Declarative Programming Techniques

Declarativeness, iterative computation (VRH 3.1-3.2)
Higher-order programming (VRH 3.6)
Abstract data types (VRH 3.7)

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April 5, 2012

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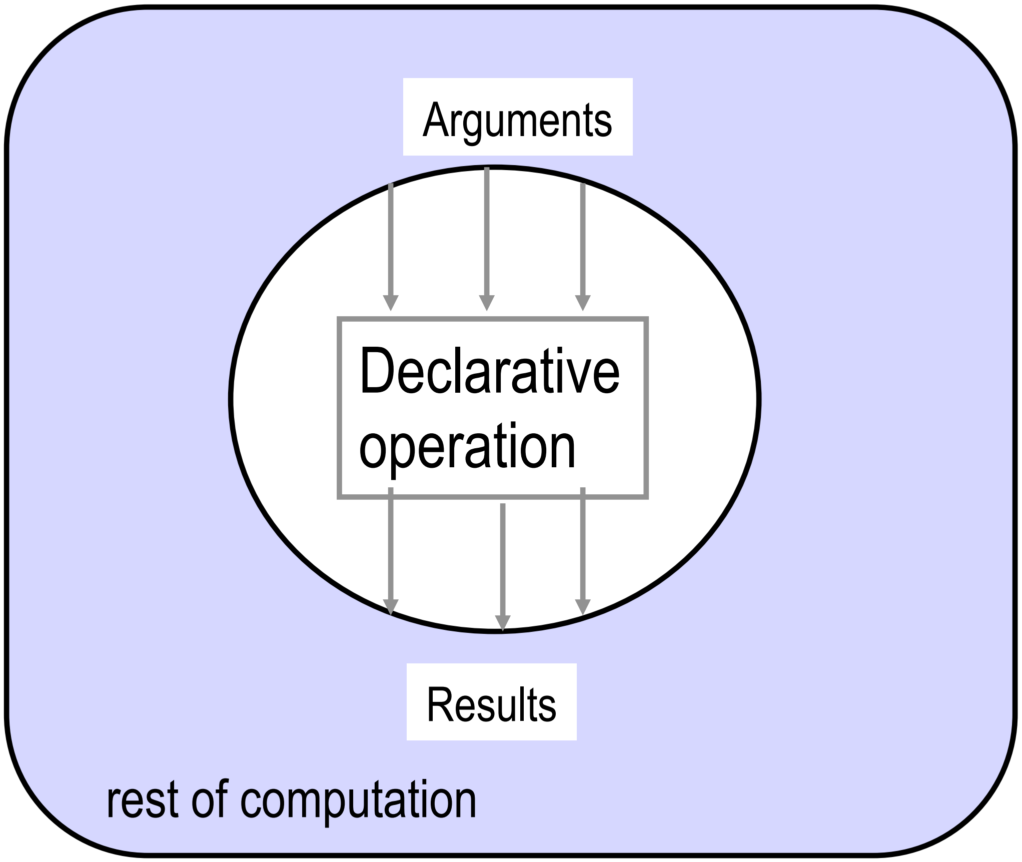
Overview

- What is declarativeness?
  - Classification
  - Advantages for large and small programs
- Control Abstractions
  - Iterative programs
- Higher-Order Programming
  - Procedural abstraction
  - Genericity
  - Instantiation
  - Embedding
- Abstract data types
  - Encapsulation
  - Security

Declarative operations (1)

- An operation is *declarative* if whenever it is called with the same arguments, it returns the same results independent of any other computation state
- A declarative operation is:
  - *Independent* (depends only on its arguments, nothing else)
  - *Stateless* (no internal state is remembered between calls)
  - *Deterministic* (call with same operations always give same results)
- Declarative operations can be composed together to yield other declarative components
  - All basic operations of the declarative model are declarative and combining them always gives declarative components

Declarative operations (2)



Why declarative components (1)

