Review of Lecture 14

- General Problem Solving Techniques:
  1. Generating and Evaluating Ideas
  2. Mapping Ideas into Code
  3. Getting the Details Right
- Small exercises to practice these techniques
- Problem Solving Strategies / Checksheet

Today!

- More on Complexity
- Problem Solving Example: Quicksort (& compare to Mergesort)
- Design Example: Conway’s Game of Life

Clearing Up Exponential Complexity

- Last time the instructors got tripped up, so let’s start by quickly fixing our understanding of $O(s^8)$ vs $O(8^s)$.
- Recall that in the non-linear word search, from any position there are a maximum of 8 choices, so any recursive call can lead to up to 8 more!
- Remember the board is $w$ wide, $h$ high, and we are searching for a word with length $s$.
- For $s=1$ and an initial position, there’s no recursion. Either we found the correct letter, or we didn’t.
- For $s=2$, and an initial position $(i, j)$, there are 8 calls: $(i-1, j-1)(i-1, j)(j-1, j)(i, j+1), (i+1, j+1), (i+1, j)(i+1, j-1), (i, j-1)$. This is $8^1 = 8$ calls.
- Now consider $s=3$. For each of the 8 positions from $s=2$, we can try 8 more positions. So that’s $8 \times 8 = 8^2 = 64$ total calls.
- For $s=i$, we could repeat this, each time we’re multiplying by another 8, because every position from $s=i-1$ can try 8 more positions.
- In general, our solution looks like $8^{(s-1)} = 8^s \times 8^{-1}$. Since $8^{-1}$ is just a constant, we can say $O(8^s)$.
- This isn’t the whole picture though. Let’s consider a few cases:
  - $w \times h = 50,000$, $s = 2$? $s = 4$? $s = 50,000$?
  - $w \times h = 4$, $s = 2$? $s = 4$? $s = 50,000$?
- How we would write a recursion to be $O(s^8)$?

```c
int func(int s, int layer){
  if(layer==0){ return 1; }
  int ret = 0;
  //Make s calls
  for(int i=0; i<s; i++){
    ret += func(s,layer-1);
  }
  return ret;
}
```

```
func(1,8); => 1
func(2,8); => 256
func(3,8); => 6561
func(4,8); => 65536
```
15.1 Example: Quicksort

- Quicksort also the partition-exchange sort is another efficient sorting algorithm. Like mergesort, it is a divide and conquer algorithm.

- The steps are:
  1. Pick an element, called a pivot, from the array.
  2. Reorder the array so that all elements with values less than the pivot come before the pivot, while all elements with values greater than the pivot come after it (equal values can go either way). After this partitioning, the pivot is in its final position. This is called the partition operation.
  3. Recursively apply the above steps to the sub-array of elements with smaller values and separately to the sub-array of elements with greater values.

```cpp
// Choose a "pivot" and rearrange the vector. Returns the location of the pivot, separating top & bottom (hopefully it's near the halfway point).
int partition(vector<double>& data, int start, int end, int& swaps) {
    int mid = (start + end)/2;
    double pivot = data[mid];
```
• What value should you choose as the pivot? What are our different options?

• What is the order notation for the running time of this algorithm?
  What is the order notation for the additional memory use of this algorithm?

• What is the best case for this algorithm? What is the worst case for this algorithm?

• Compare the design of Quicksort and Mergesort. What is the same? What is different?

15.2 Design Example: Conway's Game of Life

Let's design a program to simulate Conway's Game of Life. Initially, due to time constraints, we will focus on the main data structures of needed to solve the problem.

Here is an overview of the Game:

• We have an infinite two-dimensional grid of cells, which can grow arbitrarily large in any direction.
• We will simulate the life & death of cells on the grid through a sequence of generations.
• In each generation, each cell is either alive or dead.
• At the start of a generation, a cell that was dead in the previous generation becomes alive if it had exactly 3 live cells among its 8 possible neighbors in the previous generation.
• At the start of a generation, a cell that was alive in the previous generation remains alive if and only if it had either 2 or 3 live cells among its 8 possible neighbors in the previous generation.
  – With fewer than 2 neighbors, it dies of “loneliness”.
  – With more than 3 neighbors, it dies of “overcrowding”.
• Important note: all births & deaths occur simultaneously in all cells at the start of a generation.
• Other birth / death rules are possible, but these have proven to be a very interesting balance.
• Many online resources are available with simulation applets, patterns, and history. For example:
  http://www.bitstorm.org/gameoflife/
Applying the Problem Solving Strategies

In class we will brainstorm about how to write a simulation of the Game of Life, focusing on the representation of the grid and on the actual birth and death processes.

Understanding the Requirements

We have already been working toward understanding the requirements. This effort includes playing with small examples by hand to understand the nature of the game, and a preliminary outline of the major issues.

Getting Started

- What are the important operations?
- How do we organize the operations to form the flow of control for the main program?
- What data/information do we need to represent?
- What will be the main challenges for this implementation?

Details

- New Classes? Which STL classes will be useful?

Testing

- Test Cases?

15.3 Generating Ideas

- If running time & memory are not primary concerns, and the problems are small, what is the simplest strategy to make sure all solutions are found. Can you write a simple program that tries all possibilities?

- What variables will control the running time & memory use of this program? What is the order notation in terms of these variables for running time & memory use?

- What incremental (baby step) improvements can be made to the naive program? How will the order notation be improved?

15.4 Mapping Ideas to Code

- What are the key steps to solving this problem? How can these steps be organized into functions and flow of control for the main function?

- What information do we need to store? What C++ or STL data types might be helpful? What new classes might we want to implement?
15.5 Getting the Details Right

- What are the simplest test cases we can start with (to make sure the control flow is correct)?

- What are some specific (simple) corner test cases we should write so we won’t be surprised when we move to bigger test cases?

- What are the limitations of our approach? Are there certain test cases we won’t handle correctly?

- What is the maximum test case that can be handled in a reasonable amount of time? How can we measure the performance of our algorithm & implementation?