Image Registration Techniques
Homework 2
Due: Tuesday January 27 at the start of class

This homework explores Lectures 2 and 3. Answer each of the following questions clearly. You may submit hand-written answers, but if your handwriting isn't clear, please typeset your solutions.

1. (**5 points**) Let $\mathbf{x} = (x_1, x_2, x_3)^T$ and $\mathbf{y} = (y_1, y_2, y_3, y_4)^T$. Write out the product $\mathbf{x}\mathbf{y}^T$. This is what's known as the *outer product* of the vectors.

2. (**10 points**) Show that if $\|\mathbf{A}\mathbf{v}\| = \|\mathbf{v}\|$ for all vectors $\mathbf{v}$ then $\mathbf{A}$ must be orthonormal.

3. (**10 points**) Find the inverse of the mapping from physical to pixel coordinates. Write it using matrix and vector notation in both homogeneous and non-homogeneous forms.

4. (**10 points**) Show that the determinant of a matrix is 0 if and only if it has at least one singular value equal to 0. You may **not** use the fact that the rank of a matrix equals the number of non-zero singular values.

5. (**5 points**) Re-examine the algebraic formulation of the 2d similarity transformation from Lecture 3. Write parameters s and α as a function of a and b and write parameters a and b in terms of s and α. This will show that you can easily move back-and-forth between representations.

6. (**5 points**) Find the inverse of the similarity transformation,

$$\mathbf{x}' = s\mathbf{R}\mathbf{x} + \mathbf{t}$$

   and show that it is a similarity transformation. (You need only show that it has the form of a similarity transformation.)

7. (**10 points**) Prove that the eigenvectors associated with distinct eigenvalues of a symmetric matrix are orthogonal. **Hint:** let $\mathbf{A}$ be a symmetric matrix, let λ_1 and λ_2 be eigenvalues with $\lambda_1 \neq \lambda_2$, and let $\mathbf{v}_1$ and $\mathbf{v}_2$ be the associated eigenvectors; then consider the expression $\mathbf{v}_1^T \mathbf{A} \mathbf{v}_2$.