- There are two reasons why they are important:
- *(Programming in the large)* A declarative component can be written, tested, and proved correct independent of other components and of its own past history.
  - The complexity (reasoning complexity) of a program composed of declarative components is the *sum* of the complexity of the components
  - In general the reasoning complexity of programs that are composed of nondeclarative components explodes because of the intimate interaction between components
- *(Programming in the small)* Programs written in the declarative model are much easier to reason about than programs written in more expressive models (e.g., an object-oriented model).
  - Simple algebraic and logical reasoning techniques can be used

Why declarative components (2)

- Since declarative components are mathematical functions, algebraic reasoning is possible i.e. substituting equals for equals
- The declarative model of Chapter 2 guarantees that all programs written are declarative
- Declarative components can be written in models that allow stateful data types, but there is no guarantee

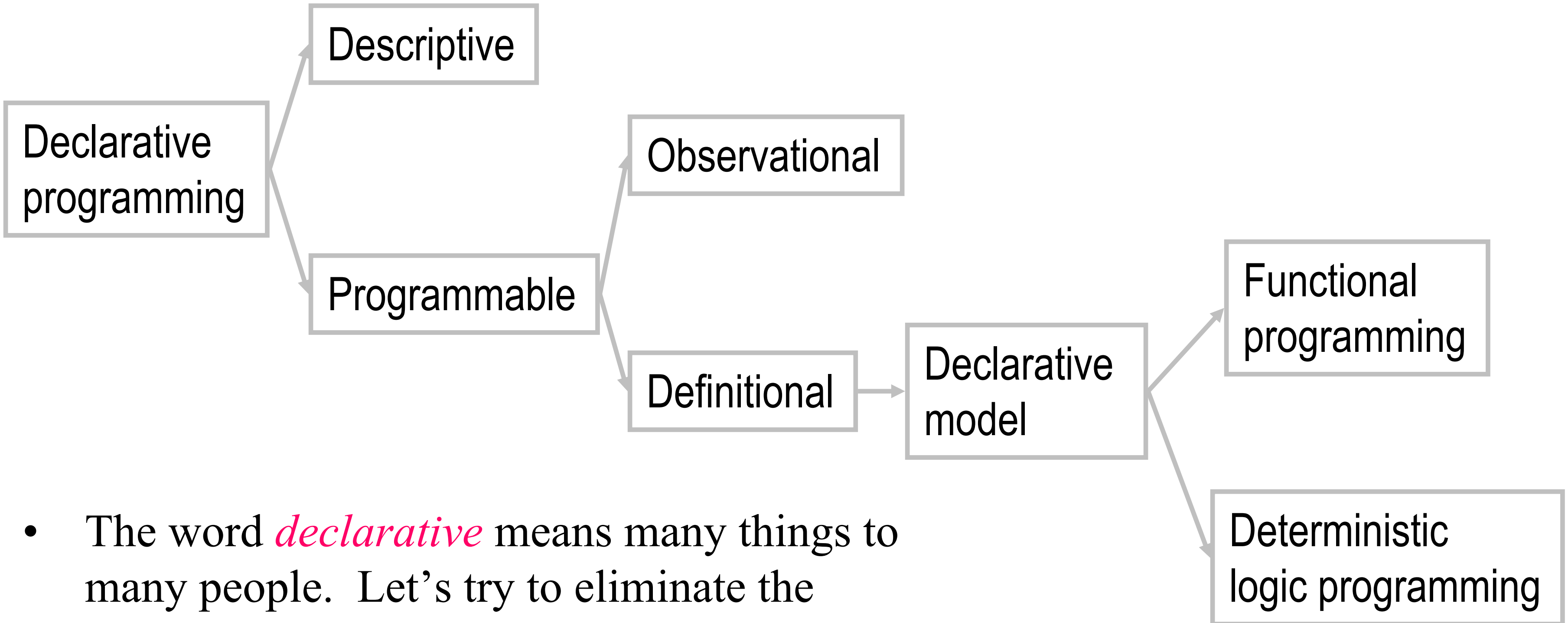
Given

$f(a) = a^2$

We can replace $f(a)$ in any other equation

$b = 7f(a)^2$ becomes $b = 7a^4$

Classification of declarative programming



- The word *declarative* means many things to many people. Let's try to eliminate the confusion.
- The basic intuition is to program by defining the *what* without explaining the *how*

