Assignment 1
CSCI-4963/6962: Geometric Algorithms
Due: Tuesday, January 25, 2000

Assignments are due at the beginning of class on January 25, and are to be done individually. Assignments will be graded on the basis of correctness, clarity, and legibility. Late assignments incur a 10% penalty.

1. The convex hull of a set S is defined to be the intersection of all convex sets that contain S. The convex hull of a set of points is the convex set with smallest perimeter. We want to show that these are equivalent definitions.

   a. Prove that the intersection of two convex sets is again convex. This implies that the intersection of a finite family of convex sets is convex as well.

   b. Prove that the smallest perimeter polygon P containing a set of points S is convex.

   c. Prove that any convex set containing the set of points S contains the smallest perimeter polygon P.

2. Let P be a nonconvex polygon with n vertices. Describe an algorithm that computes the convex hull of P in $O(n)$ time.

3. Consider the farthest-pair problem:

   a. Given a set of points P in the plane, prove that the pair of points farthest from each other must be vertices of $CH(P)$.

   b. Give an $O(n \log n)$-time algorithm to find the two points whose distance from each other is maximum.

4. On-line convex hull problem: We are given the set P of n points one at a time. After receiving each point, we are to compute the convex hull of the points seen so far. Show how to solve the on-line convex hull problem in a total of $O(n^2)$ time.

5. Here we develop a divide-and-conquer algorithm to compute the convex hull of a set of n points in the plane.

   a. Let P_1 and P_2 be two disjoint convex polygons with n vertices in total. Give an $O(n)$ time algorithm that computes the convex hull of $P_1 \cup P_2$.

   b. Use the algorithm from part a to develop an $O(n \log n)$ time divide-and-conquer algorithm to compute the convex hull of a set of n points in the plane.

6. Give an $O(n \log n)$-time algorithm to determine whether two simple polygons with a total of n vertices intersect. A *simple polygon* is a region enclosed by a single closed polygonal chain that does not intersect itself.