Declarative Computation Model

Kernel language semantics revisited (VRH 2.4.5)

From kernel to practical language (VRH 2.6)

Exceptions (VRH 2.7)

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Adapted with permission from:
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Sequential declarative computation model

- The kernel language semantics revisited.
  - Suspendable statements:
    - if,
    - case,
    - procedure application.
  - Procedure values
  - Procedure introduction
  - Procedure application.

Conditional

- The semantic statement is
  (if $\langle x\rangle$ then $\langle s_1\rangle$ else $\langle s_2\rangle$ end , E)
- If the activation condition ($E(\langle x\rangle)$ is determined) is true:
  - If $E(\langle x\rangle)$ is not Boolean (true, false), raise an error
  - $E(\langle x\rangle)$ is true, push ($\langle s_1\rangle$, E) on the stack
  - $E(\langle x\rangle)$ is false, push ($\langle s_2\rangle$, E) on the stack
- If the activation condition ($E(\langle x\rangle)$ is determined) is false:
  - Suspend

Case statement

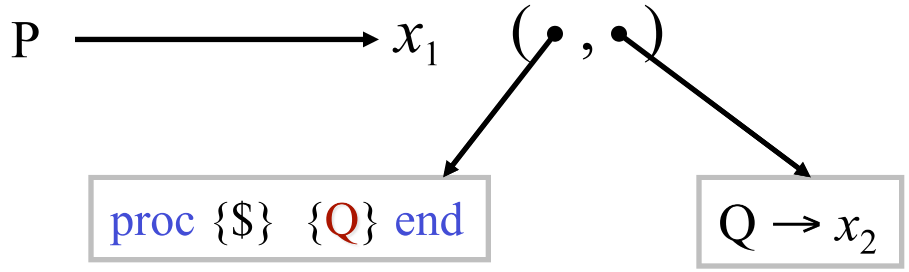
- The semantic statement is
  (case $\langle \mathsf{x} \rangle$ of $\langle l \rangle$ ($\langle f_1 \rangle : \langle x_1 \rangle \ldots \langle f_n \rangle : \langle x_n \rangle$)
  then $\langle \mathsf{s}_1 \rangle$
  else $\langle \mathsf{s}_2 \rangle$ end , E)
- If the activation condition ($E(\langle \mathsf{x} \rangle)$ is determined) is true:
  - If $E(\langle x \rangle)$ is a record, the label of $E(\langle x \rangle)$ is $\langle l \rangle$ and its arity is [$\langle f_1 \rangle \ldots \langle f_n \rangle$], and $\langle x_1 \rangle \ldots \langle x_n \rangle$ are independent:
    push (local $\langle x_1 \rangle = \langle x \rangle . \langle f_1 \rangle \ldots \langle x_n \rangle = \langle x \rangle . \langle f_n \rangle$ in $\langle \mathsf{s}_1 \rangle$ end, E)
    on the stack
  - Otherwise, push ($\langle \mathrm{s}_2 \rangle$, E) on the stack
- If the activation condition ($E(\langle \mathsf{x} \rangle)$ is determined) is false:
  - Suspend

Procedure values

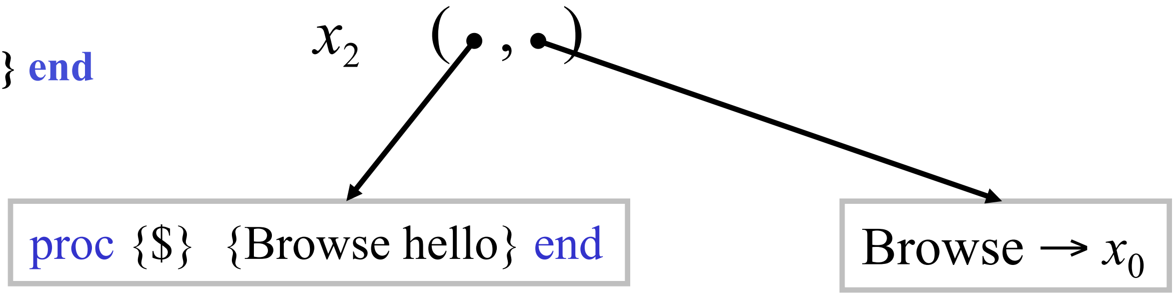
- Constructing a procedure value in the store is not simple because a procedure may have external references

```
local P Q in
   Q = proc {$} {Browse hello} end
   P = proc {$}  {Q} end
   local Q in
         Q = proc {$} {Browse hi} end
         {P}
   end
end
```

Procedure values (2)



```
local P Q in
      Q = proc {$} {Browse hello} end
      P = proc {$}  {Q} end
      local Q in
            Q = proc {$} {Browse hi} end
            {P}
      end
end
```



Procedure values (3)

- The semantic statement is
  $(\langle x \rangle = \texttt{proc}\ \{\$\ \langle y_1 \rangle \ldots \langle y_n \rangle\}\ \langle s \rangle\ \texttt{end},\ E)$
- $\langle y_1 \rangle \ldots \langle y_n \rangle$ are the (*formal*) parameters of the procedure
- Other free identifiers in $\langle s \rangle$ are called *external references* $\langle z_1 \rangle \ldots \langle z_k \rangle$
- These are defined by the environment *E* where the procedure is declared (lexical scoping)
- The contextual environment of the procedure *CE* is $E\ |_{\{\langle z_1 \rangle \ldots \langle z_k \rangle\}}$
- When the procedure is called *CE* is used to construct the environment for execution of $\langle s \rangle$

$(\texttt{proc}\ \{\$\ \langle y_1 \rangle \ldots \langle y_n \rangle\ \}$
$\quad \langle s \rangle$
$\texttt{end}\ ,$
$CE)$

Procedure introduction

- The semantic statement is

  $(\langle x\rangle = \texttt{proc} \{\$ \langle y_1\rangle \ldots \langle y_n\rangle\} \langle s\rangle \texttt{ end}, E)$

- Create a contextual environment:

  $CE = E\,|_{\{\langle z_1\rangle \ldots \langle z_k\rangle\}}$ where $\langle z_1\rangle \ldots \langle z_k\rangle$ are external references in $\langle s\rangle$.