Descriptive language

$\langle s \rangle$	::=	skip	*empty statement*
	|	$\langle x \rangle = \langle y \rangle$	*variable-variable binding*
	|	$\langle x \rangle = \langle \text{record} \rangle$	*variable-value binding*
	|	$\langle s_1 \rangle \langle s_2 \rangle$	*sequential composition*
	|	local $\langle x \rangle$ in $\langle s_1 \rangle$ end	*declaration*

Other descriptive languages include HTML and XML

Descriptive language

```
<person id = ”530101-xxx”>
    <name> Seif </name>
    <age> 48 </age>
</person>
```

Other descriptive languages include HTML and XML

Kernel language

The following defines the syntax of a statement, ⟨s⟩ denotes a statement

⟨s⟩	::=	skip	*empty statement*
	|	⟨*x*⟩ = ⟨y⟩	*variable-variable binding*
	|	⟨x⟩ = ⟨v⟩	*variable-value binding*
	|	⟨s_1⟩ ⟨s_2⟩	*sequential composition*
	|	local ⟨x⟩ in ⟨s_1⟩ end	*declaration*
	|	proc '{'⟨x⟩ ⟨y_1⟩ … ⟨y_n⟩ ' }' ⟨s_1⟩ end	*procedure introduction*
	|	if ⟨x⟩ then ⟨s_1⟩ else ⟨s_2⟩ end	*conditional*
	|	' {' ⟨x⟩ ⟨y_1⟩ … ⟨y_n⟩ ' }'	*procedure application*
	|	case ⟨x⟩ of ⟨pattern⟩ then ⟨s_1⟩ else ⟨s_2⟩ end	*pattern matching*

Why the KL is declarative

- All basic operations are declarative
- Given the components (sub-statements) are declarative,
  - sequential composition
  - local statement
  - procedure definition
  - procedure call
  - if statement
  - case statement

are all declarative (independent, stateless, deterministic).

Iterative computation

- An iterative computation is one whose execution stack is bounded by a constant, independent of the length of the computation
- Iterative computation starts with an initial state S_0, and transforms the state in a number of steps until a final state S_{final} is reached:

$$S_0 \rightarrow S_1 \rightarrow \ldots \rightarrow S_{final}$$

The general scheme

fun {Iterate S_i}
    if {*IsDone* S_i} then S_i
    else S_{i+1} in
        S_{i+1} = {*Transform* S_i}
        {Iterate S_{i+1}}
    end
end

- *IsDone* and *Transform* are problem dependent

The computation model

- STACK : [R={Iterate S_0}]
- STACK : [S_1 = {*Transform* S_0},
  R={Iterate S_1}]

- STACK : [R={Iterate S_i}]
- STACK : [S_{i+1} = {*Transform* S_i},
  R={Iterate S_{i+1}}]

- STACK : [R={Iterate S_{i+1}}]

Newton's method for the square root of a positive real number

- Given a real number x, start with a guess g, and improve this guess iteratively until it is accurate enough
- The improved guess g' is the average of g and x/g:

$$g' = (g + x / g) / 2$$

$$\varepsilon = g - \sqrt{x}$$

$$\varepsilon' = g' - \sqrt{x}$$

For g' to be a better guess than g: $\varepsilon' < \varepsilon$

$$\varepsilon' = g' - \sqrt{x} = (g + x / g) / 2 - \sqrt{x} = \varepsilon^2 / 2g$$

