Introduction to Programming Concepts (VRH 1.1-1.8)

Carlos Varela

RPI

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Adapted with permission from:

Seif Haridi

KTH

Peter Van Roy

UCL

Introduction

- An introduction to programming concepts
- Declarative variables
- Functions
- Structured data (example: lists)
- Functions over lists
- Correctness and complexity
- Lazy functions
- Concurrency and dataflow
- State, objects, and classes
- Nondeterminism and atomicity

Variables

- Variables are short-cuts for values, they cannot be assigned more than once

```
declare
V = 9999*9999
{Browse V*V}
```

- Variable identifiers: is what you type
- Store variable: is part of the memory system
- The **declare** statement creates a store variable and assigns its memory address to the identifier 'V' in the environment

Functions

- Compute the factorial function:
- Start with the mathematical definition

```
declare
fun {Fact N}
   if N==0 then 1 else N*{Fact N-1} end
end
```

- Fact is declared in the environment
- Try large factorial {Browse {Fact 100}}

$$n! = 1 \times 2 \times \cdots \times (n-1) \times n$$

$$0! = 1$$

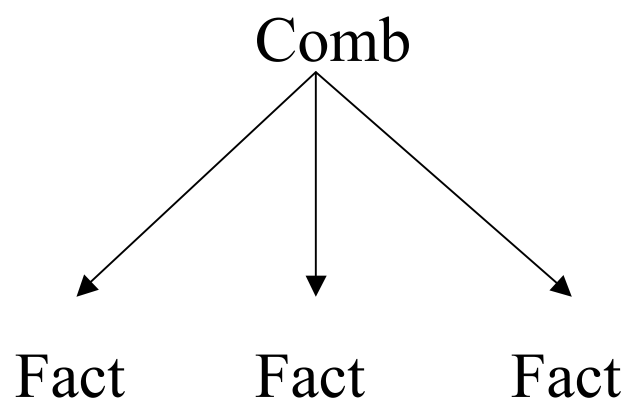
$$n! = n \times (n-1)! \text{ if } n > 0$$

Composing functions

- Combinations of r items taken from n.
- The number of subsets of size r taken from a set of size n

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

```
declare
fun {Comb N R}
   {Fact N} div ({Fact R}*{Fact N-R})
end
```



- Example of functional abstraction

Structured data (lists)

- Calculate Pascal triangle
- Write a function that calculates the nth row as one structured value
- A list is a sequence of elements:

  [1 4 6 4 1]
- The empty list is written nil
- Lists are created by means of "|" (cons)

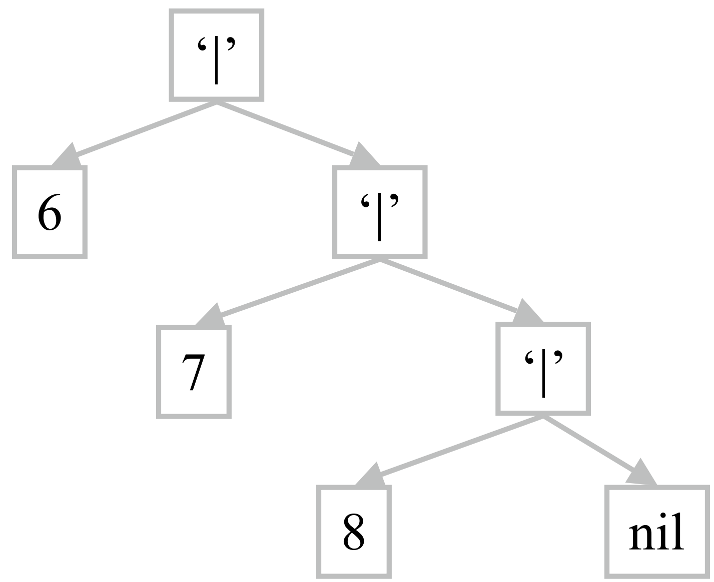
```
declare
H=1
T = [2 3 4 5]
{Browse H|T}  % This will show [1 2 3 4 5]
```

```
        1
      1   1
    1   2   1
  1   3   3   1
1   4   6   4   1
```

Lists (2)

- Taking lists apart (selecting components)
- A cons has two components: a head, and a tail

```
declare L = [5 6 7 8]
L.1 gives 5
L.2 give [6 7 8]
```