- Create a new procedure value of the form: $(\texttt{proc} \{\$ \langle y_1\rangle \ldots \langle y_n\rangle\} \langle s\rangle \texttt{ end}, CE)$, refer to it by the variable x_P
- Bind the store variable $E(\langle x\rangle)$ to x_P
- Continue to next execution step

Procedure application

- The semantic statement is
  $(\{\ \langle x\rangle\ \langle y_1\rangle \ldots \langle y_n\rangle\ \}, E)$
- If the activation condition ($E(\langle x\rangle)$ is determined) is true:
  - If $E(\langle x\rangle)$ is not a procedure value, or it is a procedure with arity that is not equal to n, raise an error
  - If $E(\langle x\rangle)$ is (proc {$ $\langle z_1\rangle \ldots \langle z_n\rangle$} $\langle s\rangle$ end, CE) , push
    $(\ \langle s\rangle, CE + \{\langle z_1\rangle \rightarrow E(\langle y_1\rangle) \ldots \langle z_n\rangle \rightarrow E(\langle y_n\rangle)\}\ )$
    on the stack
- If the activation condition ($E(\langle x\rangle)$ is determined) is false:
  - Suspend

Execution examples

```
              local Max C in
                proc {Max X Y Z}
⟨s⟩1   ⟨s⟩2       ⟨s⟩3 if X >= Y then Z=X else Z=Y end
                end
                {Max 3 5 C}
              end
```

Execution examples (2)

```
⟨s⟩1  ⎧        local Max C in
      ⎪ ⟨s⟩2 ⎧    proc {Max X Y Z}
      ⎨      ⎨     ⟨s⟩3  if X >= Y then Z=X else Z=Y end
      ⎪      ⎪    end
      ⎪      ⎩ ⟨s⟩4 {Max 3 5 C}
      ⎩        end
```

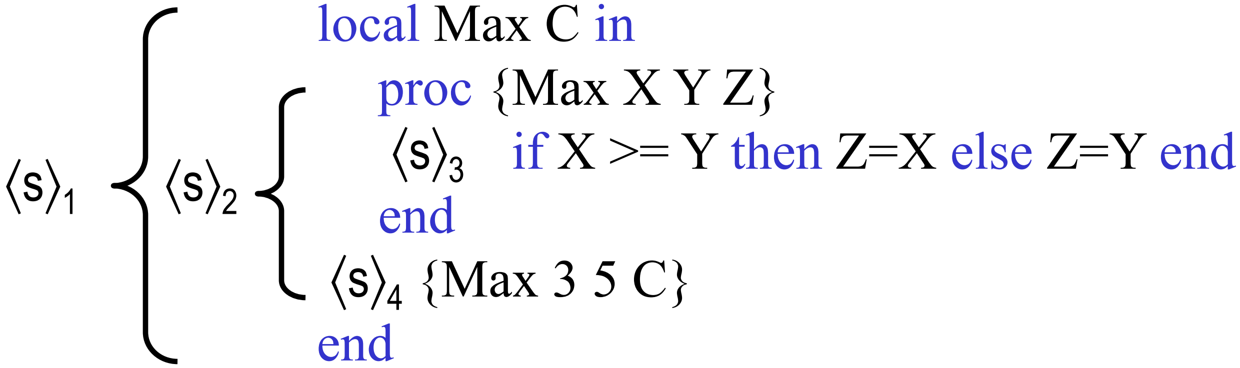
- Initial state $([(\langle s\rangle_1, \emptyset)], \emptyset)$
- After local Max C in …
  $([(\langle s\rangle_2,\ \{\mathrm{Max} \rightarrow m, \mathrm{C} \rightarrow c\})], \{m, c\})$
- After Max binding
  $([(\langle s\rangle_4,\ \{\mathrm{Max} \rightarrow m, \mathrm{C} \rightarrow c\})],$
  $\{m = (\mathrm{proc}\{\$\ X\ Y\ Z\}\ \langle s\rangle_3\ \mathrm{end}\ , \emptyset)\ , c\}\ )$

Execution examples (3)

$$\langle s\rangle_1 \left\{ \begin{array}{l} \text{local Max C in} \\ \langle s\rangle_2 \left\{ \begin{array}{l} \quad \text{proc \{Max X Y Z\}} \\ \quad \langle s\rangle_3 \quad \text{if X >= Y then Z=X else Z=Y end} \\ \quad \text{end} \\ \langle s\rangle_4 \ \text{\{Max 3 5 C\}} \end{array}\right. \\ \text{end} \end{array}\right.$$

- After Max binding
  $([(\langle s\rangle_4, \ \{\text{Max} \rightarrow m, \text{C} \rightarrow c\})],$
  $\{m = (\text{proc}\{\$ \text{ X Y Z}\} \ \langle s\rangle_3 \ \text{end} \ , \varnothing) \ , c\} \ )$
- After procedure call
  $([(\langle s\rangle_3, \ \{\text{X} \rightarrow t_1, \text{Y} \rightarrow t_2, \text{Z} \rightarrow c\}) \ ],$
  $\{m = (\text{proc}\{\$ \text{ X Y Z}\} \ \langle s\rangle_3 \ \text{end} \ , \varnothing) \ , t_1{=}3, \ t_2{=}5, \text{c}\} \ )$

