Declarative Computation Model

Single assignment store (VRH 2.2)
Kernel language syntax (VRH 2.3)

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Sequential declarative computation model

- The **single assignment store**
  - declarative (dataflow) variables
  - partial values (variables and values are also called *entities*)
- The **kernel language syntax**
- The **kernel language semantics**
  - The environment: maps textual variable names (variable identifiers) into entities in the store
  - Interpretation (execution) of the kernel language elements (statements) by the use of an abstract machine
  - Abstract machine consists of an execution stack of statements transforming the store
Single assignment store

- A single assignment store is a store (set) of variables
- Initially the variables are unbound, i.e. do not have a defined value
- Example: a store with three variables, \( x_1 \), \( x_2 \), and \( x_3 \)
Single assignment store (2)

- Variables in the store may be bound to values
- Example: assume we allow as values, integers and lists of integers
Single assignment store (3)

- Variables in the store may be bound to values
- Assume we allow as values, integers and lists of integers
- Example: $x_1$ is bound to the integer 314, $x_2$ is bound to the list [1 2 3], and $x_3$ is still unbound
Declarative (single-assignment) variables

- A declarative variable starts out as being unbound when created
- It can be bound to exactly one value
- Once bound it stays bound through the computation, and is indistinguishable from its value

The Store

\[
\begin{align*}
x_1 & \text{ 314} \\
x_2 & \text{ 1 | } 2 | 3 | \text{ nil} \\
x_3 & \text{ unbound}
\end{align*}
\]
Value store

- A store where all variables are bound to values is called a value store.
- Example: a value store where $x_1$ is bound to integer 314, $x_2$ to the list [1 2 3], and $x_3$ to the record (labeled tree) `person(name: “George” age: 25)`.
- Functional programming computes functions on values, needs only a value store.
- This notion of value store is enough for functional programming (ML, Haskell, Scheme).
Operations on the store (1)

Single assignment

\[ \langle x \rangle = \langle v \rangle \]

- \( x_1 = 314 \)
- \( x_2 = [1 \ 2 \ 3] \)
- This assumes that \( \langle x \rangle \) is unbound
Single-assignment

\[ \langle x \rangle = \langle \text{value} \rangle \]
- \( x_1 = 314 \)
- \( x_2 = [1 \ 2 \ 3] \)
\[
\langle x \rangle = \langle v \rangle \\
\text{• } x_1 = 314 \\
\text{• } x_2 = [1 \ 2 \ 3] \\
\text{• } The \ single \ assignment \ operation \ (‘=’) \ constructs \ the \ \langle v \rangle \ in \ the \ store \ and \ binds \ the \ variable \ \langle x \rangle \ to \ this \ value} \\
\text{• } If \ the \ variable \ is \ already \ bound, \ the \ operation \ will \ test \ the \ compatibility \ of \ the \ two \ values} \\
\text{• } If \ the \ test \ fails \ an \ error \ is \ raised
\]
Variable identifiers

- Variable identifiers refers to store entities (variables or values)
- The environment maps variable identifiers to variables
- `declare X`
- `local X in ...`
- "X" is a (variable) identifier
- This corresponds to 'environment' `{"X" → x₁}`
Variable-value binding revisited (1)

- $X = [1 \ 2 \ 3]$
- Once bound the variable is indistinguishable from its value
Variable-value binding revisited (2)

- \( X = [1 \ 2 \ 3] \)
- Once bound the variable is indistinguishable from its value
- The operation of traversing variable cells to get the value is known as dereferencing and is invisible to the programmer

\[
\begin{array}{c}
\text{The Store} \\
\end{array}
\]

\[
\begin{array}{c}
\text{“X”} \\
1 | \rightarrow \ 2 | \rightarrow \ 3 | \text{nil} \\
x_1 \\
\end{array}
\]
Partial Values

