Declarative Computation Model

Kernel language semantics

Basic concepts, the abstract machine (VRH 2.4.1-2.4.2)

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Sequential declarative computation model

- The single assignment store
  - declarative (dataflow) variables
  - partial values (variables and values are also called *entities*)
- The kernel language syntax
- The kernel language semantics
  - The environment: maps textual variable names (variable identifiers) into entities in the store
  - Interpretation (execution) of the kernel language elements (statements) by the use of an abstract machine
  - Abstract machine consists of an execution stack of statements transforming the store

Kernel language syntax

The following defines the syntax of a statement, ⟨s⟩ denotes a statement

⟨s⟩	::=	skip	*empty statement*
	|	⟨x⟩ = ⟨y⟩	*variable-variable binding*
	|	⟨x⟩ = ⟨v⟩	*variable-value binding*
	|	$\langle s_1 \rangle$ $\langle s_2 \rangle$	*sequential composition*
	|	local ⟨x⟩ in $\langle s_1 \rangle$ end	*declaration*
	|	if ⟨x⟩ then $\langle s_1 \rangle$ else $\langle s_2 \rangle$ end	*conditional*
	|	{ ⟨x⟩ $\langle y_1 \rangle$ … $\langle y_n \rangle$ }	*procedural application*
	|	case ⟨x⟩ of ⟨pattern⟩ then $\langle s_1 \rangle$ else $\langle s_2 \rangle$ end	*pattern matching*

⟨v⟩ ::= proc { \$ $\langle y_1 \rangle$ … $\langle y_n \rangle$ } $\langle s_1 \rangle$ end | … *value expression*

⟨pattern⟩ ::= ...

Examples

- local X in X = 1 end

- ```
  local X Y T Z in
      X = 5
      Y = 10
      T = (X>=Y)
      if T then Z = X else Z = Y end
      {Browse Z}
  end
  ```

- ```
  local S T in
      S = proc {$ X Y} Y = X*X end
      {S 5 T}
      {Browse T}
   end
  ```

Procedure abstraction

- Any statement can be abstracted to a procedure by selecting a number of the ’free’ variable identifiers and enclosing the statement into a procedure with the identifiers as parameters
- `if X >= Y then Z = X else Z = Y end`
- Abstracting over all variables

```
proc {Max X Y Z}
     if X >= Y then Z = X else Z = Y end
end
```

- Abstracting over X and Z

```
proc {LowerBound X Z}
     if X >= Y then Z = X else Z = Y end
end
```

Computations (abstract machine)

- A computation defines how the execution state is transformed step by step from the initial state to the final state
- A *single assignment store* σ is a set of store variables, a variable may be unbound, bound to a partial value, or bound to a group of other variables
- An *environment E* is mapping from variable identifiers to variables or values in σ, e.g. $\{X \rightarrow x_1, Y \rightarrow x_2\}$
- A *semantic statement* is a pair

  $(\langle s \rangle, E)$ where $\langle s \rangle$ is a statement
- *ST* is a stack of semantic statements

Computations (abstract machine)

- A computation defines how the execution state is transformed step by step from the initial state to the final state
- The *execution state* is a pair

  (ST , σ)
- *ST* is a stack of semantic statements
- A *computation* is a sequence of execution states
  $(ST_0 , \sigma_0) \rightarrow (ST_1 , \sigma_1) \rightarrow (ST_2 , \sigma_2) \rightarrow \ldots$

Semantics

- To execute a program (i.e., a statement) $\langle s \rangle$ the initial execution state is

  $$(\, [\, (\langle s \rangle \, , \varnothing) \,] \, , \varnothing \,)$$

- ST has a single semantic statement $(\langle s \rangle \, , \varnothing)$
- The environment E is empty, and the store σ is empty
- [...] denotes the stack
- At each step the first element of ST is popped and execution proceeds according to the form of the element
- The final execution state (if any) is a state in which ST is empty

skip

- The semantic statement is
  (skip, E)
- Continue to next execution step

skip

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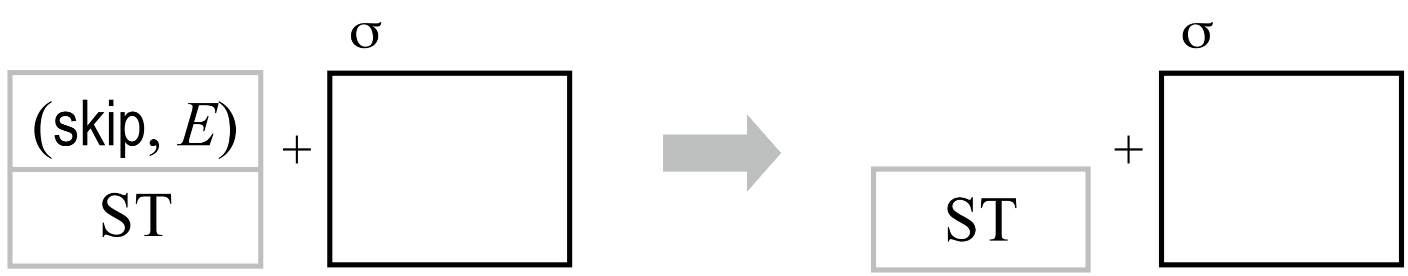
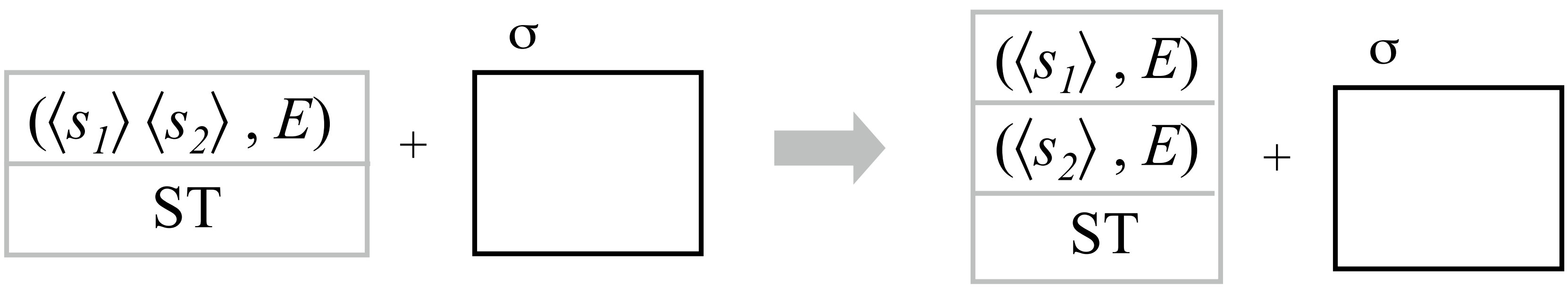
  (skip, E)
- Continue to next execution step



