Image Registration Techniques
Homework 3
Due: Tuesday February 3 at the start of class

This homework explores Lectures 4 and 5. Answer each of the following questions clearly. Submit hand-written or printed solutions at the start of lecture.

1. (**10 points**) Derive the equation for bilinear interpolation given in the Lecture 4 notes.

2. (**5 points**) Describe a simple, easily-computed method for making the SSD error measure described in Lecture 4 much less dependent on the size of the overlap region Ω.

3. (**20 points**) Write an algorithm to blend the moving image, I_m, and the fixed image, I_f, based on the backwards affine transformation $\mathbf{A}$ and $\mathbf{t}$. Note that I am not asking for a working implementation here, just a description of the algorithm. Assume the images are 2d, that each image has m rows and n columns, and (for simplicity) that image coordinates and physical coordinates coordinates coincide. Use the notation $I(x, y)$ to indicate the pixel value at row y and column x, with the first pixel at $x = y = 0$.

   There are two main parts to this.

   (a) Determine the size and origin of the resulting image (call it I_b). These will generally be different from the origin and size of either image. This will therefore necessitate transforming both the fixed and moving image, even though the fixed image will be just a translation.

   (b) Step through I_b, determining the intensity of each pixel. The intensity value will depend on whether the pixel is from I_m, from I_f, from neither, or from both. When the pixel is outside both images, the intensity should be 0. When the pixel is inside both images, the intensity should be the average of the two (mapped) intensities.

4. (**20 points**) In the least-squares derivations of Lecture 5 we used the Euclidean distance between points. In many ICP algorithms, however, other distance metrics are used. The main one is called the "normal distance". Let $\mathbf{g}_k$ and $\mathbf{f}_k$ be moving and fixed image locations, as before. In addition, let $\hat{\eta}_k = (\eta_{k1}, \eta_{k2})^T$, be a (unit) normal vector at $\mathbf{f}_k$. Think of this as the normal to the line through the image location of $\mathbf{f}_k$. The normal distance is the minimum distance from $\mathbf{T}(\mathbf{g}_k; \mathbf{\Theta})$ to this line (not to $\mathbf{f}_k$). Algebraically, this is

$$D(\mathbf{T}(\mathbf{g}_k; \mathbf{\Theta}), \mathbf{f}_k) = \left[\mathbf{T}(\mathbf{g}_k; \mathbf{\Theta}) - \mathbf{f}_k\right]^T \hat{\eta}_k.$$

We will spend more time on the geometric and mathematical intuitions behind this in a few weeks. For now, however, please derive the least-squares estimate of the 2d similarity transformation parameters using this distance measure, starting with the equation on slide 10 of Lecture 5 (but ignoring the dependence of E on $\mathcal{C}$ as done on slide 13). You may use either the component derivative or the vector-matrix form (either of the two versions presented in the lecture), although I encourage you to try both.