Rendering Lunar Eclipses

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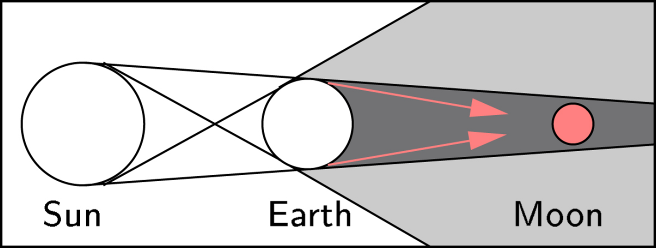
Why are Lunar Eclipses even Visible?

Earth's Atmosphere



No atmosphere

Refraction only

Refraction+scattering

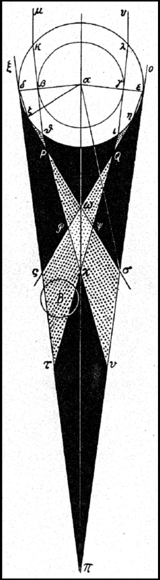
Lunar Eclipse Phenomena

First Explanation

Kepler (right) explains eclipse visibility
Attributes variations in appearance to "mists and smoke"

Danjon Scale [Danjon 1920]

L = 0	Very dark eclipse. Moon almost invisible, especially at mid-totality.
L = 1	Dark eclipse, gray or brownish in coloration. Details distinguishable only with difficulty.
L = 2	Deep red or rust-colored eclipse. Very dark central shadow, while outer edge of umbra is relatively bright.
L = 3	Brick-red eclipse. Umbral shadow usually has a bright or yellow rim.
L = 4	Very bright copper-red or orange eclipse. Umbral shadow has a bluish, very bright rim.



[Kepler 1604]

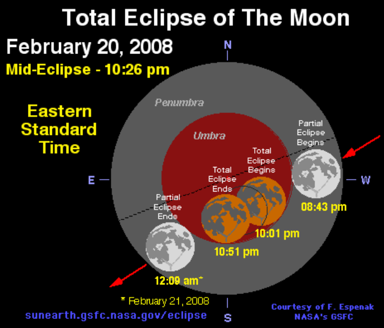
Anatomy of an Eclipse

Statistics

- Frequency: 0-3x per year
- Duration: hours
- Visibility: same as full moon

Classifications

- Penumbral
- Partial
- Total



(Our Rendering)

Related Work

Atmospheric sciences

- Modeling eclipse brightness
  [Link 1969; Hansen and Matsushima 1966; Hernitschek, et al. 2008; Vollmer, et al. 2008]
- Estimating atmospheric conditions (past/present)
  [Matsushima et al. 1966; Hoffman, et al. 2003; Stothers 2005; Keen 1983]

Inhomogeneous media

- Mirages [Berger and Trout 1990]
- General [Stam and Languenou 1996]
- Photon Mapping
  [Gutierrez, et al. 2005]

Rendering Natural Phenomena

- Night sky [Jensen, et. at 2001]
- Green flash [Gutierrez, et al. 2004]
- Sunsets [Gutierrez, et al. 2004]
- Twilight [Haber, et al. 2005]
- Solar disk [Lintu, et al. 2005]
- Various [Gutierrez, et al. 2006]

Scattering / Illumination

- Earth from Space
  [Nishita, et al. 1993]
- Daylight [Preetham, et al. 1999]

Contributions

Rendering Physical Phenomena

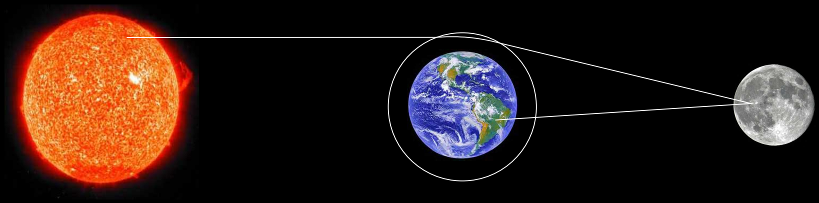
- System for rendering lunar eclipses
- Consolidates work from diverse fields
  - Astronomy
  - Atmospheric science
  - Computer graphics
- First physically-based renderings of eclipse phenomena

Verification and Comparisons

- We compare rendered images to photographs of actual eclipses
- We render images of future lunar eclipses
  ...and hope for future clear skies!

