CSCI2300 – Data Structures and Algorithms

Lab 4 – Exam 1 Review

Overview

- Exam 1 will be held in class on Friday, February 23, from 2 to 3:30pm.
- There will be no makeup exam if you miss this exam.
- The exam is closed everything – closed book, closed notes, no crib sheets, no calculators, and no aids of any kind.
- Any formulas that you may need will be given on the exam.

Coverage

- Induction and recursion.
- Order notation.
- Algorithm analysis, including recurrences.
- Linear structures: linked lists, pointers, arrays, stacks, queues, vectors.
- Trees, binary trees, binary search trees (AVL trees are not included).

Preparation for Exam

- Do not memorize formulas and theorems.
- Concentrate on lecture slides, homeworks, and labs.
- Here are a few problems to help you prepare for the exam. They do not completely cover everything that may appear in Exam 1.

Formulas

$$\sum_{i=0}^{n} i = n(n+1)/2.$$

$$\sum_{i=0}^{n} a^i = (1-a^{n+1})/(1-a), \text{ where } a \neq 1.$$

1. For each of the following sets of integers, derive a formula that gives the sum. Use induction to prove that your formula is correct.

   (a) The given numbers are $5, 6, 7, \ldots, 2n + 1$.

   (b) The given numbers are $6, 8, 10, \ldots, 2n$.

   (c) The given numbers are $5, 7, 9, \ldots, 2n + 1$.

2. (a) Use induction to prove that for all integers $n \geq 6$, $5n + 5 < n^2$.

   (b) Show that

$$\sum_{i=1}^{n} 10\, i^9 = O(n^{10}).$$

3. For each of the following, decide whether $T(n) = O(f(n))$, $T(n) = \Omega(f(n))$, or $T(n) = \Theta(f(n))$. Justify your answers.

   (a) $T(n) = 10 \log n + 10$ and $f(n) = \log(n^{100})$.

   (b) $T(n) = 100\, n^{100}$ and $f(n) = e^n$.

   (c) $T(n) = n^{100}$ and $f(n) = 100\, n^{99}$.

4. Evaluate these two recurrences. Your answers should be explicit functions of n and $T(1)$.

   (a) $T(n) = T(n - 1) + 2n$.

   (b) $T(n) = 2T(n/2) + n^2$.

5. Consider binary search trees.

   (a) Given an empty binary search tree, show the structure of the binary search tree after each of these values 5, 2, 4, 7, 8, 1, and 3 is inserted.

   (b) We want to find the maximum sum of two distinct elements on the tree. What is the worst-case running time? What is the best-case running time? Justify your answers.

6. Write a C++ program to find the second smallest element in a binary search tree.