

# Left Bit Right: For SPARQL Join Queries with OPTIONAL Patterns (Left-outer-joins)

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## ABSTRACT

SPARQL basic graph pattern (BGP) (a.k.a. SQL inner-join) query optimization is a well researched area. However, optimization of OPTIONAL pattern queries (a.k.a. SQL left-outer-joins) poses additional challenges, due to the restrictions on the *reordering* of left-outer-joins. The occurrence of such queries tends to be as high as 50% of the total queries (e.g., DBPedia query logs).

In this paper, we present *Left Bit Right* (LBR), a technique for *well-designed* nested BGP and OPTIONAL pattern queries. Through LBR, we propose a novel method to represent such queries using a graph of *supernodes*, which is used to aggressively prune the RDF triples, with the help of compressed indexes. We also propose novel optimization strategies – first of a kind, to the best of our knowledge – that combine together the characteristics of *acyclicity* of queries, *minimality*, and *nullification*, *best-match* operators. In this paper, we focus on OPTIONAL patterns without UNIONS or FILTERS, but we also show how UNIONS and FILTERS can be handled with our technique using a *query rewrite*. Our evaluation on RDF graphs of up to and over one billion triples, on a commodity laptop with 8 GB memory, shows that LBR can process *well-designed* low-selectivity complex queries up to 11 times faster compared to the state-of-the-art RDF column-stores as Virtuoso and MonetDB, and for highly selective queries, LBR is at par with them.

## Categories and Subject Descriptors

H.2.4 [Systems]: Query Processing

## Keywords

Query optimization; SPARQL OPTIONAL patterns; Left-outer-joins; Semi-joins; Compressed bitvectors.

<sup>\*</sup>Initial part of this work was completed when the author was at Rensselaer Polytechnic Institute and University of Pennsylvania.

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## 1. INTRODUCTION

Resource Description Framework (RDF) [8] is a widely accepted standard for representing *semantically linked* data on the web, and SPARQL [9] is a standard query language for it. RDF data is a directed edge-labeled multi-graph, where each unique edge (S P O) is called a *triple* – P is the label on the edge from the node S to node O.

SPARQL query language, like SQL, provides various syntactic constructs to form *structured* queries for RDF graphs. Two types of queries of our interest are *Basic Graph Pattern* (BGP) and *OPTIONAL* pattern (OPT) queries. Any SPARQL BGP query can be methodically translated into an equivalent SQL inner-join query [20], and an OPT pattern query can be methodically translated into an SQL left-outer-join query [19]. Like SQL, the SPARQL grammar allows nested OPT queries, i.e., a query composed of an intermix of BGPs and OPT patterns. Since SPARQL BGP queries are same as SQL inner-joins (denoted by symbol  $\bowtie$ ) and OPT queries are same as SQL left-outer-joins (denoted by  $\bowtie\text{L}$ ), we will use these terms, acronyms, and symbols interchangeably in the rest of the text. BGP and OPT queries tend to be performance intensive, especially for very large RDF data. An extensive amount of research has gone in the optimization of SQL inner-join queries. The database and semantic web communities have taken these optimizations further along with the novel ideas of indexing and BGP query processing over RDF data [15, 28, 29, 33, 42, 44, 45].

**Why OPT queries?** RDF is a semi-structured data, and adherence of RDF “instance data” to its Ontology specification (a.k.a. schema), and in turn its completeness, is not always enforced especially for the data published on the web (e.g., DBPedia [2]), and that which is compiled from many diverse sources of RDF graphs (e.g., Linked Open Data<sup>1</sup>). This makes OPT queries a crucial tool for the end users. For example, consider an RDF network describing movie and TV sitcom/soap actors. Not all the actors may have their contact info, such as email and telephone numbers listed. Then an OPT query (Q1) as given below fetches **all** the *actors* with their respective *name* and *address*. Along with that, it gets *email* and *tele* numbers of those who have them listed, and for the rest it marks *email* and *tele* values by *NULLs*.

```
SELECT ?actor ?name ?addr ?email ?tele WHERE {  
  ?actor :name ?name .  
  ?actor :address ?addr .  
  OPTIONAL {  
    ?actor :email ?email .  
    ?actor :telephone ?tele .}}}
```

