CSCI-1200 Data Structures — Spring 2019
Lecture 18 – Trees, Part II

Review from Lecture 17 and Lab 9

- Binary Trees, Binary Search Trees, & Balanced Trees
- STL `set` container class (like STL `map`, but without the pairs!)
- Finding the smallest element in a BST.
- Overview of the `ds_set` implementation: `begin` and `find`.

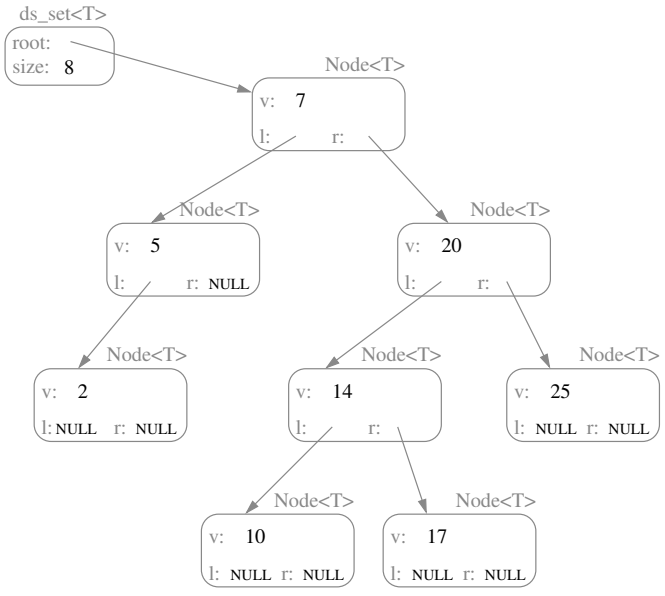
Today's Lecture

- Warmup / Review: `destroy_tree`
- A very important `ds_set` operation insert
- In-order, pre-order, and post-order traversal
- Finding the *in-order successor* of a binary tree node, tree iterator increment
- HW8 discussion

18.1 Warmup Exercise

- Write the `ds_set::destroy_tree` private helper function.

18.2 Insert



- Move left and right down the tree based on comparing keys. The goal is to find the location to do an insert *that preserves the binary search tree ordering property.*
- We will always be inserting at an empty (NULL) pointer location.
- **Exercise:** Why does this work? Is there always a place to put the new item? Is there ever more than one place to put the new item?

- IMPORTANT NOTE: Passing pointers by reference ensures that the new node is truly inserted into the tree. This is subtle but important.
- Note how the return value pair is constructed.
- **Exercise:** How does the order that the nodes are inserted affect the final tree structure? Give an ordering that produces a balanced tree and an insertion ordering that produces a highly unbalanced tree.

18.3 In-order, Pre-order, Post-order Traversal

- Reminder: For an exactly balanced binary search tree with the elements 1-7:
  - In-order: `1 2 3 (4) 5 6 7`
  - Pre-order: `(4) 2 1 3 6 5 7`
  - Post-order: `1 3 2 5 7 6 (4)`
- Now let's write code to print out the elements in a binary tree in each of these three orders. These functions are easy to write recursively, and the code for the three functions looks amazingly similar. Here's the code for an in-order traversal to print the contents of a tree:

```
void print_in_order(ostream& ostr, const TreeNode<T>* p) {
  if (p) {
    print_in_order(ostr, p->left);
    ostr << p->value << "\n";
    print_in_order(ostr, p->right);
  }
}
```

- How would you modify this code to perform pre-order and post-order traversals?

- What is the traversal order of the `destroy_tree` function we wrote earlier?

18.4 Tree Iterator Increment/Decrement - Implementation Choices

- The increment operator should change the iterator's pointer to point to the next TreeNode in an in-order traversal — the "in-order successor" — while the decrement operator should change the iterator's pointer to point to the "in-order predecessor".
- Unlike the situation with lists and vectors, these predecessors and successors are not necessarily "nearby" (either in physical memory or by following a link) in the tree, as examples we draw in class will illustrate.
- There are two common solution approaches:
  - Each node stores a parent pointer. Only the root node has a null parent pointer. [method 1]
  - Each iterator maintains a stack of pointers representing the path down the tree to the current node. [method 2]
- If we choose the parent pointer method, we'll need to rewrite the `insert` and `erase` member functions to correctly adjust parent pointers.
- Although iterator increment looks expensive in the worst case for a single application of `operator++`, it is fairly easy to show that iterating through a tree storing n nodes requires $O(n)$ operations overall.

Exercise: [method 1] Write a fragment of code that given a node, finds the in-order successor using parent pointers. Be sure to draw a picture to help you understand!

Exercise: [method 2] Write a fragment of code that given a tree iterator containing a pointer to the node *and* a stack of pointers representing path from root to node, finds the in-order successor (without using parent pointers).