Pattern matching

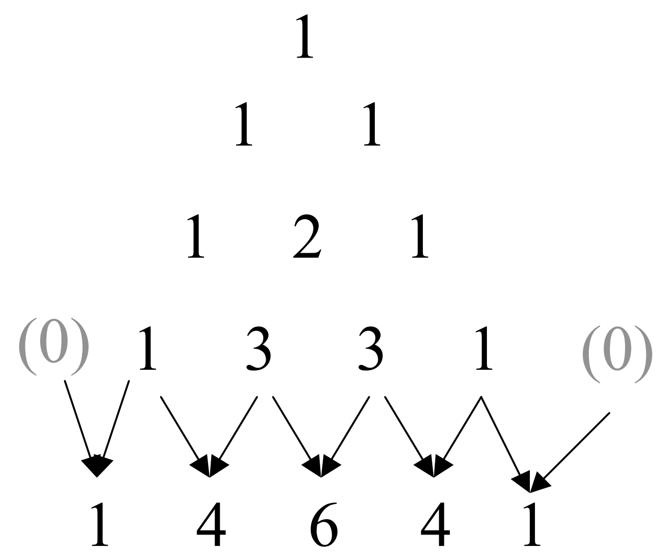
- Another way to take a list apart is by use of pattern matching with a case instruction

```
case L of H|T then {Browse H} {Browse T}
          else {Browse 'empty list'}
end
```

Functions over lists

- Compute the function {Pascal N}
- Takes an integer N, and returns the Nth row of a Pascal triangle as a list

1. For row 1, the result is [1]
2. For row N, shift to left row N-1 and shift to the right row N-1
3. Align and add the shifted rows element-wise to get row N

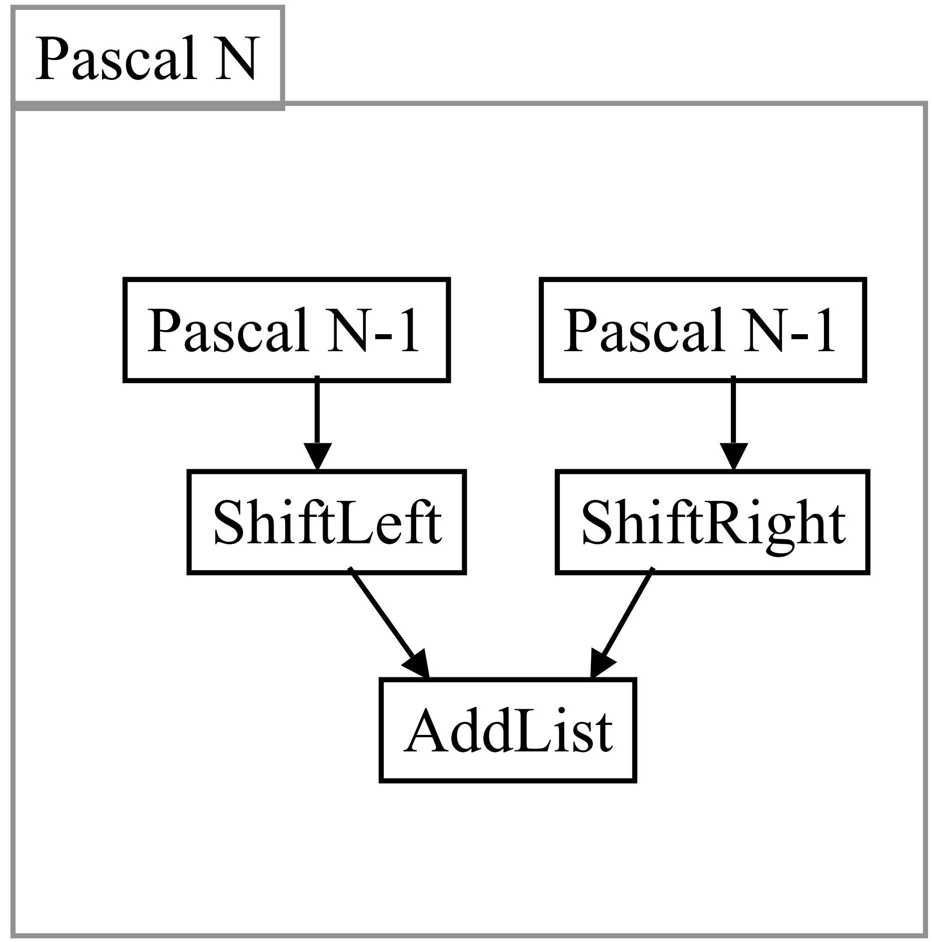


Shift right [0 1 3 3 1]

Shift left [1 3 3 1 0]

Functions over lists (2)

```
declare
fun {Pascal N}
   if N==1 then [1]
   else
      {AddList
       {ShiftLeft {Pascal N-1}}
       {ShiftRight {Pascal N-1}}}
   end
end
```