<sup>1</sup><http://linkeddata.org/>

Query logs of SPARQL endpoints on the web indeed concur with this intuition, e.g., DBpedia query logs show as high as 50% occurrence of OPT queries [27, 35], with as many as eight OPT patterns in a query. These statistics make OPT queries a non-negligible component of SPARQL “join” query optimization. Now consider a modified version (**Q2**) of the previous query:

```
SELECT ?friend ?sitcom WHERE {
  :Jerry :hasFriend ?friend .
  OPTIONAL {
    ?friend :actedIn ?sitcom .
    ?sitcom :location :NewYorkCity .}}

```

This query asks for all friends of *:Jerry* that have acted in a sitcom located in the *:NewYorkCity*. In this query, let (*:Jerry :hasFriend ?friend*) be  $tp_1$ , (*?friend :actedIn ?sitcom*)  $tp_2$ , and (*?sitcom :location :NewYorkCity*)  $tp_3$ . Then the query can be expressed as ( $Query = tp_1 \bowtie (tp_2 \bowtie tp_3)$ ), where  $tp_1$  forms a BGP (say  $P_1$ ) with only one triple pattern, and ( $tp_2 \bowtie tp_3$ ) forms another BGP (say  $P_2$ ).

BGP queries ( $\bowtie$ ) are *associative* and *commutative*, i.e., a change in the order of triple patterns and joins between them does not change the final results. But for a nested OPT query,  $\bowtie$  operator is not associative and commutative, e.g., in the case of **Q2** above, left-outer-join between  $tp_1$  and  $tp_2$  cannot be performed before the inner-join  $P_2 = (tp_2 \bowtie tp_3)$ . This limits the number of query plans an optimizer can consider.

Let us assume that this actor network has thousands of actors, and many of them have acted in sitcoms located in the *:NewYorkCity*. But *:Jerry* has just two friends, *:Larry* and *:Julia*, and among them only *:Julia* has acted in *:Seinfeld*, which has *:NewYorkCity* as the location. So the final results of this query are just two with the respective values of *?friend* and *?sitcom* set as  $\{(:Larry, NULL), (:Julia, :Seinfeld)\}$ . However, the inner-join  $P_2 = tp_2 \bowtie tp_3$  has to be evaluated before the left-outer-join  $P_1 \bowtie P_2$ , due to the restrictions on the reorderability of left-outer-joins. Since there are several actors who have acted in the sitcoms located in *:NewYorkCity*, these two triple patterns have a *low selectivity*<sup>2</sup>, which increases the evaluation time and cost of their inner-join. To overcome such a limitation, for conventional databases, Rao et al [38, 39], and Galindo-Legaria, Rosenthal [26] have proposed ways of reordering nested inner and left-outer joins using additional operators as *nullification* and *best-match* [39] (*Generalized Outerjoin* in [26]).

On this background, we propose *Left Bit Right* (LBR), and make the following main contributions.

- We propose a novel way to represent a nested BGP-OPT query with a graph of *supernodes* (Section 2).
- We extend the previously known properties of *acyclicity* of queries, *minimality* of triples, and *nullification*, *best-match* operations, to propose novel optimization strategies – first of a kind, to the best of our knowledge – for acyclic and cyclic *well-designed* OPT queries (Sections 3.2, 3.3, 5). We mainly focus on the “join” component of SPARQL, i.e., OPT patterns without UNIONS, FILTERS, or Cartesian products. Nevertheless, in Section 5.2 we show how these constructs can be handled using our technique.

- Finally, we show LBR’s performance using a commodity laptop of 8 GB memory over three popular RDF datasets, UniProt [10], LUBM [4], and DBpedia [2], with up to and

<sup>2</sup>Selectivity of a triple pattern is high if it has fewer number of triples associated with it and vice versa.

over a billion triples, in comparison with the state-of-the-art RDF column-stores like Virtuoso v7.1.0 and MonetDB v11.17.21. Through our evaluation on queries with varying degrees of selectivity, complexity, and running times, we show that for complex, low selectivity “*well-designed*” queries, LBR is up to 11 times faster than Virtuoso and MonetDB, and for highly selective queries, it is at par with them (Section 6).

## 2. QUERY GRAPH OF SUPERNODES

An OPT query with an intermix of Basic Graph Patterns (BGPs  $\bowtie$ ) and OPT patterns ( $\bowtie$ ) establishes restrictions on the order of join processing, e.g., **Q1**, **Q2** in Section 1. In this section we introduce a novel idea of constructing a *graph of supernodes* (GoSN) to capture the nesting of OPT patterns in a query. GoSN makes an important part of our query processing techniques discussed in Sections 3 and 5.