Either version can be extended to complete the implementation of increment/decrement for the `ds_set` *tree iterators.*

Exercise: What are the advantages & disadvantages of each method?

18.5 String Performance

- To motivate our new data structure for HW8, let's first think back to the idea of a string.
- Specifically, we will consider a string as either `char*` or as `vector<char>`
- Let's suppose we have a string of length 500. For each of these cases, what is the running time, using n for length of the string:
  1. `insert()/erase()` at the end, assuming we have room for at least 501 characters already
  2. `insert()/erase()` at the end, but we don't have extra room at the end of the array/vector
  3. `insert()/erase()` at middle (index 249 in our example), assuming we have room for at least 501 characters already?
- Many data structures involve a trade-off: we make something faster at the expense of additional space, or by making other operations slower. We will look at a way to speed up `insert()/erase()` but it will slow down our `operator[]`. However, it will not be as bad as if we fixed this by using a `list<char>` and we will be able to quickly use an index instead of iterators.
- Some of this material is already in the HW8 handout and will not be reprinted in the Lecture 18 handout. See handout for image source and risks with using outside resources for HW8.

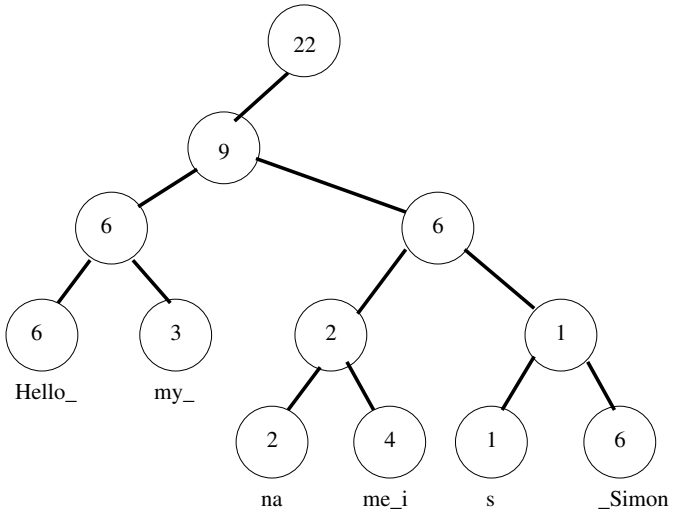
18.6 Rope Node class

- We'll use a tree-like structure. First consider this modified `Node` class:

```
class Node{
public:
    Node() : left{NULL}, right{NULL}, parent{NULL}, weight{0} {}
    Node* left;
    Node* right;
    Node* parent;
    int weight;
    std::string value;
};
```

18.7 Rope Example

- Now we can construct a `Rope` by using the following rules:
  1. Only leaves will have a non-empty `value` member.
  2. The `weight` of a leaf is the length of its `value` member.
  3. For any non-leaf, the `weight` is the sum of the length of every `value` in its left subtree.
- To the right is one possible way we could make a `Rope` for the string "Hello my name is Simon". To make it clear where spaces are, our figure uses '_', but the real `value` uses ' '.



18.8 Rope *index()*

- In binary search trees, we used `operator<` to decide whether to go left or right. In a Rope that won't work.
- See the homework handout for details
- Let's walk through calls to `index(0)`, `index(8)`, and `index(20)`, keeping track of i and whether we go left or right at every step.

18.9 Rope *concat()*

- Concatenation is a pretty simple operation.
- We must carefully consider if we want to copy nodes or just glue existing nodes together.
- We often talk about a left-hand side (LHS) and right-hand side (RHS) when we have two operands
- See the homework handout for details
- The version in our code should copy *rhs*'s values, because otherwise `~lhs()` and `~rhs()` will try to free the same nodes.
- However a version that doesn't copy anything will be useful for `split()`.

18.10 Rope *split()*

- Breaking a `Rope` into two smaller `Rope`s is another important operation.
- We'll start by considering the "main" case, where the split will happen between two nodes, for example $i = 11$
- See the homework handout for details
- During the concatenation of severed nodes, should we make copies? Why or why not? *Hint: Think about what* `~lhs()` *and* `~rhs()` *can and can't reach.*
- If the split happens in the middle of a leaf, such as with $i = 12$, we just need to split the leaf so that $i = 12$ is at the start of a leaf. This is like concatenate in reverse.