Sequential composition

- The semantic statement is
  $(\langle s_1 \rangle \langle s_2 \rangle , E)$
- Push $(\langle s_2 \rangle , E)$ and then push $(\langle s_1 \rangle , E)$ on ST
- Continue to next execution step



Calculating with environments

- E is mapping from identifiers to entities (both store variables and values) in the store
- The notation $E(\langle \mathrm{y} \rangle)$
  retrieves the entity x associated with the identifier $\langle y \rangle$ from the store
- The notation $E + \{\langle \mathrm{y} \rangle_1 \rightarrow x_1, \langle \mathrm{y} \rangle_2 \rightarrow x_2, \ldots, \langle \mathrm{y} \rangle_n \rightarrow x_n\}$
  - denotes a new environment E' constructed from E by adding the mappings
    $\{\langle \mathrm{y} \rangle_1 \rightarrow x_1, \langle \mathrm{y} \rangle_2 \rightarrow x_2, \ldots, \langle \mathrm{y} \rangle_n \rightarrow x_n\}$
  - $E'(\langle \mathrm{z} \rangle)$ is x_k if $\langle \mathrm{z} \rangle$ is equal to $\langle \mathrm{y} \rangle_k$, otherwise $E'(\langle \mathrm{z} \rangle)$ is equal to $E(\langle \mathrm{z} \rangle)$
- The notation $E|_{\{\langle \mathrm{y} \rangle_1, \langle \mathrm{y} \rangle_2, \ldots, \langle \mathrm{y} \rangle_n\}}$ denotes the projection of E onto the set $\{\langle \mathrm{y} \rangle_1, \langle \mathrm{y} \rangle_2, \ldots, \langle \mathrm{y} \rangle_n\}$, i.e., E restricted to the members of the set

Calculating with environments (2)

- $E = \{\mathsf{X} \rightarrow 1, \mathsf{Y} \rightarrow [2\ 3], \mathsf{Z} \rightarrow x_i\}$
- $E' = E + \{\mathsf{X} \rightarrow 2\}$
- $E'(\mathsf{X}) = 2$,
  $E(\mathsf{X}) = 1$
- $E|_{\{X,Y\}}$ restricts E to the 'domain' $\{X,Y\}$,
  i.e., it is equal to $\{\mathsf{X} \rightarrow 1, \mathsf{Y} \rightarrow [2\ 3]\}$

Calculating with environments (3)

```
local X in
    X = 1              (E)
    local X in
        X = 2          (E')
        {Browse X}
    end                (E)
    {Browse X}
end
```

Lexical scoping

- Free and bound identifier occurrences
- An identifier occurrence is *bound* with respect to a statement ⟨s⟩ if it is in the scope of a declaration inside ⟨s⟩
- A variable identifier is declared either by a 'local' statement, as a parameter of a procedure, or implicitly declared by a case statement
- An identifier occurrence is *free* otherwise
- In a running program every identifier is bound (i.e., declared)

Lexical scoping (2)

- proc {P X}
  local Y in Y = 1 {Browse Y} end
  X = Y
  end

Free Occurrences

Bound Occurrences

Lexical scoping (3)

- ```
  local Arg1 Arg2 in
      Arg1 = 111*111
      Arg2 = 999*999
      Res = Arg1*Arg2
  end
  ```

Free Occurrences

Bound Occurrences

This is not a runnable program!
  ```

Lexical scoping (4)

- ```
  local Res in
      local Arg1 Arg2 in
          Arg1 = 111*111
          Arg2 = 999*999
          Res = Arg1*Arg2
      end
      {Browse Res}
  end
  ```
  ```

Lexical scoping (5)

```
local P Q in
   proc {P}  {Q} end
   proc {Q} {Browse hello} end
   local Q in
        proc {Q} {Browse hi} end
        {P}
   end
end
```

Exercises

42. Translate the following function to the kernel language:

```
fun {AddList L1 L2}
  case L1 of H1|T1 then
    case L2 of H2|T2 then
        H1+H2|{AddList T1 T2}
    end
  else nil end
end
```

43. Translate the following function call to the kernel language:

```
{Browse {Max 5 7}}
```

Exercises

44. Explain the difference between static scoping and dynamic scoping. Give an example program that produces different results with static and dynamic scoping.
45. Think of a reason why static scoping may be preferable to dynamic scoping. Think of a reason why dynamic scoping may be preferable to static scoping.