Review from Lecture 16

- STL set container class (like STL map, but without the pairs!)
- Overview of the ds_set implementation and the begin and find operations.
- In-order, pre-order, and post-order traversal; Breadth-first and depth-first tree search

Today’s Lecture

- ds_set operations: insert, destroy, printing, erase; also tree height
- Finding the in-order successor of a binary tree node

17.1 ds_set Warmup/Review Exercises

- Draw a diagram of a possible memory layout for a ds_set containing the numbers 16, 2, 8, 11, and 5. Is there only one valid memory layout for this data as a ds_set? Why?

- In what order should a forward iterator visit the data? Draw an abstract table representation of this data (omits details of TreeNode memory layout).

- Write the destroy_tree member function. This should effectively be a post-order traversal, with a node being destroyed after its left and right subtrees are destroyed.

17.2 General-Purpose Breadth-First Search/Tree Traversal

- Write an algorithm to print the nodes in the tree one tier at a time, that is, in a breadth-first manner.

- What is the best/average/worst-case running time of this algorithm? What is the best/average/worst-case memory usage of this algorithm? Give a specific example tree that illustrates each case.
17.3 Insert

- Move left and right down the tree based on comparing keys. The goal is to find the location to do an insert that preserves the binary search tree ordering property.

- We will always inserting at an empty (NULL) pointer location. Why does this work? Is there always a place to put the new item? Is there ever more than one place to put the new item?

- IMPORTANT NOTE: Passing pointers by reference ensures that the new node is truly inserted into the tree. This is subtle but important.

- Note how the return value pair is constructed.

Exercise: How does the order that the nodes are inserted affect the final tree structure? Give an ordering that produces a balanced tree and an insertion ordering that produces a highly unbalanced tree.

17.4 Erase

First we need to find the node to remove. Once it is found, the actual removal is easy if the node has no children or only one child. It is harder if there are two children:

- Find the node with the greatest value in the left subtree or the node with the smallest value in the right subtree.

- The value in this node may be safely moved into the current node because of the tree ordering.

- Then we recursively apply erase to remove that node — which is guaranteed to have at most one child.

Exercise: Write a recursive version of erase.

Exercise: How does the order that nodes are deleted affect the tree structure? Starting with a mostly balanced tree, give an erase ordering that yields an unbalanced tree.
17.5 Tree Iterator Increment/Decrement - Implementation Choices

- The increment operator should change the iterator’s pointer to point to the next TreeNode in an in-order traversal — the “in-order successor” — while the decrement operator should change the iterator’s pointer to point to the “in-order predecessor”.

- Unlike the situation with lists and vectors, these predecessors and successors are not necessarily “nearby” (either in physical memory or by following a link) in the tree, as examples we draw in class will illustrate.

- There are two common solution approaches:
  - Each node stores a parent pointer. Only the root node has a null parent pointer. [method 1]
  - Each iterator maintains a stack of pointers representing the path down the tree to the current node. [method 2]

- If we choose the parent pointer method, we’ll need to rewrite the insert and erase member functions to correctly adjust parent pointers.

- Although iterator increment looks expensive in the worst case for a single application of operator++, it is fairly easy to show that iterating through a tree storing $n$ nodes requires $O(n)$ operations overall.

**Exercise:** [method 1] Write a fragment of code that given a node, finds the in-order successor using parent pointers. Be sure to draw a picture to help you understand!

**Exercise:** [method 2] Write a fragment of code that given a tree iterator containing a pointer to the node and a stack of pointers representing path from root to node, finds the in-order successor (without using parent pointers).

