# CSCI-1200 Computer Science II — Spring 2007
# Lecture 16 — Trees, Part I

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## Today's Lecture

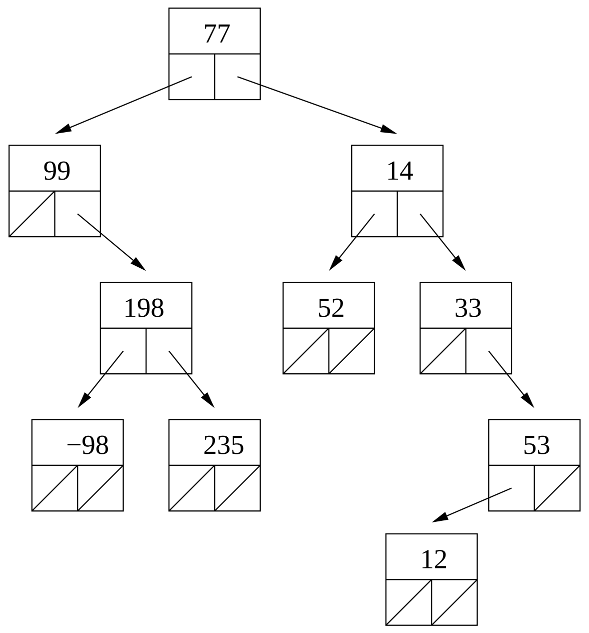
- Binary Trees and Binary Search Trees
- Definition & basic operations
- Implementation of `cs2set` class using binary search trees
- Note: Lots more tree stuff in CSCI 2300 Data Structures & Algorithms (DSA)!

## 16.1 Overview: Lists vs. Trees vs. Graphs

- Trees create a hierarchical organization of data, rather than the linear organization in linked lists (and arrays and vectors).
- Binary search trees are the mechanism underlying maps & sets (and multimaps & multisets).
- Mathematically speaking: A *graph* is a set of vertices connected by edges. And a tree is a special graph that has no *cycles*. The edges that connect nodes in trees and graphs may be *directed* or *undirected*.

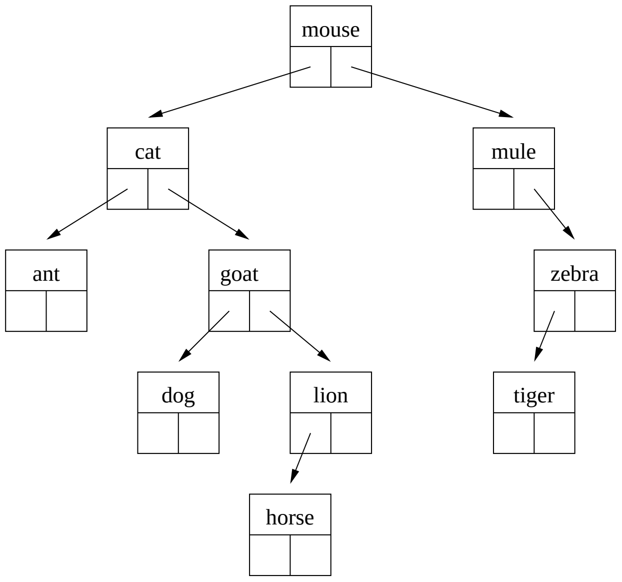
## 16.2 Definition: Binary Trees

- A binary tree (strictly speaking, a "rooted binary tree") is either empty or is a node that has pointers to two binary trees.
- Here's a picture of a binary tree storing integer values. In this figure, each large box indicates a tree node, with the top rectangle representing the value stored and the two lower boxes representing pointers. Pointers that are null are shown with a slash through the box.
- The topmost node in the tree is called the *root*.
- The pointers from each node are called *left* and *right*. The nodes they point to are referred to as that node's (left and right) *children*.
- The (sub)trees pointed to by the left and right pointers at *any* node are called the *left subtree* and *right subtree* of that node.
- A node where `both` children pointers are null is called a *leaf node*.
- A node's *parent* is the unique node that points to it. Only the root has no parent.



## 16.3 Definition: Binary Search Trees

- A *binary search tree* is a binary tree where **at each node** of the tree, the *value* stored at the node is
  - greater than or equal to all values stored in the left subtree, and
  - less than or equal to all values stored in the right subtree.
- Here is a picture of a binary search tree storing string values.



## 16.4 Exercise

Consider the following values:

```
4.5, 9.8, 3.5, 13.6, 19.2, 7.4, 11.7
```

1. Draw a binary tree with these values that *is NOT* a binary search tree.

2. Draw *two different* binary search trees with these values. Important note: This shows that the binary search tree structure for a given set of values is not unique!

## 16.5 The Tree Node Class

Here is the class definition for nodes in the tree. We will use this for the tree manipulation code we write.

```
template <class T>
class TreeNode {
public:
  TreeNode() : left(NULL), right(NULL) {}
  TreeNode(const T& init) : value(init), left(NULL), right(NULL) {}
  T value;
  TreeNode* left;
  TreeNode* right;
};
```

Sometimes a 3rd pointer — to the parent TreeNode — is added to this data structure.

## 16.6 In-order Traversal

- One of the fundamental tree operations is "traversing" the nodes in the tree and doing something at each node. The "doing something", which is often just printing, is referred to generically as "visiting" the node.
- There are three general orders in which binary trees are traversed: pre-order, in-order and post-order.
- These are usually written recursively, and the code for the three functions looks amazingly similar.
- Here's the code for an in-order traversal to print the contents of a tree:

```
void print_in_order(ostream& ostr, const TreeNode<T>* p) {
  if (p) {
    print_in_order(ostr, p->left);
    ostr << p->value << "\n";
    print_in_order(ostr, p->right);
  }
}
```

- How would you modify this code to perform pre-order and post-order traversals?