Functions over lists (3)

```
fun {ShiftLeft L}
   case L of H|T then
      H|{ShiftLeft T}
   else [0]
   end
end

fun {ShiftRight L}  0|L end
```

```
fun {AddList L1 L2}
   case L1 of H1|T1 then
      case L2 of H2|T2 then
         H1+H2|{AddList T1 T2}
      end
   else nil end
end
```

Top-down program development

- Understand how to solve the problem by hand
- Try to solve the task by decomposing it to simpler tasks
- Devise the main function (main task) in terms of suitable auxiliary functions (subtasks) that simplify the solution (ShiftLeft, ShiftRight and AddList)
- Complete the solution by writing the auxiliary functions

Is your program correct?

- "A program is correct when it does what we would like it to do"
- In general we need to reason about the program:
- **Semantics for the language**: a precise model of the operations of the programming language
- **Program specification**: a definition of the output in terms of the input (usually a mathematical function or relation)
- Use mathematical techniques to reason about the program, using programming language semantics

Mathematical induction

- Select one or more inputs to the function
- Show the program is correct for the *simple cases* (base cases)
- Show that if the program is correct for a *given case*, it is then correct for the *next case*.
- For natural numbers, the base case is either 0 or 1, and for any number n the next case is n+1
- For lists, the base case is nil, or a list with one or a few elements, and for any list T the next case is H|T

Correctness of factorial

```
fun {Fact N}
  if N==0 then 1 else N*{Fact N-1} end
end
```

$$\underbrace{1 \times 2 \times \cdots \times (n-1)}_{Fact(n-1)} \times n$$

- Base Case N=0: {Fact 0} returns 1
- Inductive Case N>0: {Fact N} returns N*{Fact N-1} assume {Fact N-1} is correct, from the spec we see that {Fact N} is N*{Fact N-1}

Complexity

- Pascal runs very slow, try {Pascal 24}
- {Pascal 20} calls: {Pascal 19} twice, {Pascal 18} four times, {Pascal 17} eight times, ..., {Pascal 1} 2^{19} times
- Execution time of a program up to a constant factor is called the program's *time complexity*.
- Time complexity of {Pascal N} is proportional to 2^N (exponential)
- Programs with exponential time complexity are impractical

```
declare
fun {Pascal N}
   if N==1 then [1]
   else
      {AddList
       {ShiftLeft {Pascal N-1}}
       {ShiftRight {Pascal N-1}}}
   end
end
```

Faster Pascal

- Introduce a local variable L
- Compute {FastPascal N-1} only once
- Try with 30 rows.
- FastPascal is called N times, each time a list on the average of size N/2 is processed
- The time complexity is proportional to N^2 (polynomial)
- Low order polynomial programs are practical.

```
fun {FastPascal N}
  if N==1 then [1]
  else
      local L in
         L={FastPascal N-1}
         {AddList {ShiftLeft L} {ShiftRight L}}
      end
  end
end
```

Lazy evaluation

- The functions written so far are evaluated eagerly (as soon as they are called)
- Another way is lazy evaluation where a computation is done only when the result is needed

- Calculates the infinite list:
  0 | 1 | 2 | 3 | ...

```
declare
fun lazy {Ints N}
   N|{Ints N+1}
end
```

Lazy evaluation (2)

- Write a function that computes as many rows of Pascal's triangle as needed
- We do not know how many beforehand
- A function is *lazy* if it is evaluated only when its result is needed
- The function PascalList is evaluated when needed

```
fun lazy {PascalList Row}
   Row | {PascalList
            {AddList
               {ShiftLeft Row}
               {ShiftRight Row}}}
end
```

Lazy evaluation (3)

- Lazy evaluation will avoid redoing work if you decide first you need the 10th row and later the 11th row
- The function continues where it left off

```
declare
L = {PascalList [1]}
{Browse L}
{Browse L.1}
{Browse L.2.1}
```

```
L<Future>
[1]
[1 1]
```

Exercises

24. Lambda Calculus Handout Exercise 7.
25. Lambda Calculus Handout Exercise 9.
26. Lambda Calculus Handout Exercise 11.
27. Lambda Calculus Handout Exercise 12.

Exercises

28. Define Add in Oz using the Zero and Succ functions representing numbers in the lambda-calculus:

```
Zero = fun {$ X} X end
Succ = fun {$ N} fun {$ X} N end
```

29. Prove that Add is correct using induction.
30. Prove the correctness of AddList and ShiftLeft using induction.
31. VRH Exercise 1.18.5.