**Exercise:** What are the advantages & disadvantages of each method?
// TREE NODE CLASS
template <class T>
class TreeNode {
public:
    TreeNode() : left(NULL), right(NULL) {}  
    TreeNode(const T& init) : value(init), left(NULL), right(NULL) {}
    T value;
    TreeNode* left;
    TreeNode* right;
};

// TREE NODE ITERATOR CLASS
template <class T>
class tree_iterator {
public:
    tree_iterator() : ptr_(NULL) {}  
    tree_iterator(TreeNode<T>* p) : ptr_(p) {}  
    tree_iterator(const tree_iterator& old) : ptr_(old.ptr_) {}  
    ~tree_iterator() {}  
    tree_iterator& operator=(const tree_iterator& old) { ptr_ = old.ptr_; return *this; }  
    // operator* gives constant access to the value at the pointer 
    const T& operator*() const { return ptr_->value; }  
    // comparions operators are straightforward 
    bool operator==(const tree_iterator& rgt) { return ptr_ == rgt.ptr_; }  
    bool operator!=(const tree_iterator& rgt) { return ptr_ != rgt.ptr_; }  
    // increment & decrement will be discussed in Lectures 17 & 18
private:
    // representation 
    TreeNode<T>* ptr_; 
};

// DS_SET CLASS
template <class T>
class ds_set {
public:
    ds_set() : root_(NULL), size_(0) {}  
    ds_set(const ds_set<T>& old) : size_(old.size_) { root_ = this->copy_tree(old.root_); }  
    ~ds_set() { this->destroy_tree(root_); }  
    ds_set& operator=(const ds_set<T>& old) { if (old != *this) { this->destroy_tree(root_); root_ = this->copy_tree(old.root_); size_ = old.size_; } return *this; }  
    typedef tree_iterator<T> iterator; 
    int size() const { return size_; }  
    bool operator==(const ds_set<T>& old) const { return (old.root_ == this->root_); }  
    // FIND, INSERT & ERASE 
    iterator find(const T& key_value) { return find(key_value, root_); }  
    std::pair< iterator, bool > insert(T const& key_value) { return insert(key_value, root_); }  
    int erase(T const& key_value) { return erase(key_value, root_); }  
    // OUTPUT & PRINTING 
    friend std::ostream& operator<<(std::ostream& ostr, const ds_set<T>& s) { 
        s.print_in_order(ostr, s.root_); 
        return ostr; 
    }  
    void print_as_sideways_tree(std::ostream& ostr) const { print_as_sideways_tree(ostr, root_, 0); }  
}
// ITERATORS
iterator begin() const {
    if (!root_) return iterator(NULL);
    TreeNode<T>* p = root_
    while (p->left) p = p->left;
    return iterator(p);
}
iterator end() const { return iterator(NULL); }

private:
  // REPRESENTATION
  TreeNode<T>* root_;  
  int size_;  
  // PRIVATE HELPER FUNCTIONS
  TreeNode<T>* copy_tree(TreeNode<T>* old_root) { /* Implemented in Lab 10 */ }  
  void destroy_tree(TreeNode<T>* p) { /* Implemented in Lecture 17 */ }  

  iterator find(const T& key_value, TreeNode<T>* p) {
    if (!p) return iterator(NULL);
    if (p->value > key_value)
       return find(key_value, p->left);
    else if (p->value < key_value)
       return find(key_value, p->right);
    else
       return iterator(p);
  }

  std::pair<iterator,bool> insert(const T& key_value, TreeNode<T>*& p) {
    if (!p) {
      p = new TreeNode<T>(key_value);
      this->size_++;
      return std::pair<iterator,bool>(iterator(p), true);
    }
    else if (key_value < p->value)
       return insert(key_value, p->left);
    else if (key_value > p->value)
       return insert(key_value, p->right);
    else
       return std::pair<iterator,bool>(iterator(p), false);
  }

  int erase(T const& key_value, TreeNode<T>* &p) {
    /* Implemented in Lecture 17 */
  }

  void print_in_order(std::ostream& ostr, const TreeNode<T>* p) const {
    if (p) {
      print_in_order(ostr, p->left);
      ostr << p->value << "\n";
      print_in_order(ostr, p->right);
    }
  }

  void print_as_sideways_tree(std::ostream& ostr, const TreeNode<T>* p, int depth) const {
    if (p) {
      print_as_sideways_tree(ostr, p->right, depth+1);
      for (int i=0; i<depth; ++i) ostr << " ";
      ostr << p->value << "\n";
      print_as_sideways_tree(ostr, p->left, depth+1);
    }
  }
};