Outline

- Solar System Model
  - Celestial Mechanics
  - Solar Irradiance
- Atmospheric Effects
  - Refraction
  - Scattering
  - Absorption
- Lunar Appearance
  - Lunar Albedo
  - Lunar BRDF
- Moon-Earth path
  - Atmospheric Optical Depth
- Photographic Comparisons
  - Future Work



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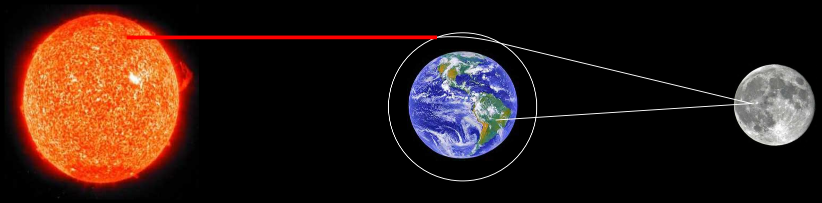


Eclipsing Configuration

- Simulation driven by:
  - Time in UTC
  - Latitude/longitude of observer
- Sun, Earth, Moon modeled as spheres
- Positions calculated [Meeus 1988] at point of deepest eclipse
- Assume Sun-Earth distance constant over eclipse duration
- Two-pass algorithm
  - Trace rays from Sun, through atmosphere to moon
  - Render moon from observer viewpoint

$\longleftarrow$ Sun is 390$\times$ this distance to the left

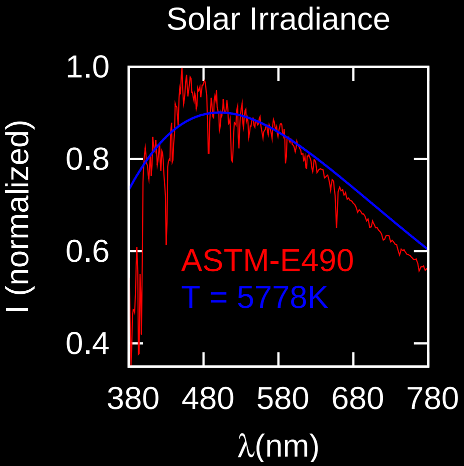
Solar Irradiance

Approximately Blackbody

- $T \approx 5778K$
- $+$ Absorption bands
- ASTM-E490 tables (right)

Spectral Ray Tracing

- Sample 200 wavelengths
- $380\text{nm} \leq \lambda \leq 780\text{nm}$
- Trace wavelengths independently
- $\sum_\lambda I(\lambda) \rightarrow {}_{CIE}XYZ \rightarrow \text{sRGB}$



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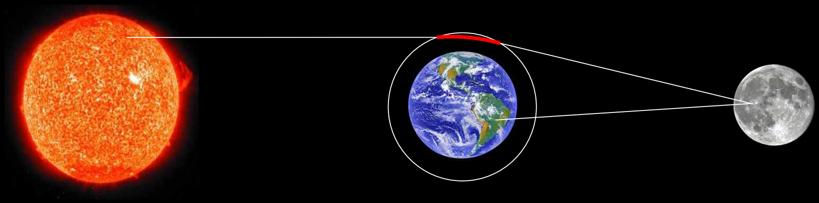
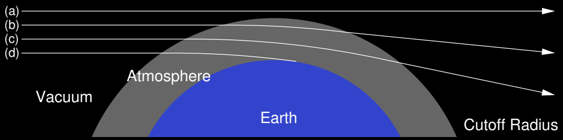


