Declarative Programming Techniques

Accumulators, Difference Lists (VRH 3.4.3-3.4.4)

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Accumulators

- *Accumulator programming* is a way to handle state in declarative programs. It is a programming technique that uses arguments to carry state, transform the state, and pass it to the next procedure.
- Assume that the state S consists of a number of components to be transformed individually:

  $$S = (X,Y,Z,...)$$

- For each predicate P, each state component is made into a pair, the first component is the *input* state and the second component is the output state after P has terminated
- S is represented as

  $$(X_{in}, X_{out}, Y_{in}, Y_{out}, Z_{in}, Z_{out},...)$$

A Trivial Example in Prolog

```
increment(N0,N) :-
    N is N0 + 1.

square(N0,N) :-
    N is N0 * N0.

inc_square(N0,N) :-
    increment(N0,N1),
    square(N1,N).
```

`increment` takes N0 as the input and produces N as the output by adding 1 to N0.

`square` takes N0 as the input and produces N as the output by multiplying N0 to itself.

`inc_square` takes N0 as the input and produces N as the output by using an intermediate variable N1 to carry N0+1 (the output of `increment`) and passing it as input to `square`. The pairs N0-N1 and N1-N are called *accumulators*.

A Trivial Example in Oz

```
proc {Increment N0 N}
    N = N0 + 1
end

proc {Square N0 N}
    N = N0 * N0
end

proc {IncSquare N0 N}
    N1 in
    {Increment N0 N1}
    {Square N1 N}
end
```

`Increment` takes N0 as the input and produces N as the output by adding 1 to N0.

`Square` takes N0 as the input and produces N as the output by multiplying N0 to itself.

`IncSquare` takes N0 as the input and produces N as the output by using an intermediate variable N1 to carry N0+1 (the output of `Increment`) and passing it as input to `Square`. The pairs N0-N1 and N1-N are called *accumulators*.

Accumulators

- Assume that the state *S* consists of a number of components to be transformed individually:

  $S = (X,Y,Z)$

- Assume P1 to Pn are procedures in Oz

$$
\begin{array}{l}
\textbf{proc}\ \{\mathrm{P}\ \overbrace{X_0\ X}^{\text{accumulator}}\ Y_0\ Y\ Z_0\ Z\} \\
\qquad\qquad \vdots \\
\qquad \{\mathrm{P1}\ X_0\ X_1\ Y_0\ Y_1\ Z_0\ Z_1\} \\
\qquad \{\mathrm{P2}\ X_1\ X_2\ Y_1\ Y_2\ Z_1\ Z_2\} \\
\qquad\qquad \vdots \\
\qquad \{\mathrm{Pn}\ X_{n-1}\ X\ Y_{n-1}\ Y\ Z_{n-1}\ Z\} \\
\textbf{end}
\end{array}
$$

The same concept applies to predicates in Prolog

- The procedural syntax is easier to use if there is more than one accumulator

MergeSort Example

- Consider a variant of MergeSort with accumulator
- proc {MergeSort1 N S0 S Xs}
  - N is an integer,
  - S0 is an input list to be sorted
  - S is the remainder of S0 after the first N elements are sorted
  - Xs is the sorted first N elements of S0
- The pair (S0, S) is an accumulator
- The definition is in a procedural syntax in Oz because it has two outputs S and Xs

Example (2)

```
fun {MergeSort Xs}
  {MergeSort1 {Length Xs} Xs _ Ys}
  Ys
end
```

```
proc {MergeSort1 N S0 S Xs}
  if N==0 then S = S0 Xs = nil
  elseif N ==1 then X in X|S = S0 Xs=[X]
  else %% N > 1
    local S1 Xs1 Xs2 NL NR in
     NL = N div 2
     NR = N - NL
     {MergeSort1 NL S0 S1 Xs1}
     {MergeSort1 NR S1 S Xs2}
     Xs = {Merge Xs1 Xs2}
   end
  end
end
```

MergeSort Example in Prolog

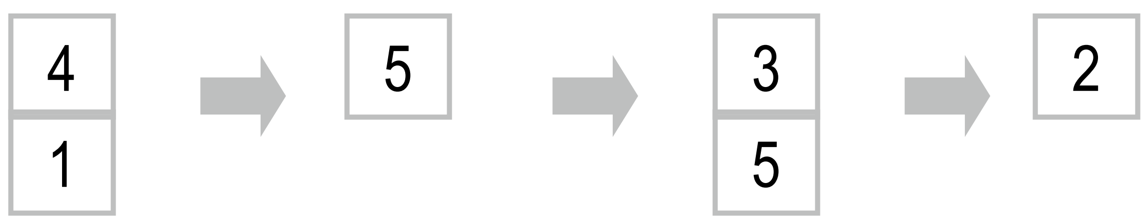
```
mergesort(Xs,Ys) :-
   length(Xs,N),
   mergesort1(N,Xs,_,Ys).
```

```
mergesort1(0,S,S,[]) :- !.
mergesort1(1,[X|S],S,[X]) :- !.
mergesort1(N,S0,S,Xs) :-
       NL is N // 2,
       NR is N - NL,
       mergesort1(NL,S0,S1,Xs1),
       mergesort1(NR,S1,S,Xs2),
       merge(Xs1,Xs2,Xs).
```