### 2.1 GoSN Construction

**Supernodes:** In a SPARQL OPT pattern of the form ( $P_1 \bowtie P_2$ ),  $P_1$  may in turn have nested BGPs and OPT patterns inside it, e.g.,  $P_1 = (P_3 \bowtie P_4)$ .  $P_2$  may have nested BGPs and OPT patterns inside it too, or either of  $P_1$  and  $P_2$  can be OPT-free. Generalizing it, if a pattern  $P_i$  does not have any OPT pattern nested inside it, we call  $P_i$  to be an *OPT-free Basic Graph Pattern*. From a given nested OPT query, first we extract all such OPT-free BGPs, and construct a *supernode* ( $SN_i$ ) for each  $P_i$ . The triple patterns (TPs) in  $P_i$  are *encapsulated* in  $SN_i$ .

Next, we serialize a nested OPT query using its OPT-free BGPs,  $\bowtie$  (inner-join),  $\bowtie$  (left-outer-join) operators, and proper parentheses. E.g., we serialize **Q2** in Section 1 as ( $P_1 \bowtie P_2$ ), where  $P_1$  and  $P_2$  are OPT-free BGPs,  $SN_1$  of  $P_1$  encapsulates just  $tp_1$ , and  $SN_2$  of  $P_2$  encapsulates  $tp_2$  and  $tp_3$  (see Figure 2.1a).

**Unidirectional edges:** From the serialized query, we consider each OPT pattern of type  $P_m \bowtie P_n$ .  $P_m$  or  $P_n$  may have nested OPT-free BGPs inside them. Using the serialized-parenthesized form of the query, we identify the *leftmost* OPT-free BGPs nested inside  $P_m$  and  $P_n$  each. E.g., if  $P_m = ((P_a \bowtie P_b) \bowtie (P_c \bowtie P_d))$ , and  $P_n = (P_e \bowtie P_f)$ ,  $P_a$  and  $P_e$  are the *leftmost* OPT-free BGPs in  $P_m$  and  $P_n$  respectively, and  $SN_a$  and  $SN_e$  are their respective supernodes. We add a directed edge  $SN_a \rightarrow SN_e$ . If either  $P_m$  or  $P_n$  does not nest any OPT-free BGPs inside it, we treat the very pattern as the *leftmost* for adding a directed edge. With this procedure, we can treat OPT patterns in a query in any order. But for all practical purposes, we start from the *innermost* OPT patterns, and recursively go on considering the outer OPT patterns using the parentheses in the serialized query. E.g., if a serialized query is  $((P_a \bowtie P_b) \bowtie (P_c \bowtie P_d)) \bowtie (P_e \bowtie P_f)$ , with  $P_a \dots P_f$  as OPT-free BGPs, we add directed edges as follows: (1)  $SN_a \rightarrow SN_b$ , (2)  $SN_c \rightarrow SN_d$ , (3)  $SN_e \rightarrow SN_f$ , (4)  $SN_a \rightarrow SN_e$ .

**Bidirectional edges:** Next we consider each inner-join of type  $P_x \bowtie P_y$  in a serialized query. If  $P_x$  or  $P_y$  has nested OPT-free BGPs inside, we add a bidirectional edge between the supernodes of *leftmost* OPT-free BGPs. E.g., if  $P_x = (P_a \bowtie P_b)$ , and  $P_y = (P_c \bowtie P_d)$ , we add a bidirectional edge  $SN_a \leftrightarrow SN_c$ . If  $P_x$  or  $P_y$  does not nest any OPT-free BGPs inside it, we consider the very pattern to be the *leftmost* for adding a bidirectional edge. We add bidirectional edges starting from the *innermost* inner-joins ( $\bowtie$ )

using the parentheses in the serialized query, and recursively go on considering the outer ones, until no more bidirectional edges can be added. Considering the same example given under unidirectional edges, we add a bidirectional edge between  $SN_a \leftrightarrow SN_c$ . The *graph of supernodes* (GoSN) for this example is shown in Figure 2.1b.

Thus we completely capture the nesting of BGPs and OPT patterns in a query using this GoSN, and establish an order among the supernodes, which is described in Section 2.2.

