The Post Correspondence Problem

continued

1. We will prove that the
   MPC problem is undecidable

2. We will prove that the
   PC problem is undecidable

1. We will prove that the
   MPC problem is undecidable

We will reduce the membership problem
to the MPC problem

Membership problem

Input: recursive language L

string w

Question: $w \in L$?

Undecidable

Membership problem

Input: unrestricted grammar G

string w

Question: $w \in L(G)$?

Undecidable

The reduction of the membership problem to the MPC problem:

For unrestricted grammar G and string w

we construct a pair A, B such that

A, B has an MPC-solution
if and only if $w \in L(G)$

A	B	Grammar G
$FS \Rightarrow$	F	S : start variable F : special symbol
a	a	For every symbol a
V	V	For every variable V

A	B	Grammar G
E	$\Rightarrow wE$	string w E : special symbol
y	x	For every production $x \rightarrow y$
$\Rightarrow$	$\Rightarrow$	

Example:

Grammar G :

$$S \to aABb \mid Bbb$$

$$Bb \to C$$

$$AC \to aac$$

String $w = aaac$

A		B	
$w_1:$	$FS \Rightarrow$	$v_1:$	F
$w_2:$	a	$v_2:$	a
	b		b
	c		c
$\vdots$	A	$\vdots$	A
	B		B
	C		C
$w_8:$	S	$v_8:$	S

A		B	
$w_9:$	$E \Rightarrow$	$v_9:$	$aaacE$
	$aABb$		S
	Bbb		S
$\vdots$		$\vdots$	
	C		Bb
	aac		AC
$w_{14}:$	$\Rightarrow$	$v_{14}:$	$\Rightarrow$