Multiple accumulators

- Consider a stack machine for evaluating arithmetic expressions
- Example: (1+4)-3
- The machine executes the following instructions

push(1)
push(4)
plus
push(3)
minus



Multiple accumulators (2)

- Example: (1+4)-3
- The arithmetic expressions are represented as trees:

  ```
  minus(plus(1 4) 3)
  ```

- Write a procedure that takes arithmetic expressions represented as trees and output a list of stack machine instructions and counts the number of instructions

  ```
  proc {ExprCode Expr Cin Cout Nin Nout}
  ```

- Cin: initial list of instructions
- Cout: final list of instructions
- Nin: initial count
- Nout: final count

Multiple accumulators (3)

```
proc {ExprCode Expr C0 C N0 N}
  case Expr
  of plus(Expr1 Expr2) then C1 N1 in
    C1 = plus|C0
    N1 = N0 + 1
    {SeqCode [Expr2 Expr1] C1 C N1 N}
  [] minus(Expr1 Expr2) then C1 N1 in
    C1 = minus|C0
    N1 = N0 + 1
    {SeqCode [Expr2 Expr1] C1 C N1 N}
  [] I andthen {IsInt I} then
    C = push(I)|C0
    N = N0 + 1
  end
end
```

Multiple accumulators (4)

```
proc {ExprCode Expr C0 C N0 N}
  case Expr
  of plus(Expr1 Expr2) then C1 N1 in
    C1 = plus|C0
    N1 = N0 + 1
    {SeqCode [Expr2 Expr1] C1 C N1 N}
  [] minus(Expr1 Expr2) then C1 N1 in
    C1 = minus|C0
    N1 = N0 + 1
    {SeqCode [Expr2 Expr1] C1 C N1 N}
  [] I andthen {IsInt I} then
    C = push(I)|C0
    N = N0 + 1
  end
end
```

```
proc {SeqCode Es C0 C N0 N}
  case Es
  of nil then C = C0 N = N0
  [] E|Er then N1 C1 in
    {ExprCode E C0 C1 N0 N1}
    {SeqCode Er C1 C N1 N}
  end
end
```

Shorter version (4)

```
proc {ExprCode Expr C0 C N0 N}
  case Expr
  of plus(Expr1 Expr2) then
    {SeqCode [Expr2 Expr1] plus|C0 C N0 + 1 N}
  [] minus(Expr1 Expr2) then
    {SeqCode [Expr2 Expr1] minus|C0 C N0 + 1 N}
  [] I andthen {IsInt I} then
    C = push(I)|C0
    N = N0 + 1
  end
end
```

```
proc {SeqCode Es C0 C N0 N}
  case Es
  of nil then C = C0 N = N0
  [] E|Er then N1 C1 in
    {ExprCode E C0 C1 N0 N1}
    {SeqCode Er C1 C N1 N}
  end
end
```

Functional style (4)

```
fun {ExprCode Expr t(C0 N0) }
  case Expr
  of plus(Expr1 Expr2) then
    {SeqCode [Expr2 Expr1] t(plus|C0 N0 + 1)}
  [] minus(Expr1 Expr2) then
    {SeqCode [Expr2 Expr1] t(minus|C0 N0 + 1)}
  [] I andthen {IsInt I} then
    t(push(I)|C0 N0 + 1)
  end
end
```

```
fun {SeqCode Es T}
  case Es
  of nil then T
  [] E|Er then
    T1 = {ExprCode E T} in
    {SeqCode Er T1}
  end
end
```

Difference lists in Oz

- A *difference list* is a pair of lists, each might have an unbound tail, with the invariant that one can get the second list by removing zero or more elements from the first list
- X # X % Represent the empty list
- nil # nil % idem
- [a] # [a] % idem
- (a|b|c|X) # X % Represents [a b c]
- [a b c d] # [d] % idem

Difference lists in Prolog

- A *difference list* is a pair of lists, each might have an unbound tail, with the invariant that one can get the second list by removing zero or more elements from the first list
- X , X % Represent the empty list
- [] , [] % idem
- [a] , [a] % idem
- [a,b,c|X] , X % Represents [a,b,c]
- [a,b,c,d] , [d] % idem

Difference lists in Oz (2)

- When the second list is unbound, an append operation with another difference list takes constant time
- ```
  fun {AppendD D1 D2}
        S1 # E1 = D1
        S2 # E2 = D2
  in    E1 = S2
        S1 # E2
  end
  ```
- `local X Y in {Browse {AppendD (1|2|3|X)#X (4|5|Y)#Y}} end`
- Displays (1|2|3|4|5|Y)#Y
  ```

Difference lists in Prolog (2)

- When the second list is unbound, an append operation with another difference list takes constant time