$$\text{i.e. } \varepsilon^2 / 2g < \varepsilon, \quad \varepsilon / 2g < 1$$

$$i.e.\ \varepsilon < 2g,\ g - \sqrt{x} < 2g,\ 0 < g + \sqrt{x}$$

Newton's method for the square root of a positive real number

- Given a real number x, start with a guess g, and improve this guess iteratively until it is accurate enough
- The improved guess g' is the average of g and x/g:
- Accurate enough is defined as:

$$| x - g^2 | / x < 0.00001$$

SqrtIter

```
fun {SqrtIter Guess X}
  if {GoodEnough Guess X} then Guess
  else
    Guess1 = {Improve Guess X} in
    {SqrtIter Guess1 X}
  end
end
```

- Compare to the general scheme:
  - The state is the pair Guess and X
  - *IsDone* is implemented by the procedure GoodEnough
  - *Transform* is implemented by the procedure Improve

The program version 1

```
fun {Sqrt X}
   Guess = 1.0
in {SqrtIter Guess X}
end
fun {SqrtIter Guess X}
   if {GoodEnough Guess X} then
      Guess
   else
      {SqrtIter {Improve Guess X} X}
   end
end
```

```
fun {Improve Guess X}
   (Guess + X/Guess)/2.0
end
fun {GoodEnough Guess X}
   {Abs X - Guess*Guess}/X < 0.00001
end
```

Using local procedures

- The main procedure Sqrt uses the helper procedures SqrtIter, GoodEnough, Improve, and Abs
- SqrtIter is only needed inside Sqrt
- GoodEnough and Improve are only needed inside SqrtIter
- Abs (absolute value) is a general utility
- The general idea is that helper procedures should not be visible globally, but only locally

Sqrt version 2

```
local
  fun {SqrtIter Guess X}
    if {GoodEnough Guess X} then Guess
    else {SqrtIter {Improve Guess X} X} end
  end
  fun {Improve Guess X}
    (Guess + X/Guess)/2.0
  end
  fun {GoodEnough Guess X}
    {Abs X - Guess*Guess}/X < 0.000001
  end
in
  fun {Sqrt X}
    Guess = 1.0
  in {SqrtIter Guess X} end
end
```

Sqrt version 3

- Define GoodEnough and Improve inside SqrtIter

```
local
  fun {SqrtIter Guess X}
    fun {Improve}
      (Guess + X/Guess)/2.0
    end
    fun {GoodEnough}
      {Abs X - Guess*Guess}/X < 0.000001
    end
  in
     if {GoodEnough} then Guess
     else {SqrtIter {Improve} X} end
  end
in fun {Sqrt X}
     Guess = 1.0 in
     {SqrtIter Guess X}
  end
end
```

Sqrt version 3

- Define GoodEnough and Improve inside SqrtIter

```
local
  fun {SqrtIter Guess X}
    fun {Improve}
      (Guess + X/Guess)/2.0
    end
    fun {GoodEnough}
      {Abs X - Guess*Guess}/X < 0.000001
    end
  in
     if {GoodEnough} then Guess
     else {SqrtIter {Improve} X} end
  end
in fun {Sqrt X}
     Guess = 1.0 in
     {SqrtIter Guess X}
  end
end
```

The program has a single drawback: on each iteration two procedure values are created, one for Improve and one for GoodEnough

Sqrt final version

```
fun {Sqrt X}
  fun {Improve Guess}
    (Guess + X/Guess)/2.0
  end
  fun {GoodEnough Guess}
    {Abs X - Guess*Guess}/X < 0.000001
  end
  fun {SqrtIter Guess}
     if {GoodEnough Guess} then Guess
     else {SqrtIter {Improve Guess} } end
  end
  Guess = 1.0
in {SqrtIter Guess}
end
```

The final version is a compromise between abstraction and efficiency

From a general scheme to a control abstraction (1)

fun {Iterate S_i}
    if {*IsDone* S_i} then S_i
    else S_{i+1} in
        S_{i+1} = {*Transform* S_i}
        {Iterate S_{i+1}}
    end
end