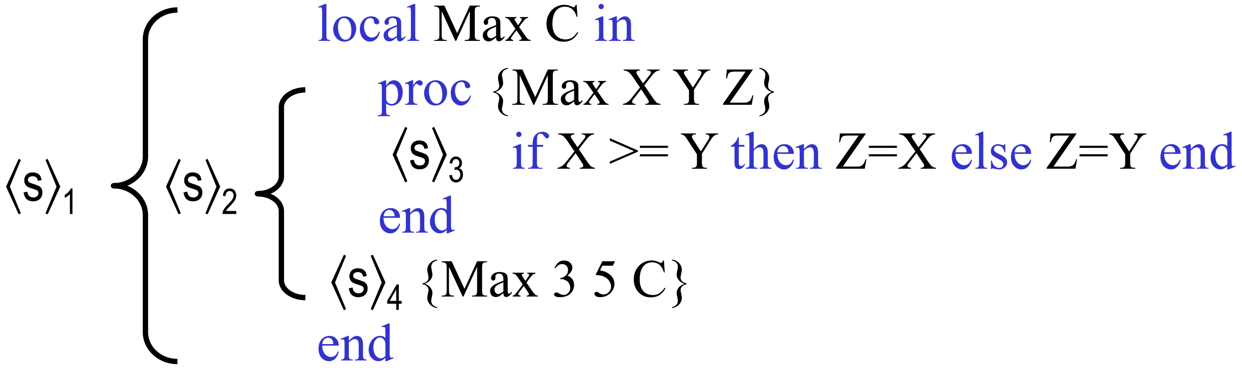
Execution examples (4)



```
local Max C in
   proc {Max X Y Z}
    ⟨s⟩3  if X >= Y then Z=X else Z=Y end
   end
⟨s⟩4 {Max 3 5 C}
end
```

- After procedure call
  $([(\langle s\rangle_3,\ \{X \rightarrow t_1, Y \rightarrow t_2, Z \rightarrow c\})],$
  $\{m = (\text{proc}\{\$\ X\ Y\ Z\}\ \langle s\rangle_3\ \text{end}\ , \emptyset)\ , t_1{=}3, t_2{=}5, c\}\ )$
- After T = (X>=Y)
  $([(\langle s\rangle_3,\ \{X \rightarrow t_1, Y \rightarrow t_2, Z \rightarrow c, T \rightarrow t\})\ ],$
  $\{m = (\text{proc}\{\$\ X\ Y\ Z\}\ \langle s\rangle_3\ \text{end}\ , \emptyset)\ , t_1{=}3, t_2{=}5, c, t{=}\text{false}\}\ )$
- $([(Z{=}Y\ ,\ \{X \rightarrow t_1, Y \rightarrow t_2, Z \rightarrow c, T \rightarrow t\})\ ],$
  $\{m = (\text{proc}\{\$\ X\ Y\ Z\}\ \langle s\rangle_3\ \text{end}\ , \emptyset)\ , t_1{=}3, t_2{=}5, c, t{=}\text{false}\}\ )$

Execution examples (5)



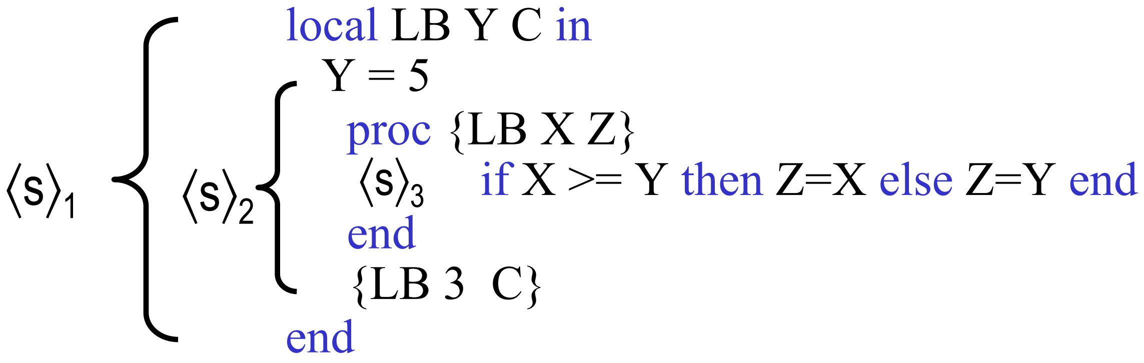
```
⟨s⟩1   local Max C in
  ⟨s⟩2   proc {Max X Y Z}
           ⟨s⟩3  if X >= Y then Z=X else Z=Y end
         end
         ⟨s⟩4 {Max 3 5 C}
       end
```

- $([(Z{=}Y ,\ \{X \rightarrow t_1, Y \rightarrow t_2, Z \rightarrow c, T \rightarrow t\})],$
  $\{m = (\mathrm{proc}\{\$\ X\ Y\ Z\}\ \langle s\rangle_3\ \mathrm{end} , \emptyset) , t_1{=}3, t_2{=}5, c, t{=}\mathrm{false}\}\ )$
- $([\ ],$
  $\{m = (\mathrm{proc}\{\$\ X\ Y\ Z\}\ \langle s\rangle_3\ \mathrm{end} , \emptyset) , t_1{=}3, t_2{=}5, c{=}5, t{=}\mathrm{false}\}\ )$

Procedures with external references

```
              local LB Y C in
                Y = 5
⟨s⟩1              proc {LB X Z}
       ⟨s⟩2        ⟨s⟩3 if X >= Y then Z=X else Z=Y end
                 end
                 {LB 3 C}
              end
```

Procedures with external references



```
              local LB Y C in
                 Y = 5
⟨s⟩1    ⟨s⟩2       proc {LB X Z}
                    ⟨s⟩3   if X >= Y then Z=X else Z=Y end
                   end
                   {LB 3  C}
              end
```

- The procedure value of LB is
- (proc{\$ X Z} $\langle s \rangle_3$ end , {Y → y})
- The store is $\{y = 5, \ldots\}$

