Connectivity-based Sensor Network Localization with Incremental Delaunay Refinement

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I. Introduction

The location of sensor nodes is an indispensable component for both network operation and sensor data integrity. In this paper we study the network localization problem for a large-scale sensor network with a complex shape. We do not assume any anchor nodes with known locations and use only network connectivity information to recover the relative positioning of all the nodes. Thus we require no extra hardware supplements (e.g., for angle or distance measurements) and investigate a fundamental problem: can the network geometry be reconstructed using network connectivity alone? This is a simple yet very challenging setting for network localization.

The Challenge of Localization A major challenge in anchor-free localization is to handle possible flip ambiguities. Two triangles sharing an edge can be embedded in two possible ways, with the two triangles on the same side, or on opposite sides of the common edge. In general, whether a graph has a unique embedding or not is investigated in graph rigidity theory [4]. A graph is rigid in 2D if a realization of the graph in the plane cannot be continuously deformed without changing the lengths of the edges. A graph is globally rigid if it has a unique embedding in the plane given the edge lengths. Graph rigidity in 2D has been relatively well understood. It is however not trivial to apply these rigidity results in the development of efficient localization algorithms. Given a graph with the edge lengths specified, finding a valid graph realization in $\mathbb{R}^d$ for a fixed dimension d is an NP-complete problem [2], [3], [6].

Pioneer Work The pioneer work of using rigidity theory in network focuses on identifying special graphs that do admit efficient localization algorithms. The first idea is to use trilateration graphs. A trilateration graph is a stronger condition than global rigidity, and thus may require more edges than necessary to uniquely embed the graph. The second idea is to examine *d-uniquely localizable graphs*. However, it is not known whether d-localizability is a generic property and it is not clear whether there is a combinatorial characterization of graphs that are d-localizable. Both approaches require that the network has sufficiently many edges to be globally rigid.

The work in this paper is a follow-up of our previous work [5] in which we proposed an algorithm for landmark selection to guarantee that the generated combinatorial Delaunay complex is globally rigid and admits a unique realization in the plane. In contrast with the previous rigidity work on *graphs*, [5] focus on the global rigidity property of the *combinatorial Delaunay complex*, that has high-order topological structures (such as Delaunay triangles) compared with graphs that do not (having only vertices and edges). The combinatorial Delaunay *complex* may be globally rigid when the combinatorial Delaunay *graph* is not. See Figure 1 for an illustration.

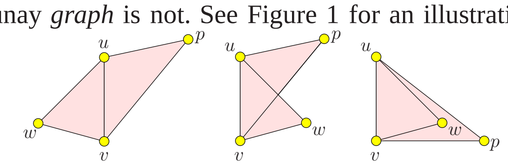

Fig. 1. Two Delaunay triangles $\triangle uvw$ and $\triangle uvp$ sharing an edge. The first figure is the only valid embedding, for in a simplicial complex two simplices can only intersect at a common face. The graph is not globally rigid.

Our work in [5] sheds some light on providing reasonable localization results when the network has low node density. But its dependency on the boundary detection algorithm, to identify the network boundary nodes first and then selects the landmarks, puts limitations on the applicability of the localization algorithm as in the case of extremely low density networks, where boundary detection algorithms do not work well. Examples of some of these cases were shown in [5].

Our Contribution The main contribution in this paper is an incremental landmark selection algorithm that does not assume knowledge of the network boundary. In particular, we start with no knowledge of the network topology (whether there are holes or how many there are, etc.) and develop local conditions to test whether a node should be included as a new landmark. The landmarks selected naturally adapt to the local geometry of the network, with a higher density of landmark nodes selected in regions with more detailed and complex features. This new landmark selection algorithm greatly enhances the robustness of our algorithm in cases of extremely sparse or even non-rigid networks, or networks with very complicated shapes that are challenging for boundary detection algorithms. We are not aware of any other localization algorithms using only connectivity information with comparable performance. We demonstrate the improved performance of our algorithm in various network settings in the simulation section.

II. Localization By Delaunay Complex

In this section we use a continuous setting to go though the framework of network localization by the combinatorial Delaunay complex and provide the theoretical foundation of the incremental Delaunay refinement algorithm. The sensor field is assumed to be a continuous domain $\mathcal{R} \in \mathbb{R}^2$ with perhaps some interior holes.

The main result in [5] is a proof that when the landmarks are selected as a γ-sample of the domain $\mathcal{R}$ with $\gamma < 1$, the Delaunay complex $DC(L)$ is globally rigid and thus admits a unique realization in the plane. This establishes the foundation of the localization algorithm as we can now embed the Delaunay complex incrementally and then localize the entire network with the Delaunay complex as a structural skeleton.

For localization, we also want that the Delaunay complex provides good 'coverage' of the sensor field in the sense that every node is not very far from the Delaunay complex, so that the Delaunay complex faithfully represents the network shape. In particular, we take $\mathcal{B}$ to denote the union of all the Voronoi balls, and $\mathcal{U}$ the shape of the union of these balls. We prove: the γ-sample guarantees that the union of Voronoi balls is a good approximation of $\mathcal{R}$ and the approximation is improved as the density of landmarks increases. See Figure 2 (ii) for an example. Using the union of the Voronoi balls to approximate the shape $\mathcal{R}$ was initially proposed in geometric processing and computer graphics [1]. However, we cannot directly apply the results in [1] as there are a couple of differences with our setting.