- *IsDone* and *Transform* are problem dependent

From a general scheme to a control abstraction (2)

```
fun {Iterate S IsDone Transform}
   if {IsDone S} then S
   else S1 in
        S1 = {Transform S}
        {Iterate S1 IsDone Transform}
   end
end
```

fun {Iterate S_i}
  if {*IsDone* S_i} then S_i
  else S_{i+1} in
    S_{i+1} = {*Transform* S_i}
    {Iterate S_{i+1}}
  end
end

Sqrt using the Iterate abstraction

```
fun {Sqrt X}
  fun {Improve Guess}
    (Guess + X/Guess)/2.0
  end
  fun {GoodEnough Guess}
    {Abs X - Guess*Guess}/X < 0.000001
  end
  Guess = 1.0
in
  {Iterate Guess GoodEnough Improve}
end
```

Sqrt using the control abstraction

```
fun {Sqrt X}
   {Iterate
     1.0
     fun {$ G} {Abs X - G*G}/X < 0.000001 end
     fun {$ G} (G + X/G)/2.0 end
   }
end
```

Iterate could become a linguistic abstraction

Higher-order programming

- Higher-order programming = the set of programming techniques that are possible with procedure values (lexically-scoped closures)
- Basic operations
  - Procedural abstraction: creating procedure values with lexical scoping
  - Genericity: procedure values as arguments
  - Instantiation: procedure values as return values
  - Embedding: procedure values in data structures
- Control abstractions
  - Integer and list loops, accumulator loops, folding a list (left and right)
- Data-driven techniques
  - List filtering, tree folding
- Explicit lazy evaluation, currying
- Higher-order programming is the foundation of component-based programming and object-oriented programming

Procedural abstraction

- Procedural abstraction is the ability to convert any statement into a procedure value
  - A procedure value is usually called a closure, or more precisely, a lexically-scoped closure
  - A procedure value is a pair: it combines the procedure code with the environment where the procedure was created (the contextual environment)
- Basic scheme:
  - Consider any statement <s>
  - Convert it into a procedure value: `P = proc {$} <s> end`
  - Executing {P} has exactly the same effect as executing <s>

Procedural abstraction

```
fun {AndThen B1 B2}
   if B1 then B2 else false
   end
end
```

Procedural abstraction

```
fun {AndThen B1 B2}
  if {B1} then {B2} else false
  end
end
```

A common limitation

- Most popular imperative languages (C, Pascal) do not have procedure values
- They have only half of the pair: variables can reference procedure code, but there is no contextual environment
- This means that control abstractions cannot be programmed in these languages
  - They provide a predefined set of control abstractions (for, while loops, if statement)
- Generic operations are still possible
  - They can often get by with just the procedure code. The contextual environment is often empty.
- The limitation is due to the way memory is managed in these languages
  - Part of the store is put on the stack and deallocated when the stack is deallocated
  - This is supposed to make memory management simpler for the programmer on systems that have no garbage collection
  - It means that contextual environments cannot be created, since they would be full of dangling pointers
- Object-oriented programming languages can use objects to encode procedure values by making external references (contextual environment) instance variables.

Genericity

- Replace specific entities (zero 0 and addition +) by function arguments
- The same routine can do the sum, the product, the logical or, etc.

```
fun {SumList L}
    case L
    of    nil then 0
    []    X|L2 then X+{SumList L2}
    end
end
```

⇩

```
fun {FoldR L F U}
    case L
    of    nil then U
    []    X|L2 then {F X  {FoldR L2 F U}}
    end
end
```

Instantiation

```
fun {FoldFactory F U}
    fun {FoldR L}
        case L
        of nil then U
        [] X|L2 then {F X {FoldR L2}}
        end
    end
in
    FoldR
end
```