Procedures with external references

```
                      local LB Y C in
                        Y = 5
                          proc {LB X Z}
⟨s⟩1        ⟨s⟩2            ⟨s⟩3    if X >= Y then Z=X else Z=Y end
                          end
                          {LB 3 C}
                      end
```

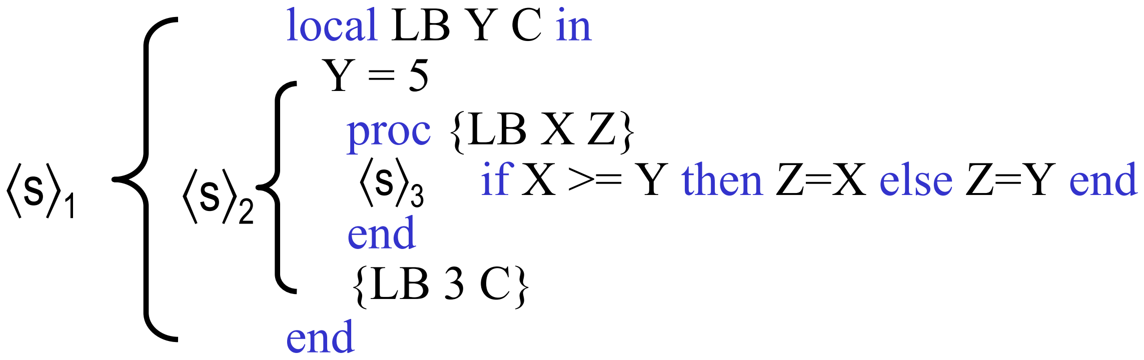
- The procedure value of LB is
- (proc{\$ X Z} $\langle s\rangle_3$ end , {Y → y})
- The store is $\{y = 5, \ldots\}$
- STACK: [({LB T C} , {Y → y , LB → lb, C → c, T → t})]
- STORE: {y = 5, lb = (proc{\$ X Z} $\langle s\rangle_3$ end , {Y → y}) , t = 3, c}

Procedures with external references

```
              local LB Y C in
                Y = 5
                  proc {LB X Z}
⟨s⟩1   ⟨s⟩2        ⟨s⟩3   if X >= Y then Z=X else Z=Y end
                  end
                  {LB 3 C}
              end
```

- STACK: $[(\ \{LB\ T\ C\}\ ,\ \{Y \rightarrow y\ ,\ LB \rightarrow lb,\ C \rightarrow c,\ T \rightarrow t\})\ ]$
- STORE: $\{y = 5,\ lb = (\text{proc}\{\$\ X\ Z\}\ \langle s\rangle_3\ \text{end}\ ,\ \{Y \rightarrow y\})\ ,\ t = 3,\ c\}$
- STACK: $[(\langle s\rangle_3\ ,\ \{Y \rightarrow y\ ,\ X \rightarrow t\ ,\ Z \rightarrow c\})\ ]$
- STORE: $\{y = 5,\ lb = (\text{proc}\{\$\ X\ Z\}\ \langle s\rangle_3\ \text{end}\ ,\ \{Y \rightarrow y\})\ ,\ t = 3,\ c\}$

Procedures with external references



```
              local LB Y C in
                 Y = 5
⟨s⟩1   ⟨s⟩2       proc {LB X Z}
                  ⟨s⟩3   if X >= Y then Z=X else Z=Y end
                  end
                  {LB 3 C}
              end
```

- STACK: $[(\langle s\rangle_3 , \{Y \rightarrow y , X \rightarrow t , Z \rightarrow c\})]$
- STORE: $\{y = 5, lb = (\text{proc}\{\$ \ X \ Z\} \ \langle s\rangle_3 \ \text{end} , \{Y \rightarrow y\}) , t = 3, c\}$
- STACK: $[(Z{=}Y , \{Y \rightarrow y , X \rightarrow t , Z \rightarrow c\})]$
- STORE: $\{y = 5, lb = (\text{proc}\{\$ \ X \ Z\} \ \langle s\rangle_3 \ \text{end} , \{Y \rightarrow y\}) , t = 3, c\}$
- STACK: $[\ ]$
- STORE: $\{y = 5, lb = (\text{proc}\{\$ \ X \ Z\} \ \langle s\rangle_3 \ \text{end} , \{Y \rightarrow y\}) , t = 3, c = 5\}$

From the kernel language to a practical language

- Interactive interface
  - the declare statement and the global environment
- Extend kernel syntax to give a full, practical syntax
  - nesting of partial values
  - implicit variable initialization
  - expressions
  - nesting the if and case statements
  - andthen and orelse operations
- Linguistic abstraction
  - Functions
- Exceptions

The interactive interface (declare)

