1 Introduction

We study a family of visibility and surveillance problems arising in data centers that are similar to art gallery problems. In traditional art gallery problems (see [3], [4], [5] and [6]) an entire polygonal region must be kept under surveillance. In our case it is a prescribed collection of line segments in the interior of the polygon which must be kept under surveillance. Czyzowicz, Rivera-Campo, Urrutia and Zacks [2] studied a similar problem to ours (an analog of Lemma 1, case 3), but without the presence of a boundary.

With power costs escalating, there has been increasing emphasis on keeping power consumption in data centers as low as possible. With this objective in mind, care has been given to modeling the flow of air around IT and facilities equipment in data centers to quickly detect emerging hot spots and optimize the use of air conditioning delivered via CRAC (Computer Room Air Conditioning) units.

One approach to understanding air flow in data centers is to deploy a large number of static temperature and air pressure sensors and feed this data into a computational fluid dynamics (CFD) model, as has been done by some of our IBM colleagues in [1]. Another approach is to deploy a significantly smaller number of infrared cameras, take temperature readings at all surfaces of interest and as a result dispense with, or at least reduce the need to use, CFD models.

Infrared cameras, however, are much more expensive than temperature or air pressure sensors so the challenge is to deploy as few of these cameras as possible. A sample, relatively small, and somewhat idealized data center, is depicted in Figure 1.

The basic problem we investigate is that of finding the minimum number of cameras needed to completely survey the entire length of all segments of interest in such a data center. Given the fact that we are dealing just with a two dimensional projection, these segments correspond more generally to surfaces in the full three dimensional problem.

2 Results

We note that the data center in Figure 1 is almost rectangular. This is typical of data centers we have studied. Hence, in what follows we consider problems with a rectangular boundary. Secondly, we note that there are certain objects in the data center that can potentially block the field of view of a camera. However, these field of view blocks tend not to impinge upon the field of view for reasonable camera placements. There is one exception, namely the objects which we must keep under surveillance also serve as field of view blocks. We thus ignore the possibility of field of view blocks which are not themselves objects which must be kept under surveillance. Next, surfaces of interest in a data center must be viewed from a prescribed side; in Figure 1 some surfaces need to be viewed from the right and some surfaces need to be viewed from the left. Finally, in the vast majority of data centers we have studied surfaces requiring thermal measurement can be thought of as having just vertical and horizontal orientations, and in many cases, as is the case in Figure 1, just one of these orientations. Hence, to begin our investigations we consider problems where the objects to be kept under surveillance are all vertical segments. Segments and cameras may come arbitrarily close to the boundary but not touch it.

We start off with some simple observations:

Lemma 1 Given n vertical segments in the plane and a bounding rectangle,

1. \([n/2]\) cameras are sufficient, and for each n sometimes necessary, to entirely see all n segments from the left.

2. n cameras are sufficient, and for each n sometimes necessary, to entirely see all segments from both sides (except for the special case where n = 1 which requires two cameras).

3. \([n/3]\) cameras are sufficient, and for each n sometimes necessary, to entirely see all segments from one side or the other (solver’s choice).

4. \([n/2] + 1\) cameras are sufficient, and for each n sometimes necessary, to entirely see all segments from one side or the other (poser’s choice).
In data centers, the most realistic scenario is that covered by the last of these cases.

In thinking about constructing a greedy heuristic for placing cameras, a natural question is the following: Given a single camera, what is the most, in terms of total length of segments, that it can see? In this case, we consider only visibility of segments from the left. If there is no minimum separation between segments, or limit to the angle of visibility of the cameras, a single camera can see arbitrarily much, as made precise by the following:

**Theorem 2** Given a bounding rectangle of width \( w \) and height \( h \), let \( d \) be the minimum horizontal separation between vertical segments with \( \frac{w}{k} < d \leq \frac{w}{k+1} \), so that, e.g., there can be at most \( k + 1 \) vertical segments. Then the maximum total length of segments visible from a single camera is \( h \ln k + O(h) \).

![Figure 2](image2.png)

Figure 2. Placement of camera (small disk at the top left) and segments, so that the camera sees the maximum total length of segments, for a minimum segment separation of \( d = \frac{w}{3} - \epsilon \) with \( \epsilon \ll \frac{w}{3} \).

An alternative to mandating a minimum separation between segments is to limit the angle of visibility of any placed camera to something less than \( \pi \) radians, in other words, when considering visibility from the left, an angle less than \( \pi/2 \) with the horizontal. See Figure 3. In this case one easily obtains:

**Lemma 3** Given a set of segments in a rectangle of width \( w \) and height \( h \) and a camera with a maximum viewing angle of \( \Theta \) radians, then the camera can see segments of total length no more than

\[
\frac{2w}{\tan \frac{\pi - \Theta}{2}} + h.
\]

This result is not tight. It would be interesting to obtain a more precise bound. In this bounded angle case we also have a hardness result:

**Theorem 4** Given \( n \) segments which must be guarded from the left in a bounding rectangle and a limited angle of visibility \( \Theta \) relative to the horizontal, it is NP-hard to decide if \( k \) cameras can entirely see all segments.

We are interested in devising practical heuristics for locating cameras. The next Lemma connects the possible approximation ratio of an algorithm that sees all segments from the left side (denoted by \( A_L \)) and another which sees all segments from both sides (\( A_B \)).

**Lemma 5** Suppose \( A_L \) is a \( f(n) \)-approximation algorithm to the problem of seeing all of \( n \) segments from the left. Let \( A_B \) be defined by applying \( A_L \) twice, once to see all segments from the left, and a second time, applying the symmetrical algorithm to see all segments from the right and taking the union of all camera locations. Then \( A_B \) is a \( 2f(n) \)-approximation algorithm to the problem of seeing all segments from both sides.

3 Future Work

We hope to extend Lemma 1 to more generically placed segments, develop practical heuristics for camera placements, potentially with approximation guarantees, and finally, extend Theorem 4 to additional cases.

References


