

1 Examples of the Assignment

Examples from the assignment description:

1. λ *Input*: $(\lambda x.y z)$ *Scheme Input*: $((\text{lambda } (x) y) z)$ *Oz Input*: $\text{apply}(\text{lambda}(x y) z)$ λ *Output*: y *Oz Output*: y
2. λ *Input*: $(\lambda x.x z)$ *Scheme Input*: $((\text{lambda } (x) x) z)$ *Oz Input*: $\text{apply}(\text{lambda}(x x) z)$ λ *Output*: z *Oz Output*: z
3. λ *Input*: $((\lambda x.x \lambda y.y) x)$ *Scheme Input*: $((\text{lambda}(x) x) (\text{lambda}(y) y)) x)$ *Oz Input*: $\text{apply}(\text{apply}(\text{lambda}(x) x) \text{lambda}(y y)) x)$ λ *Output*: x *Oz Output*: x

2 Trivial Cases

Test cases that have same outputs as inputs:

1. λ *Input*: x *Scheme Input*: x *Oz Input*: x λ *Output*: x *Oz Output*: x
2. λ *Input*: $(x y)$ *Scheme Input*: $(x y)$ *Oz Input*: $\text{apply}(x y)$ λ *Output*: $(x y)$ *Oz Output*: $\text{apply}(x y)$
3. λ *Input*: $\lambda x.(x x)$ *Scheme Input*: $(x (x x))$ *Oz Input*: $\text{lambda}(x \text{apply}(x x))$ λ *Output*: $\lambda x.(x x)$ *Oz Output*: $\text{lambda}(x \text{apply}(x x))$

3 β Reduction

Simple β Reduction test cases:

1. λ *Input*: $(\lambda x.x (y w))$ *Scheme Input*: $((\text{lambda } (x) x) (y w))$ *Oz Input*: $\text{apply}(\text{lambda}(x x) \text{apply}(y w))$ λ *Output*: $(y w)$ *Oz Output*: $\text{apply}(y w)$
2. λ *Input*: $(\lambda p.(p j) \lambda x.(q x))$ *Scheme Input*: $((\text{lambda}(p) (p j)) (\text{lambda}(x) (q x)))$ *Oz Input*: $\text{apply}(\text{lambda}(p \text{apply}(p j)) \text{lambda}(x \text{apply}(q x)))$ λ *Output*: $(q j)$ *Oz Output*: $\text{apply}(q j)$
3. λ *Input*: $(\lambda x.\lambda x.(x y) z)$ *Scheme Input*: $((\text{lambda}(x) (\text{lambda}(x) (x y))) z)$ *Oz Input*: $\text{apply}(\text{lambda}(x \text{lambda}(x \text{apply}(x y))) z)$ λ *Output*: $\lambda x.(x y)$ *Oz Output*: $\text{lambda}(x \text{apply}(x y))$ *Note*: For β reduction, only x that appears free in E can be replaced by M in $(\lambda x.M E)$

4 η Conversion

1. λ *Input*: $\lambda x.(y x)$ *Scheme Input*: $(\text{lambda}(x) (y x))$ *Oz Input*: $\text{lambda}(x \text{apply}(y x))$ λ *Output*: y *Oz Output*: y
2. λ *Input*: $\lambda x.((y w) x)$ *Scheme Input*: $(\text{lambda } (x) ((y w) x))$ *Oz Input*: $\text{lambda}(x \text{apply}(\text{apply}(y w) x))$ λ *Output*: $(y w)$ *Oz Output*: $\text{apply}(y w)$
3. λ *Input*: $\lambda x.((y x) x)$ *Scheme Input*: $(\text{lambda } (x) ((x y) x))$ *Oz Input*: $\text{lambda}(x \text{apply}(\text{apply}(x y) x))$ λ *Output*: $\lambda x.((y x) x)$ *Oz Output*: $\text{lambda}(x \text{apply}(\text{apply}(x y) x))$
Note: This formula cannot be η converted.

5 Complex cases

Test cases that may use any of β Reduction, η Conversion and α Renaming.

1. λ *Input*: $(\lambda x.\lambda y.(x y) (x y))$ *Scheme Input*: $((\text{lambda}(x) (\text{lambda}(y) (x y))) (x y))$ *Oz Input*: $\text{apply}(\text{lambda}(x \text{lambda}(y \text{apply}(x y))) \text{apply}(x y))$ λ *Output*: $(x y)$ *Oz Output*: $\text{apply}(x y)$
Note: This formula needs α , β and η .

2. λ *Input:* $(\lambda x.\lambda y.\lambda z.(y z) (y z))$ *Scheme Input:* $((\text{lambda}(x) (\text{lambda}(y) (\text{lambda}(z) (y z)))) (y z))$ *Oz Input:* $\text{apply}(\text{lambda}(x \text{ lambda}(y) \text{ lambda}(z) \text{ apply}(y z)))) \text{ apply}(y z)$ λ *Output:* $\lambda y.y$ *Oz Output:* $\text{lambda}(y y)$
Note: This formula needs β and η .
3. λ *Input:* $(\lambda x.\lambda y.\lambda z.((x z) y) \lambda x.\lambda y.y)$ *Scheme Input:* $((\text{lambda}(x) (\text{lambda}(y) (\text{lambda}(z) ((x z) y)))) (\text{lambda}(x) (\text{lambda}(y) y)))$ *Oz Input:* $\text{apply}(\text{lambda}(x \text{ lambda}(y) \text{ lambda}(z) \text{ apply}(\text{apply}(x z) y)))) \text{ lambda}(x \text{ lambda}(y y))$ λ *Output:* $\lambda y.\lambda z.y$ *Oz Output:* $\text{lambda}(y \text{ lambda}(z y))$
Note: This formula needs α , β and η .

6 Loop

1. λ *Input:* $(\lambda x.y (\lambda x.(x x) \lambda x.(x x)))$ *Scheme Input:* $((\text{lambda}(x) y) ((\text{lambda}(x) (x x)) (\text{lambda}(x) (x x))))$ *Oz Input:* $\text{apply}(\text{lambda}(x y) \text{ apply}(\text{lambda}(x \text{ apply}(x x)) \text{ lambda}(x \text{ apply}(x x))))$ λ *Output:* INFINITE LOOP (call by value) or y (call by name) *Oz Output:* INFINITE LOOP (call by value) or y (call by name)