Atmosphere Model

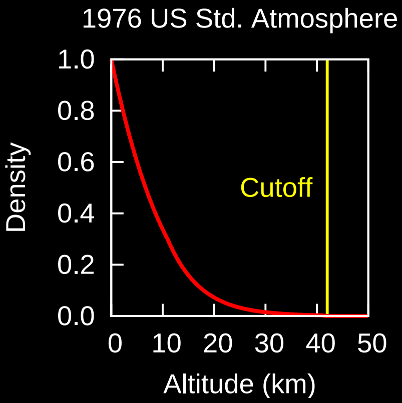
Varying Density

- Refractive index
  - Bends light rays
- Scattering ability

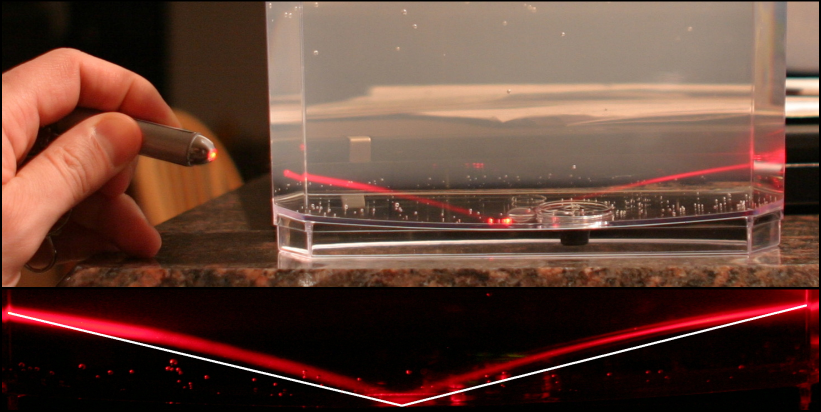


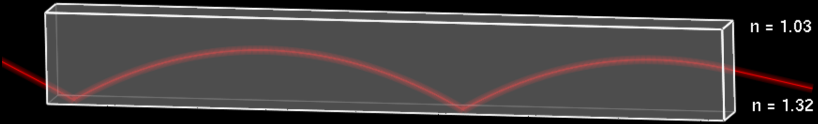
Our Model

- US 1976 Std. Atmosphere
- Spherically symmetric
- Inputs
  - Surface temperature
  - Surface pressure
- Outputs
  - Density: $\rho(h)$
  - Refractive index: $n(h)$
  - Rayleigh scattering cross-section $\sigma(h, \lambda)$



Inhomogeneous Refraction
Demonstration: Sucrose Solution





Inhomogeneous Refraction

Numerical Solution of PDE

Ray trajectory described by [Seron, et al. 2004]:

$$\frac{d}{dl}\left(n\frac{dx_j}{dl}\right) - \frac{\partial n}{\partial x_j} = 0,\ j \in \{0,1,2\}$$

(l = arc length, x = spatial coordinate, n = refractive index)
Discretizing, we obtain:

$$\frac{dx_{j,i+1}}{dl} = \frac{n_i \frac{dx_{j,i}}{dl} + \frac{\partial n}{\partial x_j}\Delta l_i}{n_{i+1}}$$

We use the approximation:

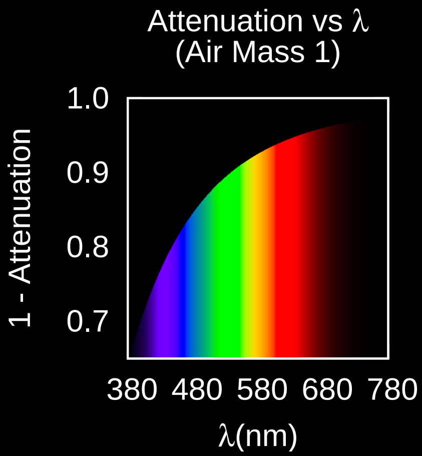
$$\frac{\partial n}{\partial x_j} \approx \frac{n(x_j + \Delta x_j) - n(x_j)}{\Delta x_j}$$

$\Delta x_j \approx 10$m works well in practice

Scattering

Rayleigh Scattering

- Small particles
- Function of $(\frac{1}{\lambda^4})$
  - Attenuates shorter λ
- Mie scattering not significant [Hansen 1966]