```
append_dl(S1,E1, S2,E2, S1,E2)  :-  E1 = S2.
```

- ?- append_dl([1,2,3|X],X, [4,5|Y],Y, S,E).

Displays

X = [4, 5|_G193]

Y = _G193

S = [1, 2, 3, 4, 5|_G193]

E = _G193 ;

A FIFO queue with difference lists (1)

- A *FIFO queue* is a sequence of elements with an insert and a delete operation.
  - Insert adds an element to one end and delete removes it from the other end
- Queues can be implemented with lists. If L represents the queue content, then inserting X gives X|L and deleting X gives {ButLast L X} (all elements but the last).
  - Delete is inefficient: it takes time proportional to the number of queue elements
- With difference lists we can implement a queue with constant-time insert and delete operations
  - The queue content is represented as q(N S E), where N is the number of elements and S#E is a difference list representing the elements

A FIFO queue with difference lists (2)

```
fun {NewQueue} X in q(0 X X) end

fun {Insert Q X}
   case Q of q(N S E) then E1 in E=X|E1 q(N+1 S E1) end
end

fun {Delete Q X}
   case Q of q(N S E) then S1 in X|S1=S q(N-1 S1 E) end
end

fun {EmptyQueue} case Q of q(N S E) then N==0 end end
```

- Inserting ‘b’:
  - In: q(1 a|T T)
  - Out: q(2 a|b|U U)
- Deleting X:
  - In: q(2 a|b|U U)
  - Out: q(1 b|U U) and X=a
- Difference list allows operations at both ends
- N is needed to keep track of the number of queue elements

Flatten (revisited)

```
fun {Flatten Xs}
case Xs
   of nil then nil
   [] X|Xr andthen {IsLeaf X} then
     X|{Flatten Xr}
   [] X|Xr  andthen {Not {IsLeaf X}} then
      {Append {Flatten X} {Flatten Xr}}
   end
end
```

Flatten takes a list of elements and sub-lists and returns a list with only the elements, e.g.:

{Flatten [1 [2] [[3]]]} = [1 2 3]

Let us replace lists by difference lists and see what happens.

Flatten with difference lists (1)

- Flatten of nil is X#X
- Flatten of X|Xr is Y1#Y where
  – flatten of X is Y1#Y2
  – flatten of Xr is Y3#Y
  – equate Y2 and Y3
- Flatten of a leaf X is (X|Y)#Y

Here is what it looks like as text

Flatten with difference lists (2)

```
proc {FlattenD Xs Ds}
case Xs
  of nil then Y in Ds = Y#Y
  [] X|Xr  then Y0 Y1 Y2 in
      Ds = Y0#Y2
     {FlattenD X Y0#Y1}
     {FlattenD Xr Y1#Y2}
  [] X andthen {IsLeaf X} then Y in (X|Y)#Y
  end
end
fun {Flatten Xs}  Y in {FlattenD Xs Y#nil}  Y end
```

Here is the new program. It is much more efficient than the first version.

Reverse (revisited)

- Here is our recursive reverse:

```
fun {Reverse Xs}
   case Xs
   of nil then nil
   [] X|Xr then {Append {Reverse Xr} [X]}
   end
end
```

- Rewrite this with difference lists:
  - Reverse of nil is X#X
  - Reverse of X|Xs is Y1#Y, where
    - reverse of Xs is Y1#Y2, and
    - equate Y2 and X|Y

Reverse with difference lists (1)

- The naive version takes time proportional to the square of the input length
- Using difference lists in the naive version makes it linear time
- We use two arguments Y1 and Y instead of Y1#Y
- With a minor change we can make it iterative as well

```
fun {ReverseD Xs}
   proc {ReverseD Xs Y1 Y}
      case Xs
      of nil then Y1=Y
      [] X|Xr then Y2 in
         {ReverseD Xr Y1 Y2}
         Y2 = X|Y
      end
   end
   R in
   {ReverseD Xs R nil}
   R
end
```

Reverse with difference lists (2)

```
fun {ReverseD Xs}
     proc {ReverseD Xs Y1 Y}
          case Xs
          of nil then Y1=Y
          [] X|Xr then
              {ReverseD Xr Y1 X|Y}
          end
     end
     R in
     {ReverseD Xs R nil}
     R
end
```

Difference lists: Summary

- Difference lists are a way to represent lists in the declarative model such that one append operation can be done in constant time
  - A function that builds a big list by concatenating together lots of little lists can usually be written efficiently with difference lists
  - The function can be written naively, using difference lists and append, and will be efficient when the append is expanded out
- Difference lists are declarative, yet have some of the power of destructive assignment
  - Because of the single-assignment property of dataflow variables
- Difference lists originated from Prolog and are used to implement, e.g., definite clause grammar rules for natural language parsing.

Exercises

15. Draw the search trees for Prolog queries:
    - `append([1,2],[3],L).`
    - `append(X,Y,[1,2,3]).`
    - `append_dl([1,2|X],X,[3|Y],Y,S,E).`
16. Rewrite the multiple accumulators example in Prolog.
17. VRH Exercise 3.10.11 (page 232)
18. VRH Exercise 3.10.14 (page 232)
19. VRH Exercise 3.10.15 (page 232)