## 16.7 Exercises

1. Write a templated function to find the smallest value stored in a binary search tree whose root node is pointed to by `p`.

2. Write a function to count the number of odd numbers stored in a binary tree (not necessarily a binary search tree) of integers. The function should accept a `TreeNode<int>` pointer as its sole argument and return an integer. Hint: think recursively!

## 16.8 `cs2set` and Binary Search Tree Implementation

- A partial implementation of a set using a binary search tree is in the code attached. We will continue to study this implementation in the next lecture & lab.
- The increment and decrement operations for iterators have been omitted from this implementation. We will discuss a couple strategies for adding these operations later.
- We will use this as the basis both for understanding an initial selection of tree algorithms and for thinking about how standard library sets really work.

## 16.9 `cs2set`: Class Overview

- The classes are templated.
- There is an auxiliary `TreeNode` class
- The only member variables of the `cs2set` class are the root and the size (number of tree nodes).
- The iterator class is declared internally, and is effectively a wrapper on the TreeNode pointers.
  - Note that `operator*` returns a `const` reference because the keys can't change.
  - As just discussed the increment and decrement operators are missing.
- The main public member functions just call a private (and often recursive) member function (passing the root node) that does all of the work.
- Because the class stores and manages dynamically allocated memory, a copy constructor, `operator=`, and destructor must be provided.

## 16.10 Exercises

1. Provide the implementation of the member function `cs2set<T>::begin`. This is essentially the problem of finding the node in the tree that has stores the smallest value.

2. Write a recursive version of the function `find`.

```
// Partial implementation of binary-tree based set class similar to std::set.
// The iterator increment & decrement operations have been omitted.
#ifndef cs2set_h_
#define cs2set_h_
#include <iostream>
#include <utility>

// -------------------------------------------------------------------
// TREE NODE CLASS
template <class T>
class TreeNode {
public:
  TreeNode() : left(NULL), right(NULL) {}
  TreeNode(const T& init) : value(init), left(NULL), right(NULL) {}
  T value;
  TreeNode* left;
  TreeNode* right;
};

template <class T> class cs2set;

// -------------------------------------------------------------------
// TREE NODE ITERATOR CLASS
template <class T>
class tree_iterator {
public:
  tree_iterator() : ptr_(NULL) {}
  tree_iterator(TreeNode<T>* p) : ptr_(p) {}
  tree_iterator(const tree_iterator& old) : ptr_(old.ptr_) {}
  ~tree_iterator() {}
  tree_iterator& operator=(const tree_iterator& old) { ptr_ = old.ptr_;  return *this; }

  // operator* gives constant access to the value at the pointer
  const T& operator*() const { return ptr_->value; }
  // comparions operators are straightforward
  friend bool operator==(const tree_iterator& l, const tree_iterator& r) { return l.ptr_ == r.ptr_; }
  friend bool operator!=(const tree_iterator& l, const tree_iterator& r) { return l.ptr_ != r.ptr_; }

private:
  // representation
  TreeNode<T>* ptr_;
};

// -------------------------------------------------------------------
// CS2 SET CLASS
template <class T>
class cs2set {
public:
  cs2set() : root_(NULL), size_(0) {}
  cs2set(const cs2set<T>& old) : size_(old.size_) {
    root_ = this->copy_tree(old.root_); }
  ~cs2set() { this->destroy_tree(root_);  root_ = NULL; }
  cs2set& operator=(const cs2set<T>& old) {
    if (old != *this) {
      this->destroy_tree(root_);
      root_ = this->copy_tree(old.root_);
      size_ = old.size_;
    }
    return *this;
  }

  typedef tree_iterator<T> iterator;

  int size() const { return size_; }
  bool operator==(const cs2set<T>& old) const { return (old.root_ == this->root_); }
  // FIND, INSERT & ERASE
  iterator find(const T& key_value) { return find(key_value, root_); }
  std::pair< iterator, bool > insert(T const& key_value) { return insert(key_value, root_); }
  int erase(T const& key_value) { return erase(key_value, root_); }

  // OUTPUT & PRINTING
  friend std::ostream& operator<< (std::ostream& ostr, const cs2set<T>& s) {
    s.print_in_order(ostr, s.root_);
    return ostr;
  }
  void print_as_sideways_tree(std::ostream& ostr) const { print_as_sideways_tree(ostr, root_, 0); }

  // ITERATORS
  iterator begin() const {
    // lecture exercise








  }
  iterator end() const { return iterator(NULL); }

private:
  // REPRESENTATION
  TreeNode<T>* root_;
  int size_;

  // PRIVATE HELPER FUNCTIONS
  TreeNode<T>*  copy_tree(TreeNode<T>* old_root) {  /* next lecture */  }
  void destroy_tree(TreeNode<T>* p) {  /* next lecture */  }

  iterator find(const T& key_value, TreeNode<T>* p) {
    // lecture exercise









  }

  std::pair<iterator,bool> insert(const T& key_value, TreeNode<T>*& p) {  /* next lecture */  }
  int erase(T const& key_value, TreeNode<T>* &p) {  /* next lecture */  }

  void print_in_order(std::ostream& ostr, const TreeNode<T>* p) const {
    if (p) {
      print_in_order(ostr, p->left);
      ostr << p->value << "\n";
      print_in_order(ostr, p->right);
    }
  }

  void print_as_sideways_tree(std::ostream& ostr, const TreeNode<T>* p, int depth) const {
    /* next lecture */  }
};

#endif
```