Modeling

- Beer-Lambert law

$$I = I_0 e^{-\tau}$$

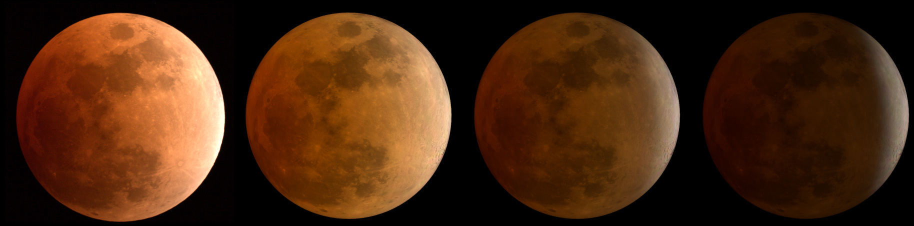
τ: dimensionless "optical depth"
function of λ, n, ρ

Atmospheric Dusts

Stratospheric Dust Parameter (α)

- Models volcanic aerosols [Keen 1983]
- Uniform attenuation for 15km $\leq h \leq$ 25km; zero elsewhere
- Flat spectrum; not λ-dependent
- Beer-Lambert Law: $I = I_0 e^{-\alpha \Delta x}$

Photograph — Our Simulations



$\alpha = 0$ — $\alpha = 4.7 \times 10^{-6}$ — $\alpha = 1.0 \times 10^{-5}$

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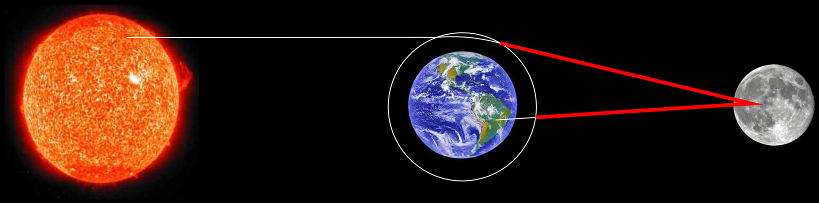


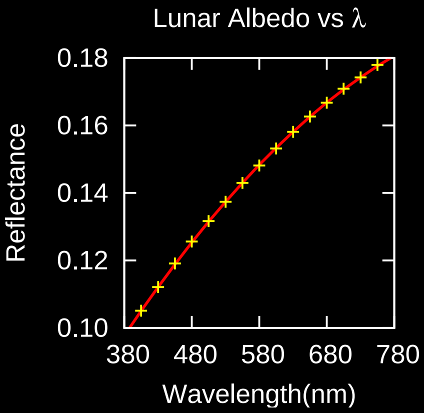
Lunar Albedo

What color is the Moon?

- Actually, dark, reddish brown
- Reflects only 10-20% of incident sunlight

Our Model

- Apollo 11 samples [Pieters 1999]
- Fit polynomial to visible spectrum
- Normalize to $k_r(780nm) = 1$



Lunar albedo rendered in sRGB (brightened 4×)

Lunar BRDF

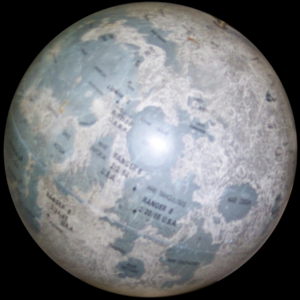
Why does the Moon look flat?

- Lunar BRDF is *not* Lambertian!
- Moon does not show "limb darkening"
- High degree of retro-reflection

Photon Tracing Issues

- Photon tracing simulates Lambert's law
- Ideal solution
  - Invert Lambertian function
  - Apply lunar BRDF
- Our Approximation

$$I'_{ij} = I_{ij}\left(\frac{1}{0.1 + 0.9\cos\theta}\right)$$



Photograph of globe

Photograph of Moon

http://www.murrayhudson.com/antique_globes/10358g.jpg

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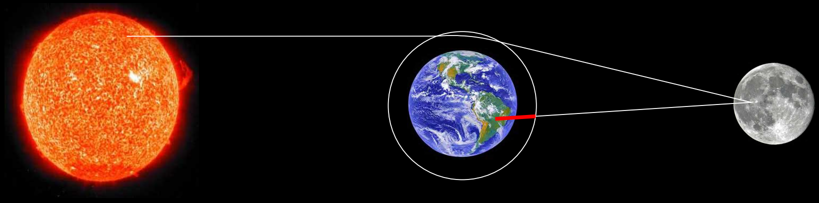