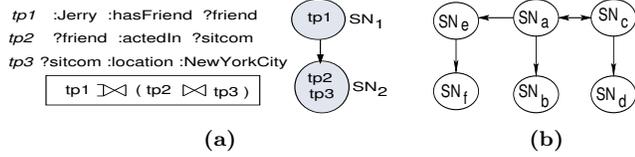


Figure 2.1: GoSN for – (a) Q2 in Section 1, (b)  $((P_a \bowtie P_b) \bowtie (P_c \bowtie P_d)) \bowtie (P_e \bowtie P_f)$

## 2.2 Nomenclature

In this section, we highlight the nomenclature that we use in the context of GoSN, OPT patterns, and RDF graphs.

**Master-Slave:** In an OPT pattern  $P_c \bowtie P_d$ , we call pattern  $P_c$  to be a *master* of  $P_d$ , and  $P_d$  a *slave* of  $P_c$ . This master-slave relationship is *transitive*, i.e., if a supernode  $SN_f$  is reachable from another supernode  $SN_c$  by following *at least one* unidirectional edge in GoSN, then  $SN_c$  is called a master of  $SN_f$  (see Figure 2.1b).

**Peers:** We call two supernodes to be peers if they are connected to each other through a bidirectional edge, or they can be reached from each other by following *only* bidirectional edges in GoSN, e.g.,  $SN_a$  and  $SN_c$  in Figure 2.1b.

**Absolute masters:** Supernodes that are *not* reachable from any other supernode through a path involving *at least one* unidirectional edge are called the *absolute masters*, e.g.,  $SN_a$  and  $SN_c$  in Figure 2.1b are absolute masters.

These master-slave, peer, and absolute master nomenclatures and relationships apply to any triple patterns enclosed within the respective supernodes too.

**Well-designed patterns:** As per the definition given by Pérez et al [34], a *well-designed* OPT query is – for every sub-pattern of type  $P' = P_k \bowtie P_l$  in the query, if a join variable “?j” in  $P_l$  appears outside  $P'$ , then “?j” also appears in  $P_k$ . A query that violates this condition is said to be *non-well-designed*. In this paper, we have focused on well-designed queries, because they occur most commonly for RDF graphs, and remain unaffected by the difference between SPARQL and SQL algebra over treatment of NULLs. Nevertheless, we discuss *non-well-designed* queries in Appendices B and C for the completeness of the text.

**NULLs and blank nodes:** Unlike relational tables, RDF graphs do not have NULLs. Note that NULLs represent *non-existence* of entities, whereas “blank nodes” in an RDF graph have “blank node identifiers” to represent entities without distinct URIs<sup>3</sup>, and in SPARQL queries, they are treated similar to the entities that have URIs. An OPT query may generate NULLs, and joins over NULLs happen only in *non-well-designed* patterns. SPARQL and SQL algebra handles joins over NULLs differently, and Appendix C elaborates on this issue. Well-designed patterns, however, remain unaffected by this – which are the focus of this paper.

<sup>3</sup> [www.w3.org/TR/REC-rdf-syntax/#section-Syntax-blank-nodes](http://www.w3.org/TR/REC-rdf-syntax/#section-Syntax-blank-nodes)

## 3. OPTIMIZATION STRATEGIES

In Figure 3.1 we point out that LBR can process all the nested BGP-OPT queries, but not all of them can avoid *nullification* and *best-match*, and we mainly focus on the *well-designed* queries in this paper. Our technique can be applied to *non-well-designed* queries too, but we have not focused on them due to lack of evidence of such queries in practice, disparity between pure SPARQL and SPARQL-over-SQL engines for joins over NULLs (refer to Appendices B and C), and space limitations.