- The interactive interface is a program that has a single global environment

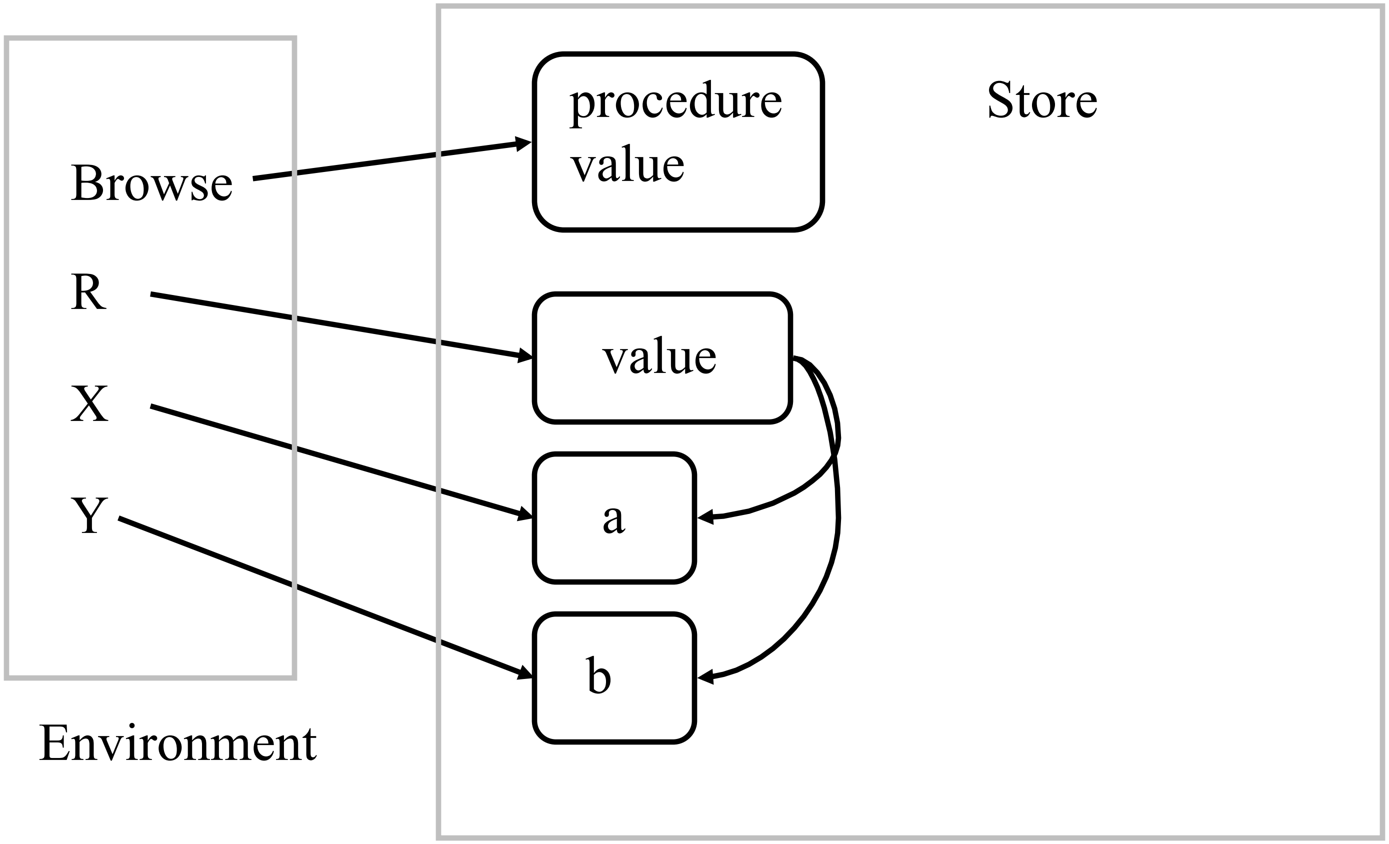
```
declare X Y
```

- Augments (and overrides) the environment with new mappings for X and Y

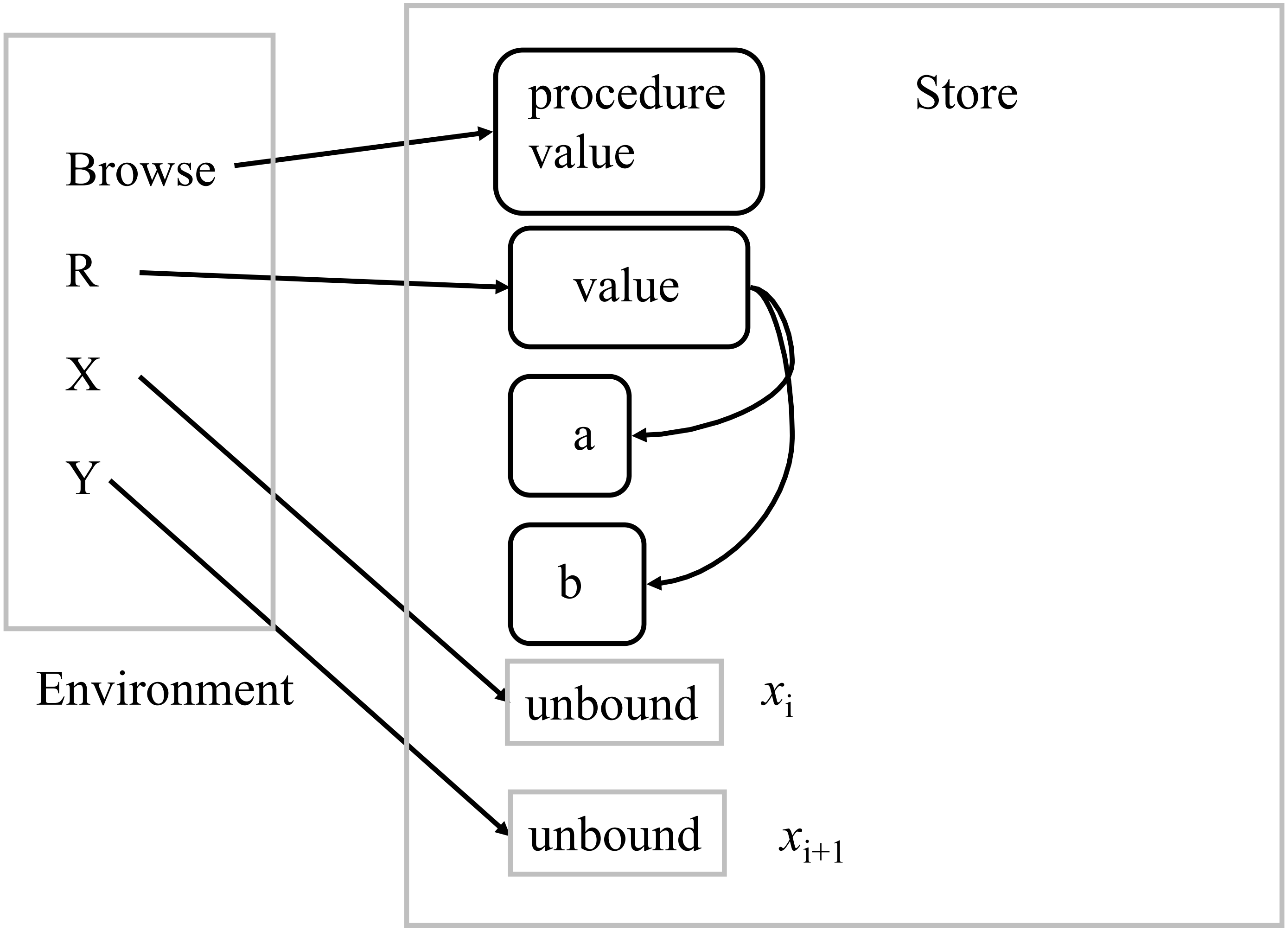
```
{Browse X}
```

- Inspects the store and shows partial values, and incremental changes

The interactive interface (declare)



declare X Y



Browse

R

X

Y

Environment

procedure value

value

a

b

unbound x_i

unbound x_{i+1}

Store

Syntactic extensions

- **Nested partial values**
  - person(name: “George” age:25)

    local A B in A= “George” B=25 person(name:A age:B) end
- **Implicit variable initialization**
  - local ⟨pattern⟩ = ⟨expression⟩ in ⟨statement⟩ end
- **Example:**

  assume T has been defined, then

  ```
  local tree(key:A left:B right:C value:D) = T in ⟨statement⟩ end
  ```

  is the same as:

  ```
  local A B C D in
        T = tree(key:A left:B right:C value:D) <statement>
  end
  ```

Extracting fields in local statement

```