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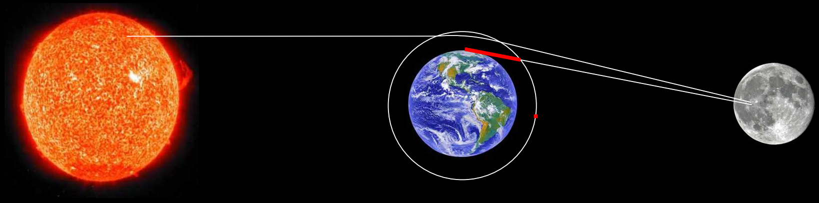


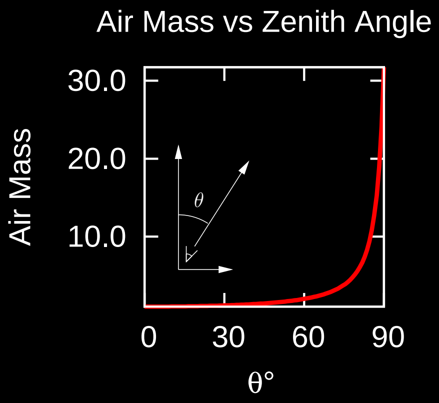
Atmospheric Optical Depth

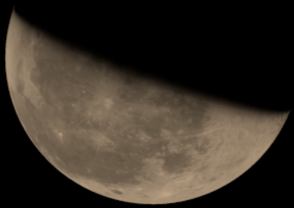
Path length varies

- $> 30\times$ at horizon!
- Sunrise/sunset color
- Moon color near horizon

Return path approximation

- Could use photon tracing
  - Expensive
  - "Backwards"
- Approximation
  - Calculate air mass [Young 1994]
  - Apply equivalent spectral filter
  - Efficient, correct (colors)

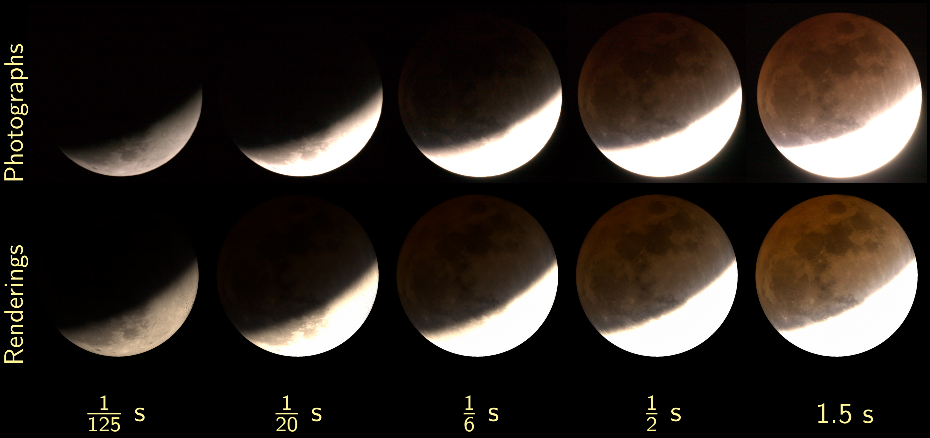




Moonset (rendering)

Photographic Comparison: February 21, 2008 Eclipse

Upstate New York, c. 04:12:44 UTC



Future Work

- Rendering historical eclipses
  - May yield clues about past climate
- Inverse model
  - Inferring atmospheric conditions from eclipse photographs

Prediction of December 21, 2010 Total Eclipse

07:22:17 UTC (0.8s) | 07:40:21 UTC (1.0s) | 08:16:56 UTC (1.5s)