- Instantiation is when a procedure returns a procedure value as its result
- Calling {FoldFactory fun {$ A B} A+B end 0} returns a function that behaves identically to SumList, which is an « instance » of a folding function

Embedding

- Embedding is when procedure values are put in data structures
- Embedding has many uses:
  - Modules: a module is a record that groups together a set of related operations
  - Software components: a software component is a generic function that takes a set of modules as its arguments and returns a new module. It can be seen as specifying a module in terms of the modules it needs.
  - Delayed evaluation (also called explicit lazy evaluation): build just a small part of a data structure, with functions at the extremities that can be called to build more. The consumer can control explicitly how much of the data structure is built.

Control Abstractions

```
declare
proc {For I J P}
  if I >= J then skip
  else {P I} {For I+1 J P}
  end
end

{For 1 10 Browse}

for I in 1..10 do {Browse I} end
```

Control Abstractions

```
proc {ForAll Xs P}
   case Xs
   of nil then skip
   [] X|Xr then
      {P X} {ForAll Xr P}
   end
end

{ForAll [a b c d]
  proc{$ I} {System.showInfo "the item is: " # I} end}

for I in [a b c d] do
   {System.showInfo "the item is: " # I}
end
```

Control abstractions

```
fun {FoldL Xs F U}
   case Xs
   of nil then U
   [] X|Xr then {FoldL Xr F {F X U}}
   end
end
```

Assume a list [*x1 x2 x3* ....]

S0 → *S1* → *S2*

U → {F *x1* U}→ {F *x2* {F *x1* U}} → ....→

Control abstractions

```
fun {FoldL Xs F U}
   case Xs
   of nil then U
   [] X|Xr then {FoldL Xr F {F X U}}
   end
end
```

What does this program do ?

```
{Browse {FoldL [1 2 3]
     fun {$ X Y} X|Y end nil}}
```

List-based techniques

```
fun {Map Xs F}
   case Xs
   of nil then nil
   [] X|Xr then
      {F X}|{Map Xr F}
   end
end
```

```
fun {Filter Xs P}
   case Xs
   of nil then nil
   [] X|Xr andthen {P X} then
      X|{Filter Xr P}
   [] X|Xr then {Filter Xr P}
   end
end
```

Tree-based techniques

```
proc {DFS Tree}
   case Tree  of tree(node:N sons:Sons …) then
        {Browse N}
         for T in Sons do {DFS T} end
   end
end
```

Call {P T} at each node T

```
proc {VisitNodes Tree P}
   case Tree  of tree(node:N sons:Sons …) then
        {P N}
         for T in Sons do {VisitNodes T P} end
   end
end
```

Explicit lazy evaluation

- Supply-driven evaluation. (e.g.The list is completely calculated independent of whether the elements are needed or not.)
- Demand-driven execution.(e.g. The consumer of the list structure asks for new list elements when they are needed.)
- Technique: a programmed trigger.
- How to do it with higher-order programming? The consumer has a function that it calls when it needs a new list element. The function call returns a pair: the list element and a new function. The new function is the new trigger: calling it returns the next data item and another new function. And so forth.

Currying

- Currying is a technique that can simplify programs that heavily use higher-order programming.
- The idea:function of n arguments $\Rightarrow$ n nested functions of one argument.
- Advantage: The intermediate functions can be useful in themselves.

```
fun {Max X Y}
   if X>=Y then X else Y end
end
```

$\Rightarrow$

```
fun {Max X}
   fun {$ Y}
      if X>=Y then X else Y end
   end
end
```

Abstract data types

- A datatype is a set of values and an associated set of operations
- A datatype is abstract only if it is completely described by its set of operations regardless of its implementation
- This means that it is possible to change the implementation of the datatype without changing its use
- The datatype is thus described by a set of procedures
- These operations are the only thing that a user of the abstraction can assume