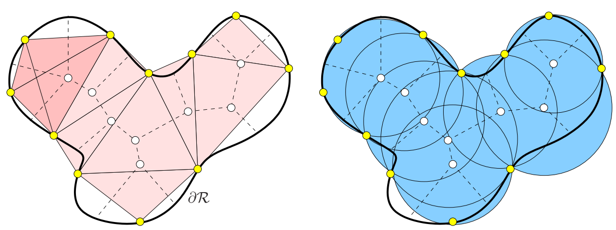

Fig. 2. Left: The Voronoi graph (shown in dashed lines) and the Delaunay complex for a set of landmarks on the boundary $\partial\mathcal{R}$. The Delaunay simplices (vertices, edges, triangles, tetrahedrons) are shaded. Right: The union of Voronoi balls approximately covers the domain $\mathcal{R}$.

Based on the previous discussion, there are two desirable criterions, namely, global rigidity and coverage, for the final Delaunay complex. Now, we investigate *local* conditions for landmark selection to guarantee both rigidity and good coverage of the induced Delaunay complex:

1) **Local Voronoi edge connectivity:** The Voronoi edges for each landmark u form a connected set.
2) **Local Voronoi ball coverage:** Each node x inside a Voronoi cell $V(u)$ is δ-covered by a Voronoi ball $B_r(p)$, where p is a Voronoi vertex with landmark u.

We show if both conditions are satisfied for a set of landmarks L, then the Delaunay complex $DC(L)$ satisfies both the global rigidity and coverage property.

We also remark in the paper that every landmark q added by the incremental algorithm is not sufficiently covered by existing landmarks, i.e., the distance to its closest landmark is at least $\gamma \cdot ILFS(q)$ for an appropriate parameter $\gamma < 1/3$. And the algorithm will certainly terminate when the landmark set is a γ-sample for any $\gamma < 1$.

Lemma 2.1. *If a Voronoi cell $V(u)$ violates the local Voronoi edge connectivity condition, the new landmark q selected is not covered by any landmark within $\gamma \cdot ILFS(q)$, for any $\gamma < 1/3$.*

Lemma 2.2. *If a Voronoi cell $V(u)$ for a landmark u violates the local Voronoi ball coverage condition, the new landmark q selected is not covered by any landmark within $\gamma \cdot ILFS(q)$, $\gamma = \delta/(2+\delta)$.*

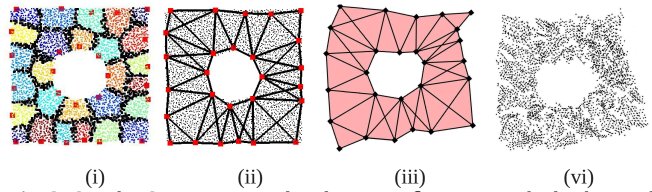

Fig. 3. Step by Step Incremental Delaunay Refinement Method. The number of nodes is 3887. The connectivity follows a unit disk graph model with average node degree 7.5. (i) The Voronoi diagram when no more landmarks can be selected. (ii) The Delaunay edges extracted from the Voronoi cells of the landmarks. (iv) Embedding Result. (v) All nodes localized.

III. Incremental Delaunay Refinement

The basic idea of the algorithm implementation is to select landmarks incrementally in the network until both the global rigidity and the coverage property are satisfied as described in Section II. The biggest difference between this paper and our previous one is that the new landmark selection algorithm does not depend on the success of boundary detection or the knowledge of the local feature size. Thus the new algorithm is more robust in practice, and yet still captures the geometry of the network. The main steps (Figure 3) include: 1). Select Initial landmarks; 2). Compute Voronoi diagram and Select more landmarks incrementally until no more landmarks can be selected; 3). Extract Delaunay complex; 4). Embed Delaunay complex; 5). Network localization.

IV. Simulation

We conducted extensive simulations under various scenarios to evaluate how well our algorithm extracts the network topology and how performance is effected by different factors such as node density, or communication model (quasi-UDG, probabilistic model, etc.). Typically our examples have an average node degree of around 10, but we even get good performance for average degree as low as 6. We also demonstrate a good result for a special case where nodes are aligned on a perfect grid having an average degree of 4. We evaluate the communication cost of our algorithm at the end.

References

[1] N. Amenta and R. K. Kolluri. Accurate and efficient unions of balls. In *SCG '00: Proceedings of the sixteenth annual symposium on Computational geometry*, pages 119–128, New York, NY, USA, 2000. ACM.

[2] J. Aspnes, D. Goldenberg, and Y. R. Yang. On the computational complexity of sensor network localization. In *The First International Workshop on Algorithmic Aspects of Wireless Sensor Networks (ALGOSENSORS)*, pages 32–44, 2004.

[3] M. Badoiu, E. D. Demaine, M. T. Hajiaghayi, and P. Indyk. Low-dimensional embedding with extra information. In *SCG '04: Proceedings of the twentieth annual symposium on Computational geometry*, pages 320–329, 2004.

[4] J. E. Graver, B. Servatius, and H. Servatius. *Combinatorial Rigidity*. Graduate Studies in Math., AMS, 1993.

[5] S. Lederer, Y. Wang, and J. Gao. Connectivity-based localization of large scale sensor networks with complex shape. In *Proc. of the 27th Annual IEEE Conference on Computer Communications (INFOCOM'08)*, pages 789–797, May 2008. http://www.cs.sunysb.edu/∼jgao/paper/layout.pdf.

[6] J. B. Saxe. Embeddability of weighted graphs in k-space is strongly NP-hard. In *Proceedings of the 17th Allerton Conference in Communications, Control and Computing*, pages 480–489, 1979.