that positive wheel velocities result in positive (i.e. counterclockwise) rotation.

   You may either:

   - Solve this in vector form, using only the vectors $\vec{R}_1$, $\vec{R}_2$, and $\vec{R}_3$ (from the center of the robot to the center of each wheel), the velocities $\vec{v}$ and $\omega$, the unit vectors $\hat{R}_1$, $\hat{R}_2$, $\hat{R}_3$, and $\hat{z}$, and the cross product, dot product, and vector addition, subtraction, and scalar multiplication.
   - Solve this in algebraic form. Assume wheel 1 is at 0 degrees (relative to the world frame), wheel 2 is at 120 degrees, and wheel 3 is at -120 degrees. Your answer should then be in terms of the velocities $v_x$, $v_w$, and $\omega$, angle constants (e.g. $\frac{2\pi}{3}$), the robot radius $R$, and should include $\sin$ and $\cos$ terms.

⋆4. Suppose the wheel velocities (as a function of time) for a three-roller wheeled robot (as described in the previous problem) are:

$$\begin{aligned} v_1 &= R + \cos(t + \frac{\pi}{2}) \\ v_2 &= R + \cos(t - \frac{5\pi}{6}) \\ v_3 &= R + \cos(t - \frac{\pi}{6}) \end{aligned}$$

   Describe the resulting motion of the robot.