Example: A Stack

- Assume we want to define a new datatype ⟨stack T⟩ whose elements are of any type T

  fun {NewStack}: ⟨Stack T⟩

  fun {Push ⟨Stack T⟩ ⟨T⟩ }: ⟨Stack T⟩

  fun {Pop ⟨Stack T⟩ ⟨T⟩ }: ⟨Stack T⟩

  fun {IsEmpty ⟨Stack T⟩ }: ⟨Bool⟩

- These operations normally satisfy certain conditions:

  {IsEmpty {NewStack}} = true

  for any *E* and *S0*, *S1*={Push *S0 E*} and *S0* ={Pop *S1 E*} hold

  {Pop {NewStack} E} raises error

Stack (implementation)

```
fun {NewStack} nil end
fun {Push S E} E|S end
fun {Pop S E} case S of  X|S1 then E = X  S1 end end
fun {IsEmpty S} S==nil end
```

Stack (another implementation)

```
fun {NewStack} nil end
fun {Push S E} E|S end
fun {Pop S E} case S of  X|S1 then E = X  S1 end end
fun {IsEmpty S} S==nil end
```

```
fun {NewStack} emptyStack end
fun {Push S E} stack(E S) end
fun {Pop S E} case S of stack(X S1) then E = X S1 end end
fun {IsEmpty S} S==emptyStack end
```

Dictionaries

- The datatype dictionary is a finite mapping from a set *T* to ⟨value⟩, where T is either ⟨atom⟩ or ⟨integer⟩
- fun {NewDictionary}
  - returns an empty mapping
- fun {Put D Key Value}
  - returns a dictionary identical to D except Key is mapped to Value
- fun {CondGet D Key Default}
  - returns the value corresponding to Key in D, otherwise returns Default
- fun {Domain D}
  - returns a list of the keys in D

Implementation

```
fun {Put Ds Key Value}
   case Ds
   of nil then [Key#Value]
   [] (K#V)|Dr andthen Key==K then
      (Key#Value) | Dr
   [] (K#V)|Dr andthen K>Key then
      (Key#Value)|(K#V)|Dr
   [] (K#V)|Dr andthen K<Key then
      (K#V)|{Put Dr Key Value}
   end
end
```

Implementation

```
fun {CondGet Ds Key Default}
  case Ds
  of nil then Default
  [] (K#V)|Dr andthen Key==K then
    V
  [] (K#V)|Dr andthen K>Key then
    Default
  [] (K#V)|Dr andthen K<Key then
    {CondGet Dr Key Default}
  end
end
fun {Domain Ds}
  {Map Ds fun {$ K#_} K end}
end
```

Further implementations

- Because of abstraction, we can replace the dictionary ADT implementation using a list, whose complexity is linear (i.e., O(n)), for a binary tree implementation with logarithmic operations (i.e., O(log(n)).
- Data abstraction makes clients of the ADT unaware (other than through perceived efficiency) of the internal implementation of the data type.
- It is important that clients do not use anything about the internal representation of the data type (e.g., using `{Length Dictionary}` to get the size of the dictionary). Using only the interface (defined ADT operations) ensures that different implementations can be used in the future.

Secure abstract data types: Stack is not secure

```
fun {NewStack} nil end
fun {Push S E} E|S end
fun {Pop S E}
    case S of X|S1 then E=X  S1 end
end
fun {IsEmpty S} S==nil end
```

Secure abstract data types II

- The representation of the stack is visible:

  [a b c d]

- Anyone can use an incorrect representation, i.e., by passing other language entities to the stack operation, causing it to malfunction (like a|b|X or Y=a|b|Y)
- Anyone can write new operations on stacks, thus breaking the abstraction-representation barrier
- How can we guarantee that the representation is invisible?