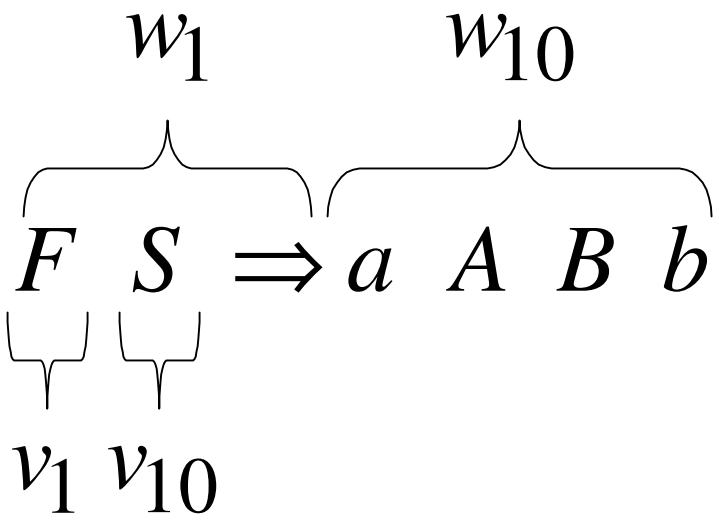
$$w = aaac$$

$$aaac \in L(G)$$

$$S \Rightarrow aABb \Rightarrow aAC \Rightarrow aaac$$

$$S \Rightarrow aABb$$

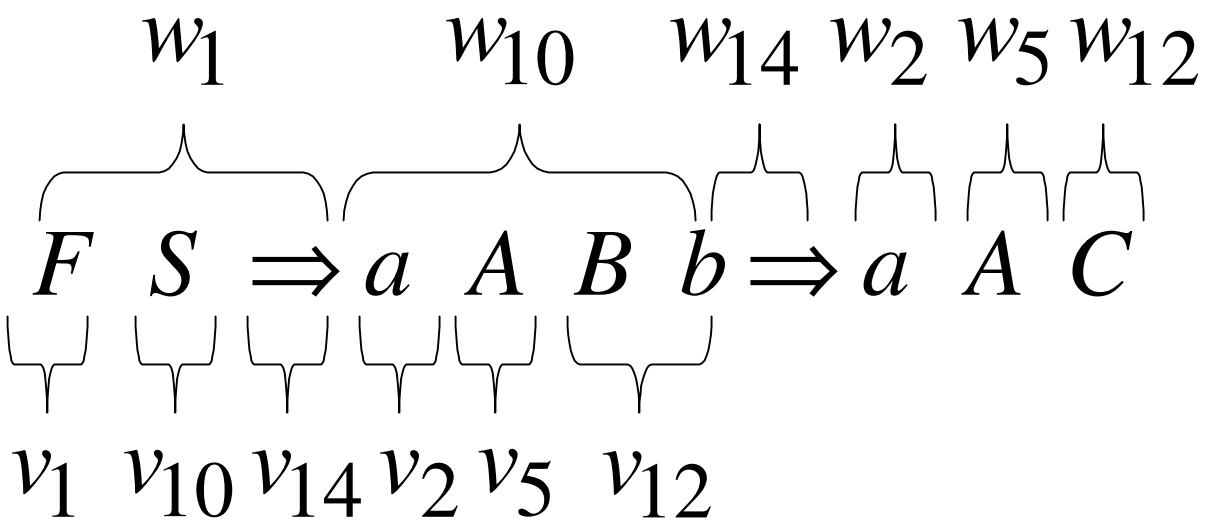
$$A$$



$$B$$

$$S \Rightarrow aABb \Rightarrow aAC$$

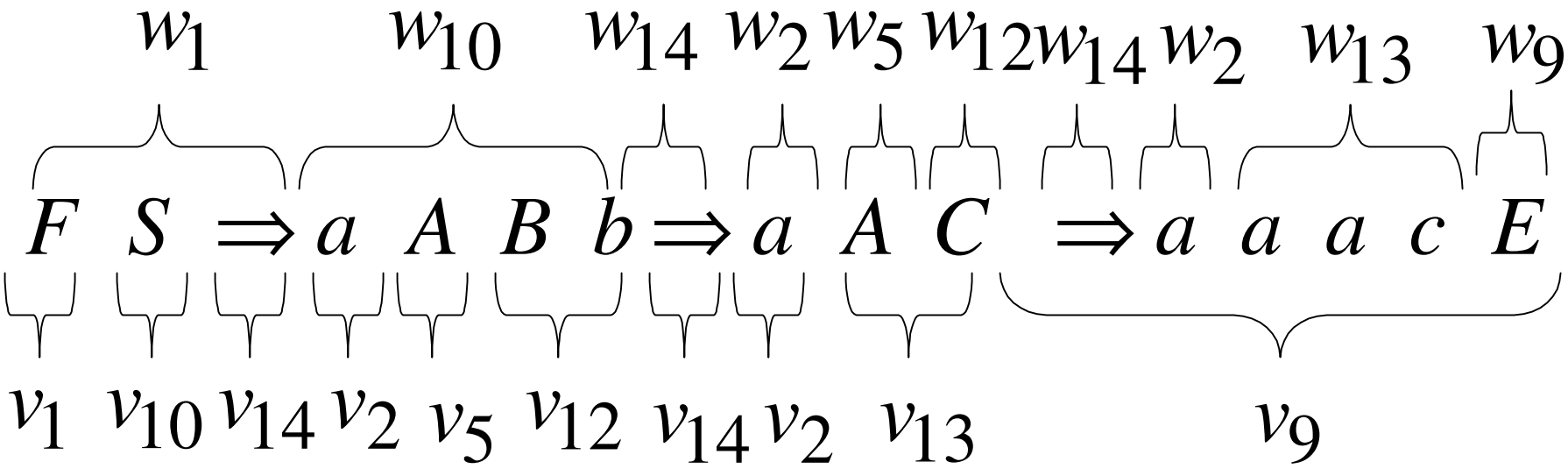
A



B

$$S \Rightarrow aABb \Rightarrow aAC \Rightarrow aaac$$

A



B

Theorem:

(A, B) has an MPC-solution

if and only if $w \in L(G)$

Algorithm for membership problem:

Input: unrestricted grammar G

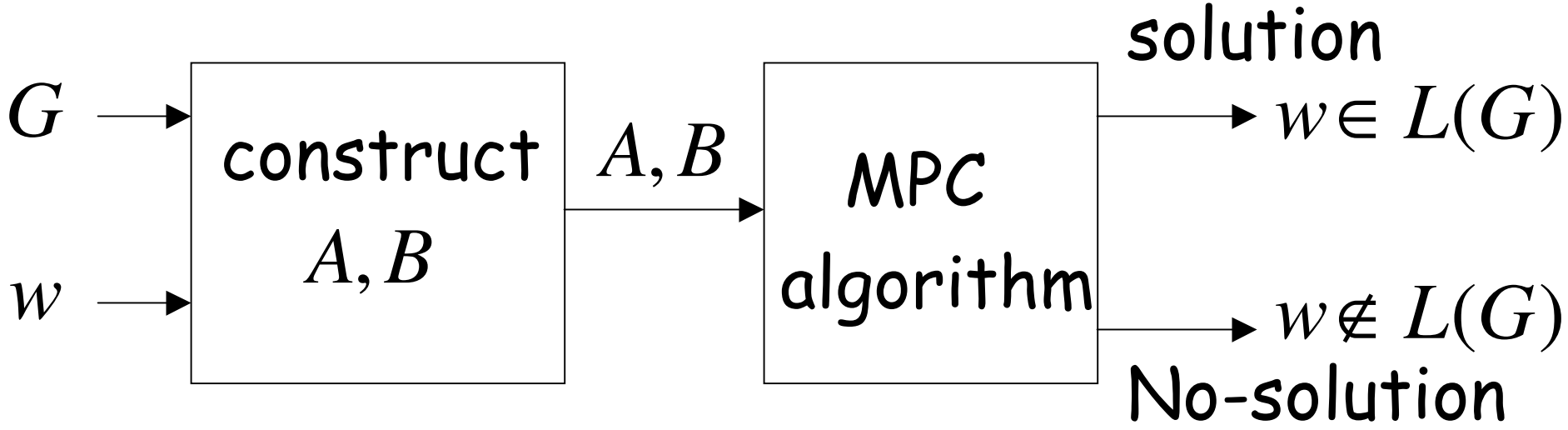
string w

Construct the pair A, B

If A, B has an MPC-solution then $w \in L(G)$

else $w \notin L(G)$

Membership machine



2. We will prove that the
   PC problem is undecidable

We will reduce the MPC problem
to the PC problem

A, B : input to the MPC problem

$$A = w_1, w_2, \ldots, w_n$$

$$B = v_1, v_2, \ldots, v_n$$

We construct a new sequences C, D

$$C = w'_0, w'_1, \ldots, w'_n, w'_{n+1}$$

$$D = v'_0, v'_1, \ldots, v'_n, v'_{n+1}$$

$$A = w_1, w_2, \ldots, w_n$$

$$B = v_1, v_2, \ldots, v_n$$

A $w_i = abcad$

C $w_i' = a * b * c * a * d *$

We insert a special symbol between any two symbols

B $v_i = abcad$

D $v_i' = *a*b*c*a*d$

Special Cases

C

$$w'_0 = * \, w_1$$

$$w'_{n+1} = \Diamond$$

D

$$v'_0 = v'_1$$

$$v'_{n+1} = *\Diamond$$

Observation:

There is a PC-solution for C, D
if and only if
there is a MPC-solution for A, B

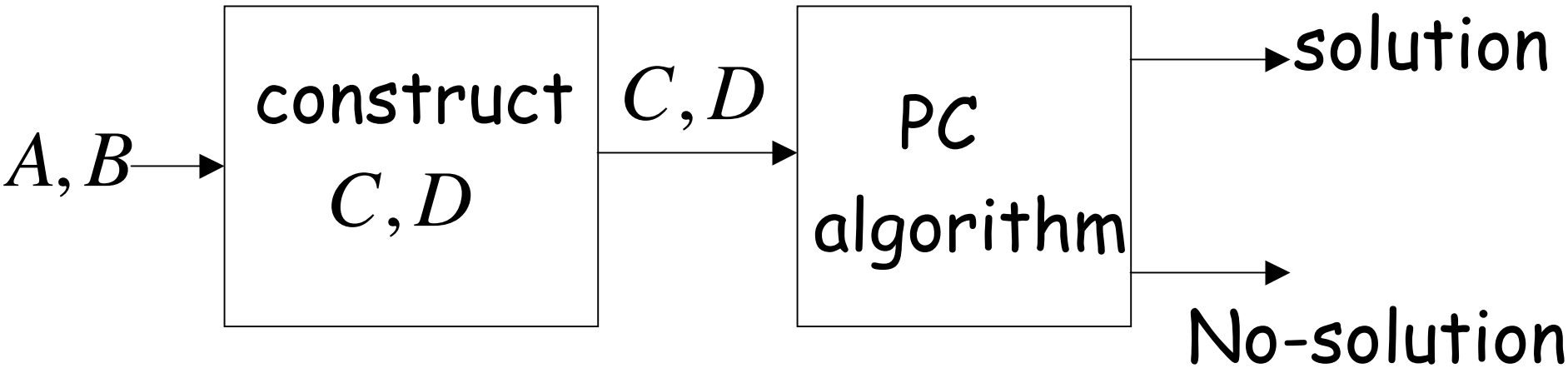
PC-solution $w'_0 \cdots w'_k w'_{n+1} = v'_0 \cdots w'_k v'_{n+1}$

MPC-solution $w_1 \cdots w_k = v_1 \cdots v_k$

MPC-algorithm

Input: sequences A, B

Construct sequences C, D

Solve the PC problem for C, D

MPC algorithm