- A partial value is a data structure that may contain unbound variables.
- The store contains the partial value: `person(name: “George” age: \(x_2\))`.
- `declare Y X` where `X = person(name: “George” age: Y)`.
- The identifier ’Y’ refers to \(x_2\).
Partial Values may be complete

- declare Y X
  X = person(name: “George” age: Y)
- Y = 25
Variable to variable binding

\[ \langle x_1 \rangle = \langle x_2 \rangle \]

- It is to perform the bind operation between variables
- Example:
  - \( X = Y \)
  - \( X = [1 \ 2 \ 3] \)
- The operations equates (merges) the two variables
Variable to variable binding (2)

$\langle x_1 \rangle = \langle x_2 \rangle$

- It is to perform a single assignment between variables
- Example:
  - $X = Y$
  - $X = [1 \ 2 \ 3]$
- The operations equates the two variables (forming an equivalence class)
Variable to variable binding (3)

\[ \langle x_1 \rangle = \langle x_2 \rangle \]

- It is to perform a single assignment between variables
- Example:
  - \( X = Y \)
  - \( X = [1 \ 2 \ 3] \)
- All variables (X and Y) are bound to \([1 \ 2 \ 3]\)
Summary

Variables and partial values

- **Declarative variable:**
  - is an entity that resides in a single-assignment store, that is initially unbound, and can be bound to exactly one (partial) value
  - it can be bound to several (partial) values as long as they are compatible with each other

- **Partial value:**
  - is a data-structure that may contain unbound variables
  - when one of the variables is bound, it is replaced by the (partial) value it is bound to
  - a complete value, or value for short is a data structure that does not contain any unbound variables
Declaration and use of variables

• Assume that variables can be declared (introduced) and used separately
• What happens if we try to use a variable before it is bound?
  1. Use whatever value happens to be in the memory cell occupied by the variable (C, C++)
  2. The variable is initialized to a default value (Java, SALSA), use the default
  3. An error is signaled (Prolog). Makes sense if there is a single activity running (pure sequential programs)
  4. An attempt to use the variable will wait (suspends) until another activity binds the variable (Oz/Mozart)
Declaration and use of variables (2)

- An attempt to use the variable will wait (suspends) until another activity binds the variable (Oz/Mozart)
- Declarative (single assignment) variables that have this property are called *dataflow* variables
- It allows multiple operations to proceed concurrently giving the correct result
- Example: \( A = 23 \) running concurrently with \( B = A+1 \)
- Functional (concurrent) languages do not allow the separation between declaration and binding (ML, Haskell, and Erlang)
Kernel language syntax

The following defines the syntax of a statement, $\langle s \rangle$ denotes a statement

\[
\langle s \rangle ::= \text{skip} \quad \text{empty statement}
\]
\[
| \langle x \rangle = \langle y \rangle \quad \text{variable-variable binding}
\]
\[
| \langle x \rangle = \langle v \rangle \quad \text{variable-value binding}
\]
\[
| \langle s_1 \rangle \langle s_2 \rangle \quad \text{sequential composition}
\]
\[
| \text{local } \langle x \rangle \text{ in } \langle s_1 \rangle \text{ end} \quad \text{declaration}
\]
\[
| \text{if } \langle x \rangle \text{ then } \langle s_1 \rangle \text{ else } \langle s_2 \rangle \text{ end} \quad \text{conditional}
\]
\[
| \{' \langle x \rangle \langle y_1 \rangle \ldots \langle y_n \rangle \}' \quad \text{procedural application}
\]
\[
| \text{case } \langle x \rangle \text{ of } \langle \text{pattern} \rangle \text{ then } \langle s_1 \rangle \text{ else } \langle s_2 \rangle \text{ end} \quad \text{pattern matching}
\]

\[
\langle v \rangle ::= ... \quad \text{value expression}
\]

\[
\langle \text{pattern} \rangle ::= ... \quad 
\]

