CSCI 2400 – Models of Computation, Section 3

Homework 4

Return by: Thursday, October 12

1. Use the pumping lemma to prove that the following languages are not regular. (The alhabet is $\Sigma = \{a, b\}$.)

(a) $L = \{a^n b^l a^k : k \geq n + l\}$

(b) $L = \{ww : w \in \Sigma^*\}$

(c) $L = \{a^{k^2}\}$

(d) $L = \{uww^R v : u, v, w \in \Sigma^+, |u| \geq |v|\}$

2. Which of the following languages are regular? Give short explanations. (The alphabet is $\Sigma = \{a, b\}$.)

(a) $L = \{w : n_a(w) \neq n_b(w)\}$

(b) $L = \{a^n b^l : n \geq 100, l \leq 100\}$

(c) $L = \{uww^R v : u, v, w \in \Sigma^+\}$

(d) $L = \{b^n a^l b^k : n > 5, l > 3, l \geq k\}$

3. Find context-free grammars for the following languages. Explain your answers.

(a) $L = \{w :$ w starts and ends with the same symbol, and $w \in \{a, b\}^*\}$

(b) The complement of the language $L = \{a^n b^n\}$.

(c) $L = \{w : n_a(w) = 2n_b(w),$ where $w \in \{a, b\}^*\}$

(d) $L = \{w\#x : w^R$ is a substring of x, where $w, x \in \{a, b\}^*\}$

4. You are given the following ambiguous context-free grammar:

$$\begin{aligned} S &\rightarrow AB|aaB \\ A &\rightarrow a|Aa \\ B &\rightarrow b \end{aligned}$$

(a) Find a string s generated by the grammar that has two leftmost derivations. Show the derivations.

(b) Show the two derivation trees for the string s.

(c) Find an equivalent unambiguous context-free grammar. Explain your answer.

(d) Give the unique leftmost derivation and derivation tree for the string s generated from the unambiguous grammar of part (c).