18.11 HW8 Rope Limitations

- Normally to stay efficient, a balanced tree is used
- Whenever we do an operation like `split()`, `insert()`, or `erase()`, we would normally rebalance the tree.
- We're going to skip that for now, but try and think about how this might impact your rope's performance.
- Another issue is iterators - we will be using the same iterator we used today for BSTs.
- A common task is to just move between leaves in a rope, so real implementations often have a seperate iterator for leaf traversal
- This still requires a fair amount of work, but for very large strings with many operations performed on them, n vs. $\frac{n}{2}$ can be a huge savings.

```
// ---------------------------------------------------------------------
// TREE NODE CLASS
template <class T>
class TreeNode {
public:
  TreeNode() : left(NULL), right(NULL)/*, parent(NULL)*/ {}
  TreeNode(const T& init) : value(init), left(NULL), right(NULL)/*, parent(NULL)*/ {}
  T value;
  TreeNode* left;
  TreeNode* right;
  // one way to allow implementation of iterator increment & decrement
  // TreeNode* parent;
};

// ---------------------------------------------------------------------
// TREE NODE ITERATOR CLASS
template <class T>
class tree_iterator {
public:
  tree_iterator() : ptr_(NULL) {}
  tree_iterator(TreeNode<T>* p) : ptr_(p) {}
  tree_iterator(const tree_iterator& old) : ptr_(old.ptr_) {}
  ~tree_iterator() {}
  tree_iterator& operator=(const tree_iterator& old) { ptr_ = old.ptr_;  return *this; }
  // operator* gives constant access to the value at the pointer
  const T& operator*() const { return ptr_->value; }
  // comparions operators are straightforward
  bool operator== (const tree_iterator& rgt) { return ptr_ == rgt.ptr_; }
  bool operator!= (const tree_iterator& rgt) { return ptr_ != rgt.ptr_; }
  // increment & decrement operators
  tree_iterator<T> & operator++() { /* discussed & implemented in Lecture 19 */









    return *this;
  }
  tree_iterator<T> operator++(int) {  tree_iterator<T> temp(*this);  ++(*this);  return temp;  }
  tree_iterator<T> & operator--() { /* implementation omitted */ }
  tree_iterator<T> operator--(int) {  tree_iterator<T> temp(*this);  --(*this);  return temp;  }

private:
  // representation
  TreeNode<T>* ptr_;
};

// ---------------------------------------------------------------------
// DS_SET CLASS
template <class T>
class ds_set {
public:
  ds_set() : root_(NULL), size_(0) {}
  ds_set(const ds_set<T>& old) : size_(old.size_) { root_ = this->copy_tree(old.root_,NULL); }
  ~ds_set() { this->destroy_tree(root_); root_ = NULL; }
  ds_set& operator=(const ds_set<T>& old) { /* implementation omitted */ }

  typedef tree_iterator<T> iterator;

  int size() const { return size_; }
  bool operator==(const ds_set<T>& old) const { return (old.root_ == this->root_); }
```

```
  // FIND, INSERT & ERASE
  iterator find(const T& key_value) { return find(key_value, root_); }
  std::pair< iterator, bool > insert(T const& key_value) { return insert(key_value, root_); }
  int erase(T const& key_value) { return erase(key_value, root_); }

  // OUTPUT & PRINTING
  friend std::ostream& operator<< (std::ostream& ostr, const ds_set<T>& s) {
    s.print_in_order(ostr, s.root_);
    return ostr;
  }

  // ITERATORS
  iterator begin() const {
    if (!root_) return iterator(NULL);
    TreeNode<T>* p = root_;
    while (p->left) p = p->left;
    return iterator(p);
  }
  iterator end() const { return iterator(NULL); }

private:
  // REPRESENTATION
  TreeNode<T>* root_;
  int size_;

  // PRIVATE HELPER FUNCTIONS
  TreeNode<T>*  copy_tree(TreeNode<T>* old_root) { /* Implemented in Lab 9 */ }
  void destroy_tree(TreeNode<T>* p) {
     /* Implemented in Lecture 18 */









  }

  iterator find(const T& key_value, TreeNode<T>* p) {  /* Implemented in Lecture 17 */  }

  std::pair<iterator,bool> insert(const T& key_value, TreeNode<T>*& p) {
    // NOTE: will need revision to support & maintain parent pointers
    if (!p) {
      p = new TreeNode<T>(key_value);
      this->size_++;
      return std::pair<iterator,bool>(iterator(p), true);
    }
    else if (key_value < p->value)
      return insert(key_value, p->left);
    else if (key_value > p->value)
      return insert(key_value, p->right);
    else
      return std::pair<iterator,bool>(iterator(p), false);
  }

  int erase(T const& key_value, TreeNode<T>* &p) { /* Implemented in Lecture 19 */ }

  void print_in_order(std::ostream& ostr, const TreeNode<T>* p) const {
    if (p) {
      print_in_order(ostr, p->left);
      ostr << p->value << "\n";
      print_in_order(ostr, p->right);
    }
  }
};
```