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Variable identifiers

• \( \langle x \rangle \), \( \langle y \rangle \), \( \langle z \rangle \) stand for variables
• In the concrete kernel language variables begin with upper-case letter followed by a (possibly empty) sequence of alphanumeric characters or underscore
• Any sequence of printable characters within back-quote
• Examples:
  - X
  - Y1
  - Hello_World
  - `hello this is a $5 bill` (back-quote)
Values and types

- A *data type* is a set of values and a set of associated operations
- Example: `Int` is the data type "Integer", i.e. set of all integer values
- 1 is *of type* `Int`
- `Int` has a set of operations including +,-,*,div, etc
- The model comes with a set of basic types
- Programs can define other types, e.g., *abstract data types* (ADT)
Data types

- **Value**
  - **Number**
    - Int
    - Float
    - Char
  - **Record**
  - **Procedure**
  - **Tuple**
    - **Literal**
      - Atom
    - **Boolean**
      - true
      - false
    - List
      - String
Data types (2)
Value expressions

\[
\langle v \rangle ::= \langle \text{procedure} \rangle | \langle \text{record} \rangle | \langle \text{number} \rangle
\]

\[
\langle \text{procedure} \rangle ::= \text{proc} \{ \$ \langle y_1 \rangle \ldots \langle y_n \rangle \} \langle s \rangle \text{ end}
\]

\[
\langle \text{record}, \langle \text{pattern} \rangle \rangle ::= \langle \text{literal} \rangle \\
| \langle \text{literal} \rangle \langle \langle \text{feature}_1 \rangle : \langle x_1 \rangle \ldots \langle \text{feature}_n \rangle : \langle x_n \rangle \rangle \rangle
\]

\[
\langle \text{literal} \rangle ::= \langle \text{atom} \rangle | \langle \text{bool} \rangle
\]

\[
\langle \text{feature} \rangle ::= \langle \text{int} \rangle | \langle \text{atom} \rangle | \langle \text{bool} \rangle
\]

\[
\langle \text{bool} \rangle ::= \text{true} | \text{false}
\]

\[
\langle \text{number} \rangle ::= \langle \text{int} \rangle | \langle \text{float} \rangle
\]

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Numbers

• Integers
  – 314, 0
  – ~10 (minus 10)

• Floats
  – 1.0, 3.4, 2.0e2, 2.0E2 (2×10^2)
Atoms and booleans

• A sequence starting with a lower-case character followed by characters or digits, …
  – person, peter
  – ‘Seif Haridi’

• Booleans:
  – true
  – false
Records

• Compound representation (data-structures)
  - \langle l \rangle (\langle f_1 : x_1 \rangle \ldots \langle f_n : x_n \rangle)
  - \langle l \rangle is a literal

• Examples
  - person(age:X1 name:X2)
  - person(1:X1 2:X2)
  - ‘\|’(1:H 2:T)
  - nil
  - person
Syntactic sugar (tuples)

- Tuples
  \[ \langle l \rangle (\langle x_1 \rangle \ldots \langle x_n \rangle) \]  
  (tuple)
- This is equivalent to the record
  \[ \langle l \rangle (1: \langle x_1 \rangle \ldots n: \langle x_n \rangle) \]
- Example:
  \[ \text{person('George' 25)} \]
- This is the record
  \[ \text{person(1:'George' 2:25)} \]
Syntactic sugar (lists)

- Lists
  \( \langle x_1 \rangle | \langle x_2 \rangle \) (a cons with the infix operator ‘|’)
- This is equivalent to the tuple
  ‘|’(\( \langle x_1 \rangle \langle x_2 \rangle \))

- Example:
  \( H | T \)
- This is the tuple
  ‘|’(\( H \ T \))
Syntactic sugar (lists)

• Lists
  \( \langle x_1 \rangle | \langle x_2 \rangle | \langle x_3 \rangle \)
• ‘|’ associates to the right
  \( \langle x_1 \rangle | (\langle x_2 \rangle | \langle x_3 \rangle) \)