declare T
   :
   T = tree(key:seif age:48 profession:professor)
   :
local
   tree(key:A ...) = T
in
   ⟨statement⟩
end
```

Nested if and case statements

- Observe a pair notation is: 1 # 2, is the tuple '#'(1 2)

```
case Xs # Ys
of  nil # Ys then ⟨s⟩1
[]  Xs # nil then ⟨s⟩2
[]  (X|Xr) # (Y|Yr) andthen X=<Y then ⟨s⟩3
else ⟨s⟩4 end
```

- Is translated into (assuming X,Xr,Y,Yr not free in ⟨s⟩$_4$)

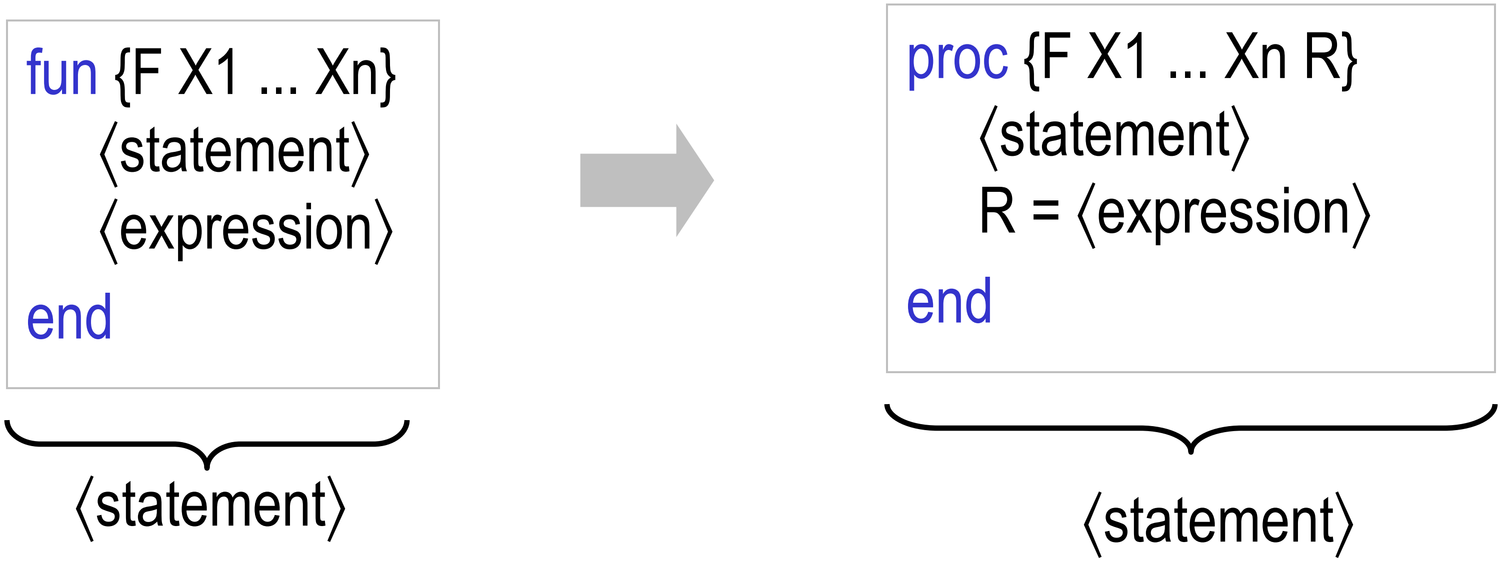
```
case Xs of nil then ⟨s⟩1
else
    case Ys of nil then ⟨s⟩2
    else
         case Xs of X|Xr then
            case Ys of Y|Yr then
                     if  X=<Y then ⟨s⟩3 else ⟨s⟩4 end
            else ⟨s⟩4 end
        else ⟨s⟩4 end
    end
end
```

Expressions

- An expression is a sequence of operations that returns a value
- A statement is a sequence of operations that does not return a value. Its effect is on the store, or outside of the system (e.g. read/write a file)
- $\underbrace{11*11}_{\text{expression}}$ $\qquad\qquad$ $\underbrace{X=11*11}_{\text{statement}}$