Secure abstract data types III

- The model can be extended. Here are two ways:
  - By adding a new basic type, an unforgeable constant called a **name**
  - By adding encapsulated state.
- A **name** is like an atom except that it cannot be typed in on a keyboard or printed!
  - The only way to have a name is if one is given it explicitly
- There are just two operations on names:

  N={NewName} : returns a fresh name

  N1==N2 : returns true or false

Secure abstract datatypes IV

- We want to « wrap » and « unwrap » values
- Let us use names to define a wrapper & unwrapper

```
proc {NewWrapper ?Wrap ?Unwrap}
   Key={NewName}
in
   fun {Wrap X}
      fun {$ K} if K==Key then X end end
   end
   fun {Unwrap C}
      {C Key}
   end
end
```

Secure abstract data types: A secure stack

With the wrapper & unwrapper we can build a secure stack

```
local Wrap Unwrap in
    {NewWrapper Wrap Unwrap}
    fun {NewStack} {Wrap nil} end
    fun {Push S E} {Wrap E|{Unwrap S}} end
    fun {Pop S E}
        case {Unwrap S} of X|S1 then E=X  {Wrap S1} end
    end
    fun {IsEmpty S} {Unwrap S}==nil end
end
```

Capabilities and security

- We say a computation is secure if it has well-defined and controllable properties, independent of the existence of other (possibly malicious) entities (either computations or humans) in the system
- What properties must a language have to be secure?
- One way to make a language secure is to base it on capabilities
  - A **capability** is an unforgeable language entity (« ticket ») that gives its owner the right to perform a particular action and only that action
  - In our model, all values are capabilities (records, numbers, procedures, names) since they give the right to perform operations on the values
  - Having a procedure gives the right to call that procedure. Procedures are very general capabilities, since what they do depends on their argument
  - Using names as procedure arguments allows very precise control of rights; for example, it allows us to build secure abstract data types
- Capabilities originated in operating systems research
  - A capability can give a process the right to create a file in some directory

Secure abstract datatypes V

- We add two new concepts to the computation model
- {NewChunk Record}
  - returns a value similar to record but its arity cannot be inspected
  - recall {Arity foo(a:1 b:2)} is [a b]
- {NewName}
  - a function that returns a new symbolic (unforgeable, i.e. cannot be guessed) name
  - foo(a:1 b:2 {NewName}:3) makes impossible to access the third component, if you do not know the arity
- {NewChunk foo(a:1 b:2 {NewName}:3) }
  - Returns what ?

Secure abstract datatypes VI

```
proc {NewWrapper ?Wrap ?Unwrap}
   Key={NewName}
in
   fun {Wrap X}
      {NewChunk foo(Key:X)}
   end
   fun {Unwrap C}
      C.Key
   end
end
```

Secure abstract data types: Another secure stack

With the new wrapper & unwrapper we can build another secure stack (since we only use the interface to wrap and unwrap, the code is identical to the one using higher-order programming)

```
local Wrap Unwrap in
    {NewWrapper Wrap Unwrap}
    fun {NewStack} {Wrap nil} end
    fun {Push S E} {Wrap E|{Unwrap S}} end
    fun {Pop S E}
        case {Unwrap S} of X|S1 then E=X  {Wrap S1} end
    end
    fun {IsEmpty S} {Unwrap S}==nil end
end
```

Exercises

58. Modify the Pascal function to use local functions for AddList, ShiftLeft, ShiftRight. Think about the abstraction and efficiency tradeoffs.
59. VRH Exercise 3.10.2 (page 230)
60. VRH Exercise 3.10.3 (page 230)
61. Develop a control abstraction for iterating over a list of elements.

Exercises

62. Implement the function {FilterAnd Xs P Q} that returns all elements of Xs in order for which P and Q return true. Hint: Use {Filter Xs P}.
63. Compute the maximum element from a nonempty list of numbers by folding.
64. Suppose you have two sorted lists. Merging is a simple method to obtain an again sorted list containing the elements from both lists. Write a Merge function that is generic with respect to the order relation.
65. VRH Exercise 3.10.17 (pg. 232). You do not need to implement it using gump, simply specify how you would add currying to Oz (syntax and semantics).