• Example:
  1 | 2 | 3 | nil
• Is
  1 | (2 | (3 | nil))
Syntactic sugar (complete lists)

- Complete lists
- Example:
  \[ [1 \ 2 \ 3] \]
- Is
  \[ 1 \mid (2 \mid (3 \mid \text{nil} )) \]
Strings

- A string is a list of character codes enclosed with double quotes
- Ex: "E=mc^2"
- Means the same as [69 61 109 99 94 50]
Procedure declarations

- According to the kernel language
  \[ \langle x \rangle = \text{proc} \{ \langle y_1 \rangle \ldots \langle y_n \rangle \} \langle s \rangle \text{ end} \]
  is a legal statement
- It binds \( \langle x \rangle \) to a procedure value
- This statement actually declares (introduces) a procedure
- Another syntactic variant which is more familiar is
  \[ \text{proc} \{ \langle x \rangle \langle y_1 \rangle \ldots \langle y_n \rangle \} \langle s \rangle \text{ end} \]
  This introduces (declares) the procedure \( \langle x \rangle \)
Operations of basic types

• Arithmetics
  – Floating point numbers: +,-,*, and /  
  – Integers: +,-,*,div (integer division, i.e. truncate fractional part), mod (the remainder after a division, e.g. 10 mod 3 = 1)

• Record operations
  – Arity, Label, and "."
  – X = person(name:"George" age:25)
  – {Arity X} = [age name]
  – {Label X} = person, X.age = 25

• Comparisons
  – Boolean comparisons, including ==, \= (equality)
  – Numeric comparisons, =\<, \<, \>, \>=, compares integers, floats, and atoms
Value expressions

\langle v \rangle ::= \langle \text{procedure} \rangle \mid \langle \text{record} \rangle \mid \langle \text{number} \rangle \mid \langle \text{basicExpr} \rangle

\langle \text{basicExpr} \rangle ::= \ldots \mid \langle \text{numberExpr} \rangle \mid \ldots

\langle \text{numberExpr} \rangle ::= \langle x \rangle_1 + \langle x \rangle_2 \mid \ldots

\ldots
Syntactic sugar (multiple variables)

- Multiple variable introduction

\[
\text{local } X \ Y \text{ in } \langle \text{statement} \rangle \text{ end}
\]

- is transformed to

\[
\begin{align*}
\text{local } X \text{ in} \\
\text{local } Y \text{ in } \langle \text{statement} \rangle \text{ end} \\
\text{end}
\end{align*}
\]
Syntactic sugar (basic expressions)

• Basic expression nesting

\[
\text{if } \langle \text{basicExpr} \rangle \ \text{then } \langle \text{statement} \rangle_1 \ \text{else } \langle \text{statement} \rangle_2 \ \text{end}
\]

• is transformed to

\[
\text{local } T \ \text{in} \\
T = \langle \text{basicExpr} \rangle \\
\text{if } T \ \text{then } \langle \text{statement} \rangle_1 \ \text{else } \langle \text{statement} \rangle_2 \ \text{end}
\]

• where T is a fresh ('new') variable identifier
Syntactic sugar (variables)

- Variable initialization

  \[
  \text{local } X = \langle \text{value} \rangle \text{ in } \langle \text{statement} \rangle \text{ end}
  \]

- Is transformed to

  \[
  \text{local } X \text{ in }
  \begin{align*}
  X &= \langle \text{value} \rangle \\
  \langle \text{statement} \rangle \\
  \text{end}
  \end{align*}
  \]
Exercises

38. Using Oz, perform a few basic operations on numbers, records, and booleans (see Appendix B1-B3)

39. Explain the behavior of the declare statement in the interactive environment. Give an example of an interactive Oz session where “declare” and “declare ... in” produce different results. Explain why.

40. VRH Exercise 2.9.1

41. Describe what an anonymous procedure is, and write one in Oz. When are anonymous procedures useful?