Functions as linguistic abstraction

- R = {F X1 ... Xn}
- {F X1 ... Xn R}



```
fun {F X1 ... Xn}
   ⟨statement⟩
   ⟨expression⟩
end
```

⟨statement⟩

```
proc {F X1 ... Xn R}
   ⟨statement⟩
   R = ⟨expression⟩
end
```

⟨statement⟩

Nesting in data structures

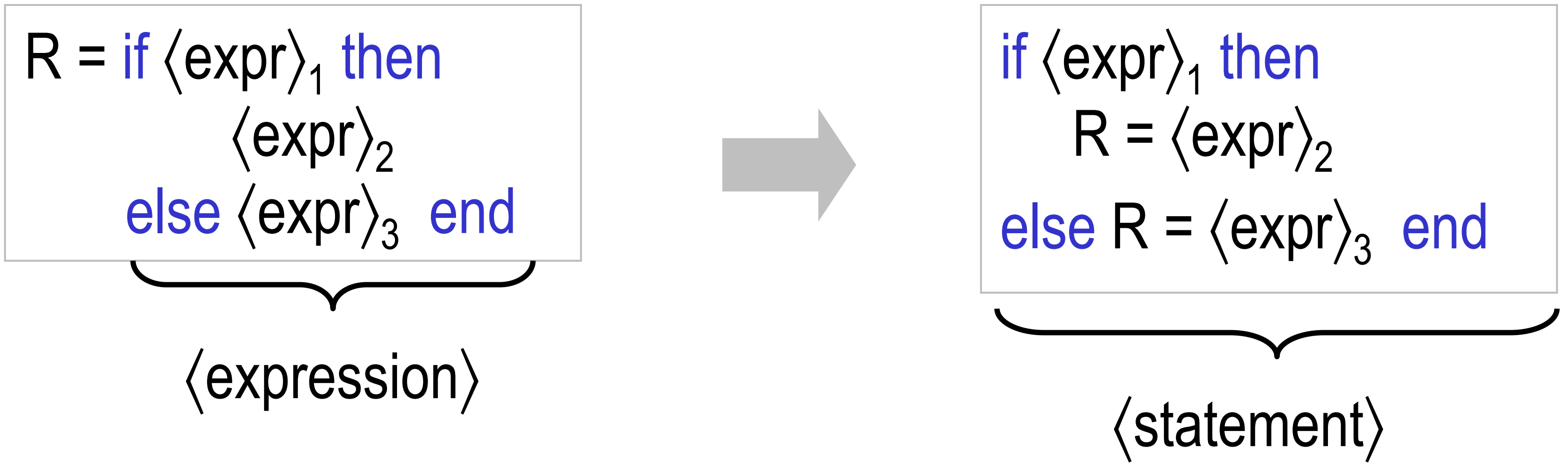
- `Ys = {F X}|{Map Xr F}`
- Is unnested to:
-
```
local Y Yr in
    Ys = Y|Yr
    {F X Y}
    {Map Xr F Yr}
end
```
- The unnesting of the calls occurs after the data structure

Functional nesting

- Nested notations that allows expressions as well as statements

-
  ```
  local R in
          {F X1 ... Xn R}
          {Q R ...}
  end
  ```
- Is written as (equivalent to):
- $$\underbrace{\{Q\ \underbrace{\{F\ X1\ \ldots\ Xn\}}_{\text{expression}}\ \ldots\}}_{\text{statement}}$$

Conditional expressions



R = if ⟨expr⟩$_1$ then
⟨expr⟩$_2$
else ⟨expr⟩$_3$ end

⟨expression⟩

if ⟨expr⟩$_1$ then
R = ⟨expr⟩$_2$
else R = ⟨expr⟩$_3$ end

⟨statement⟩

```
fun {Max X Y}
    if X>=Y then X
    else Y end
end
```

```
proc {Max X Y R}
    R =  ( if X>=Y then X
              else Y end )
end
```

Example

```
fun {Max X Y}
    if X>=Y then X
    else Y end
end
```

```
proc {Max X Y R}
    R =  ( if X>=Y then X
              else Y end )
end
```

```
proc {Max X Y R}
    if X>=Y then R = X
    else R = Y end
end
```

andthen and orelse

$\langle\text{expr}\rangle_1$ andthen $\langle\text{expr}\rangle_2$

→

if $\langle\text{expr}\rangle_1$ then
$\langle\text{expr}\rangle_2$
else false end

$\langle\text{expr}\rangle_1$ orelse $\langle\text{expr}\rangle_2$

→

if $\langle\text{expr}\rangle_1$ then
true
else $\langle\text{expr}\rangle_2$ end

Function calls

Observe

```
{F1 {F2 X} {F3 Y}}
```

```
local R1 R2 in
    R1 = {F2 X}
    R2 = {F3 Y}
    {F1 R1 R2}
end
```

The arguments of a function are evaluated first from left to right

A complete example

```
fun {Map Xs F}
   case Xs
   of nil then nil
   []  X|Xr then {F X}|{Map Xr F}
   end
end
```

```
proc {Map Xs F Ys}
   case Xs
   of nil then Ys = nil
   []  X|Xr then Y Yr in
         Ys = Y|Yr
         {F X Y}
         {Map Xr F Yr}
   end
end
```

Exceptions

- How to handle exceptional situations in the program?
- Examples:
  - divide by 0
  - opening a nonexistent file
- Some errors are programming errors
- Some errors are imposed by the external environment
- Exception handling statements allow programs to handle and recover from errors

Exceptions

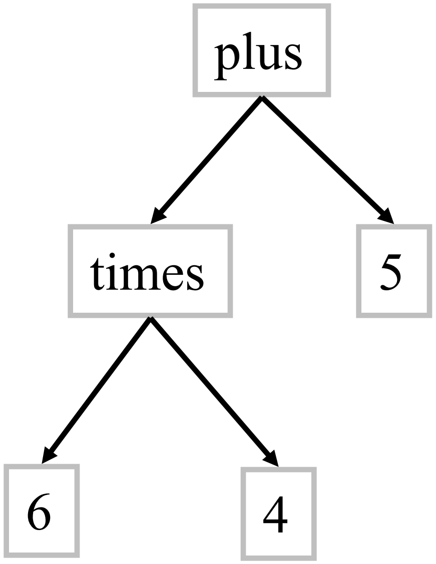
- The error confinement principle:
  - Define your program as a structured layers of components
  - Errors are visible only internally and a recovery procedure corrects the errors: either errors are not visible at the component boundary or are reported (nicely) to a higher level
- In one operation, exit from arbitrary depth of nested contexts
  - Essential for program structuring; else programs get complicated (use boolean variables everywhere, etc.)

