

BISIMILARITY

When are two processes equivalent?

Bisimulation

A symmetric binary relation R on agents, s.t.:

$$P R Q \text{ and } P \xrightarrow{\alpha} P'$$

implies

$$\exists Q' : Q \xrightarrow{\alpha} Q' \wedge P' R Q'$$

Two agents are bisimilar if they can indefinitely mimic the transitions of each other.

STRONG BISIMULATION

A symmetric binary relation R on agents, s.t.:

$P R Q$ and $P \xrightarrow{\alpha} P'$ where $\text{bn}(\alpha)$ is fresh

implies that

(i) If $\alpha = a(x)$ then

$\exists Q' : Q \xrightarrow{a(x)} Q' \wedge \forall u : P' \{u/x\} R Q' \{u/x\}$

(ii) If α is not an input then

$\exists Q' : Q \xrightarrow{\alpha} Q' \wedge P' R Q'$

P and Q are strongly bisimilar, written

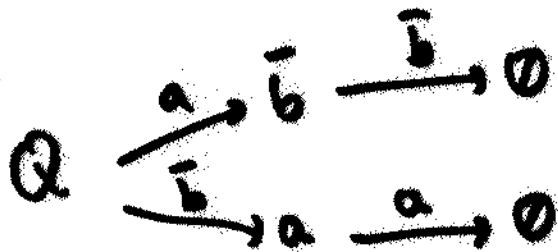
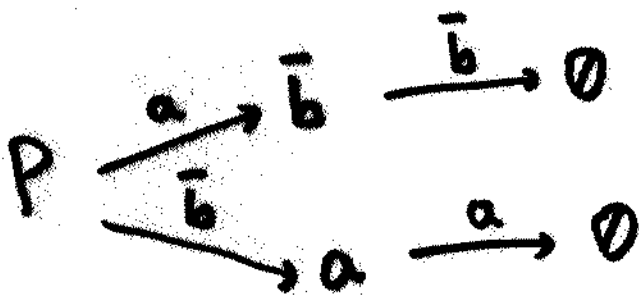
$P \dot{\sim} Q$

if they are related by a bisimulation.

Example 1.

$$P = a\bar{b}$$

$$Q = a\bar{b} + \bar{b}a$$



So, we say $P \sim Q$.

Example 2

$$P = a(u)$$

$$Q = a(x). (\lambda v) \bar{v} u$$

④

Transition $P \xrightarrow{a(u)} \emptyset$ cannot be simulated by Q because x cannot be α -converted to u in Q , since u appears free in Q .

But intuitively P and Q behave the same.

Example 3

$$P = a(x). \bar{b}x$$

$$Q = a(y). \bar{b}y$$

$$P \xrightarrow{a(x)} \bar{b}x$$

$$Q \xrightarrow{a(x)} \bar{b}x \quad (\text{after } \alpha\text{-conversion: } a(y). \bar{b}y \equiv a(x). \bar{b}x) \\ \text{by STRUCT rule.}$$

$$\forall u: \bar{b}x \{u/x\} R \bar{b}x \{u/x\}$$

$$\text{since } \bar{b}u \sim \bar{b}u.$$

In general,

$$P \equiv Q \text{ implies } P \dot{\sim} Q.$$

Example 9.

$$P_1 = a(x).P + a(x).0$$

$$P_2 = a(x).P + a(x).\text{if } x=u \text{ then } P$$

If we assume $P \neq 0$ then

$$P_1 \neq P_2$$

since the transition

$$P_1 \xrightarrow{a(x)} 0$$

cannot be simulated by P_2 .

$$P_2 \xrightarrow{a(x)} \text{if } x=u \text{ then } P$$

for the substitution $\{u/x\}$.

□

Example 5.

$$P = a / \bar{a}$$

$$Q_1 = a.\bar{a} + \bar{a}.a + \epsilon$$

$$Q_2 = a.\bar{a} + \bar{a}.a$$

$P \sim Q_2$ but $P \not\sim Q_1$ since
the empty transition cannot be
simulated by Q_2 .

So, in general, \sim is not closed under
substitutions. i.e., from $P \sim Q$ we cannot
conclude $P\sigma \sim Q\sigma$.

Example 6

$$P = c(a). (a | \bar{b})$$

$$Q = c(a). (a.\bar{b} + \bar{b}.a)$$

$P \not\sim Q$ since for input substitution $\{b/a\}$,

$$(a | \bar{b}) \not\sim (a.\bar{b} + \bar{b}.a)$$

$$\text{i.e. } (b | \bar{b}) \not\sim (b.\bar{b} + \bar{b}.b)$$

In general, \sim is not preserved by input prefix.

i.e. from $P \sim Q$ we cannot conclude

$$a(x).P \sim a(x).Q.$$

PROPOSITION 1 EXAMPLE

$$P = \exists x (Q \mid y(a).R)$$

$$\equiv \exists x' (Q \{x'/x\} \mid y(a).R)$$

for $x' \notin P$.

Proof.

$$\exists x (Q \mid y(a).R)$$

$$\equiv \exists x' (Q \{x'/x\} \mid y(a).R)$$

by α -conversion
by commutativity

$$\equiv y(a).R \mid \exists x' (Q \{x'/x\})$$

$$\equiv \exists x' (y(a).R \mid Q \{x'/x\})$$

by scope extension
law 2,
right to left

$$\equiv \exists x' (Q \{x'/x\} \mid y(a).R)$$

by commutativity

D.

PROPOSITION 2.

If $P \sim Q$ and σ is injective,
then $P\sigma \sim Q\sigma$.

PROPOSITION 3.

\sim is an equivalence.

(Reflexive, symmetric, and transitive).

PROPOSITION 4.

\sim is preserved by all operators except
input prefix.

CONGRUENCE

Two agents P and Q are (strongly) congruent, written $P \sim Q$, if $P\sigma \sim Q\sigma$ for all substitutions σ .

Example 7

$$P = a \mid \bar{b}$$

$$Q = a.\bar{b} + \bar{b}.a + \text{if } a=b \text{ then } \tau.$$

$P \sim Q$ since every substitution σ (including $\{b/a\}$ and $\{a/b\}$) preserves $P\sigma \sim Q\sigma$.

EQUATIONAL THEORY -- AXIOMS FOR STRING DISIMILARITY

- STR If $P \equiv Q$ then $P = Q$
- CONG1 If $P = Q$ then $\bar{a}u.P = \bar{a}u.Q$
 $\tau.P = \tau.Q$
 $P + R = Q + R$
 $(\nu x)P = (\nu x)Q$
- CONG2 If $P\{y/x\} = Q\{y/x\}$ for all $y \in \text{fn}(P, Q, x)$
then $a(x).P = a(x).Q$
- S $P + P = P$
- M1 if $x = x$ then $P = P$
- M2 if $x = y$ then $P = \emptyset$ if $x \neq y$
- MM1 if $x \neq x$ then $P = \emptyset$
- MM2 if $x \neq y$ then $P = P$ if $x \neq y$
- R1 $(\nu x)\alpha.P = \alpha.(\nu x)P$ if $x \notin \alpha$
- R2 $(\nu x)\alpha.P = \emptyset$ if x is the subject of α .
- R3 $(\nu x)(P + Q) = (\nu x)P + (\nu x)Q$