

Assignment 3

CSCI-4963/6962: Geometric Algorithms

Due: Thursday, February 24, 2000

Assignments are due at the beginning of class on February 24, and are to be done individually. Assignments will be graded on the basis of correctness, clarity, and legibility. Late assignments incur a 10% penalty. Assignments handed in more than a week late will receive no credit.

This assignment focuses on Voronoi diagrams and Delaunay triangulations.

1. Let P be a set of n points in the plane. Give an $O(n \log n)$ time algorithm to find two points in P that are closest together. Show that your algorithm is correct.
2. Let P be a set of n points in the plane. Give an $O(n \log n)$ time algorithm to find for each point p in P another point in P that is closest to p .
3. The *medial axis* of a simple polygon P is the set of points μ inside P such that each point in μ has more than one closest point among the points on the boundary of P . So every point of the medial axis is the center of a circle that touches the boundary in at least two points. Give an efficient algorithm to compute the medial axis of a convex polygon.
4. Show that no set of n points can be triangulated in more than $2^{\binom{n}{2}}$ ways.
5. Prove that the smallest angle of any triangulation of a convex polygon whose vertices lie on a circle is the same. This implies that any completion of the Delaunay triangulation of a set of points maximizes the minimum angle.
6. A *Euclidean Minimum Spanning Tree* (EMST) of a set of points in the plane is a tree of minimum total edge length connecting all the points.
 - a. Prove that the set of edges of a Delaunay triangulation of a set of points P in the plane contains an EMST for P .
 - b. Use this result to give an $O(n \log n)$ algorithm to compute an EMST for a given set of n points.