Obstacle-Avoiding Fastest Paths in Anisotropic Media

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Shortest-path planning among obstacles is an extensively studied problem in the field of computational geometry, however most of the research to date is restricted to minimizing the Euclidean length of a path [4]. Even when the problem is generalized, such as in the case of minimum weighted Euclidean distances, the cost function is still assumed to be isotropic (i.e., invariant with respect to direction). In this talk, we address the problems of optimal path finding in anisotropic (or direction-dependent) media. We consider the fastest-path finding problems where speed is defined as a function of heading, yet without restriction on the structure of the speed function itself. The presented analysis combines the authors' earlier work on fastest-path finding in an obstacle-free domain, with the 'visibility graph' approach traditionally applied to Euclidean shortest-path finding problems. Separate path-finding algorithms are presented for a special case corresponding to a simpler procedure, as well as for a problem with an arbitrary anisotropic speed function.

In this talk, we analyze obstacle-avoiding fastest-path finding problems in anisotropic media, that is, corresponding to direction-dependent cost functions. We let $\mathcal{P}$ denote a set of open polygonal obstacles, such that their closures do not intersect, or in another words, the distance between any two obstacles is assumed to be greater than zero. Note that since each obstacle is assumed to be an open set, movement along its edges is permitted. Then, for a given time and space homogeneous speed function, $V(\theta)$, defined for $\theta \in [0, 2\pi]$, our problem objective is to find a fastest path from the start point s to the target point t. All the feasible paths, including the points s and t, are assumed to lie in the *free space*, which we define as the compliment of the obstacles, or $\mathbb{R}^2 \backslash \mathcal{P}$. Consequently, the cost function, determined by the time it takes to traverse a path from s to t, is explicitly dependent on the heading direction along the path.

The anisotropic nature of our problem adds a new layer of difficulty to a widely studied obstacle-avoiding shortest path problem for Euclidean metrics. The direction-dependent structure of the speed $V(\theta)$ results in an asymmetric cost function, where the cost of traversing a straight line segment ab, is not necessarily equal to that of a reversed link ba. Thus, the travel time function is not a metric, consequently restricting the set of mathematical tools available for our use. Furthermore, the triangle inequality, commonly exploited in Euclidean analysis, does not hold for a general anisotropic medium. Because of this fact, we are no longer guaranteed that one of the 'taut-string' paths has to be optimal for our problem. Thus, the traditional approach of searching among the taut-string paths, might not deliver the desired results.

Direction-dependent cost functions are a common occurrence in a wide range of applications. For example, waves at sea have anisotropic effects on a vessel speed, while airplanes and unmanned aerial vehicles are constantly subjected to direction-dependent winds. Friction and gravity forces result in anisotropic speed, as well as direction-dependent fuel consumption of a robot as it explores uneven terrains. Likewise, submarines often encounter direction-dependent currents during their operation. Since the work presented in this talk makes no assumption on the structure of the speed function, our results can be applied to all of the aforementioned problems, as well as any other obstacle-avoiding fastest-path finding problems with direction-dependent speed functions.

Fastest-path finding for anisotropic speed functions has been studied in several specific areas of application. However most of the analysis and results found in literature are particular to each application at hand and can not be easily extended to other areas. Furthermore, the presence of obstacles is not commonly addressed in the published studies. Only a hand full of researchers looked at the path finding problems in anisotropic medium for the general case. Nevertheless, they limit their research to the cases where the speed function has very specific structures. In this talk, we deliver closed form fastest paths in the presence of obstacles for a very general anisotropic speed function.

This talk finds closed form solutions to obstacle-avoiding fastest path problems and presents algorithms used to determine the optimal paths. We extend the visibility graph search method [1, 3], developed for Euclidean shortest path problems, to anisotropic media. In our earlier work [2], we found analytical solutions to fastest-path finding problems in the obstacle-free anisotropic domain. Here, we merge these results with the visibility graph technique to develop an obstacle-avoiding fastest path finding algorithm for anisotropic speed function.

We have previously shown that in the case that when a speed polar plot encloses a convex region, the straight line path is the fastest path in $\mathbb{R}^2$. As a result, the triangle inequality holds true for this special case of problems. Consequently, fastest-path finding in a polygonal domain can be restricted to a modified visibility graph, similarly to Euclidian shortest-path problems. However, the triangle inequality might not hold for a general speed function which does not have the property of a convex polar plot. In that case, an augmented speed function, corresponding to the convex hull of the original speed polar plot, is used to find a lower bound on the minimum travel time for our problem. Then, we use results from the authors' earlier work to construct an obstacle-avoiding path that achieves this lower bound, implying its optimality. An application to vessel routing is introduced throughout the talk to motivate the problem.

References

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