Basic concepts

- A program that encounters an error (*exception*) should transfer execution to another part, the *exception handler* and give it a (partial) value that describes the error
- try $\langle s \rangle_1$ catch $\langle x \rangle$ then $\langle s \rangle_2$ end
- raise $\langle x \rangle$ end
- Introduce an exception marker on the semantic stack
- The execution is equivalent to $\langle s \rangle_1$ if it executes without raising an error
- Otherwise, $\langle s \rangle_1$ is aborted and the stack is popped up to the marker, the error value is transferred through $\langle x \rangle$, and $\langle s \rangle_2$ is executed

Exceptions (Example)

```
fun {Eval E}
   if {IsNumber E} then E
   else
      case E
      of plus(X Y) then {Eval X}+{Eval Y}
      []  times(X Y) then {Eval X}*{Eval Y}
      else raise illFormedExpression(E) end
      end
   end
end
```



Exceptions (Example)

```
try
   {Browse {Eval plus(5 6) }}
   {Browse {Eval plus(times(5 5) 6) }}
   {Browse {Eval plus(minus(5 5) 6) }}
catch illFormedExpression(E) then
   {System.showInfo  ”**** illegal expresion ****” # E}
end
```

Try semantics

- The semantic statement is

  (try $\langle s \rangle_1$ catch $\langle \mathrm{y} \rangle$ then $\langle s \rangle_2$ end, E)
- Push the semantic statement (catch $\langle \mathrm{y} \rangle$ then $\langle s \rangle_2$ end, E) on ST
- Push ($\langle s \rangle_1$, E) on ST
- Continue to next execution step

Raise semantics

- The semantic statement is

  (raise ⟨x⟩ end, E)
- Pop elements off ST looking for a catch statement:
  - If a catch statement is found, pop it from the stack
  - If the stack is emptied and no catch is found, then stop execution with the error message ”Uncaught exception”
- Let (catch ⟨y⟩ then ⟨s⟩ end, E_c) be the catch statement that is found
- Push (⟨s⟩ , $E_c + \{<y> \rightarrow E(<x>)\}$) on ST
- Continue to next execution step

C. Varela; Adapted w/permission from S. Haridi and P. Van Roy

Catch semantics

- The semantic statement is

  (catch ⟨x⟩ then ⟨*s*⟩ end, *E*)

- Continue to next execution step (like skip)

Full exception syntax

- Exception statements (expressions) with multiple patterns and finally clause

- Example:

```
:
FH = {OpenFile ”xxxxx”}
:
try
   {ProcessFile FH}
catch X then
   {System.showInfo ”***** Exception when processing *****” # X}
finally {CloseFile FH} end
```

finally syntax

try $\langle s \rangle_1$ finally $\langle s \rangle_2$ end

is converted to:

```
try ⟨s⟩1
catch X then
   ⟨s⟩2
   raise X end
end
⟨s⟩2
```

Case statement revisited

- The semantic statement is

  (case $\langle \mathrm{x} \rangle$ of $\langle l \rangle$ $(\langle f_1 \rangle : \langle x_1 \rangle \ldots \langle f_n \rangle : \langle x_n \rangle)$
  then $\langle \mathrm{s}_1 \rangle$
  else $\langle \mathrm{s}_2 \rangle$ end , E)
- If the activation condition ($E(\langle \mathrm{x} \rangle)$ is determined) is true:
  - If $E(\langle x \rangle)$ is a record, the label of $E(\langle x \rangle)$ is $\langle l \rangle$ and its arity is $[\langle f_1 \rangle \ \ldots \langle f_n \rangle]$:
    push (try local $\langle x_1 \rangle = \langle x \rangle.\ \langle f_1 \rangle \ldots \langle x_n \rangle = \langle x \rangle.\ \langle f_n \rangle$ in $\langle \mathrm{s}_1 \rangle^*$ end
    catch user(E) then raise E end [] E then $\langle \mathrm{s}_2 \rangle$ end, E)
    on the stack, where $\langle \mathrm{s}_1 \rangle^*$ is the same as $\langle \mathrm{s}_1 \rangle$ except
    raise E end is changed to raise user(E) end
  - Otherwise, push ($\langle \mathrm{s}_2 \rangle$, E) on the stack
- If the activation condition ($E(\langle \mathrm{x} \rangle)$ is determined) is false:
  - Suspend

Exercises

49. VRH Exercise 2.9.3 (page 107).
50. VRH Exercise 2.9.7 (page 108) –translate example to kernel language and execute using operational semantics.
51. Write an example of a program that suspends. Now, write an example of a program that never terminates. Use the operational semantics to prove suspension or non-termination.
52. VRH Exercise 2.9.12 (page 110).
53. Change the semantics of the `case` statement, so that patterns can contain variable labels and variable feature names.

Exercises

54. Restrict the kernel language to make it strictly functional (i.e., without dataflow variables)
    - Language similar to Scheme (dynamically typed functional language)

    This is done by disallowing variable declaration (without initialization) and disallowing procedural syntax
    - Only use implicit variable initialization
    - Only use functions