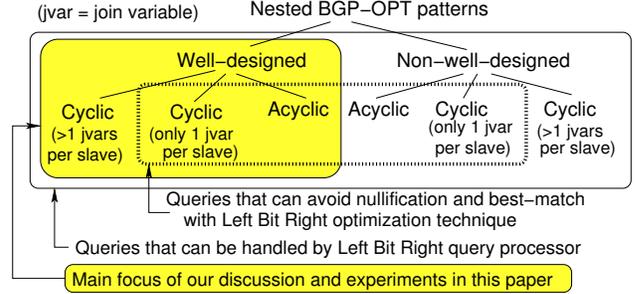


Figure 3.1: Classification of OPT queries

In Section 3.1, briefly ignoring GoSN, we describe the conceptual foundation of our technique by combining the properties of *nullification*, *best-match*, *acyclicity* of queries, and *minimality* of triples. Then in Section 3.2 we present our optimization strategies to “prune” the triples for *acyclic* well-designed patterns using GoSN, and in Section 3.3 we discuss pruning for *cyclic* well-designed patterns. For the discussion in Sections 3.1, 3.2, 3.3 we do not take into consideration the underlying indexes on RDF graphs, because the optimization strategies are agnostic to them. Then in Section 4 we describe our indexes, and in Section 5 we show how to prune the triples using our indexes, and generate the final query results using *multi-way-pipelined* join for both, *acyclic* as well as *cyclic* well-designed queries.

### 3.1 Preliminaries

Typically, for a pairwise join processing plan, if nested inner and left-outer joins are reordered, *nullification* and *best-match* (a.k.a. *minimum-union*) operations are required. We explain them here with a brief example for the completeness of the text, and refer the reader to [24, 39] for the details.

**Nullification, Best-match:** Consider the same query given in Figure 2.1a, along with the sample data associated with it in Figure 3.2. *:NewYorkCity* has been the location for a lot of American sitcoms, and a lot of actors have acted in them (they are not shown in the sample data for conciseness). But, among all such actors, *:Jerry* has only two friends, *:Julia* and *:Larry*. Hence,  $tp_1$  is more *selective* than  $tp_2$  and  $tp_3$ . A left-outer-join reordering algorithm as proposed in [26, 39] will typically reorder these joins as  $(tp_1 \bowtie tp_2) \bowtie tp_3$ . Due to this reordering, all four sitcoms that *:Julia* has acted in show up as bindings of *?sitcom* (see Res1 in Fig. 3.2), although only *:Seinfeld* was located in the *:NewYorkCity*. To fix this, *nullification* operator is used, which ensures that variable bindings across the reordered joins are consistent with the original join order in the query (see Res2 in Figure 3.2).

We can see that the *nullification* operation caused results that are *subsumed* within other results. A result  $r_1$

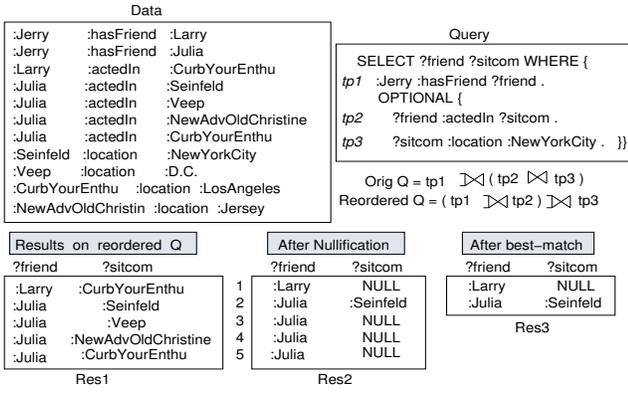


Figure 3.2: Nullification and best-match example

is said to be subsumed within another result  $r_2$  ( $r_1 \sqsubset r_2$ ), if for every non-null variable binding in  $r_1$ ,  $r_2$  has the same binding, and  $r_2$  has more non-null variable bindings than  $r_1$ . Thus results 3–5 in Res2 are subsumed within result 2. The *best-match* operator removes all the subsumed results (see Res3). Final results of the query are given as  $\text{best-match}(\text{nullification}((tp_1 \bowtie tp_2) \bowtie tp_3))$ .

**Semi-join** ( $\bowtie$ ) is a well-known concept in databases, with which triples associated with a triple pattern (TP) can be removed due to restrictions on the variable bindings coming from another TP, without actually performing a join between the two.  $tp_2 \bowtie_{?j} tp_1 = \{t \mid t \in tp_2, t.?j \in (\pi_{?j}(tp_1) \cap \pi_{?j}(tp_2))\}$ . Here  $t$  is a triple matching  $tp_2$ , and  $t.?j$  is a variable binding (value) of variable  $?j$  in  $t$ . After this semi-join,  $tp_2$  is left with only triples whose  $?j$  bindings are also in  $tp_1$ , and all other triples are removed. Now let triple patterns,  $tp_1, tp_2, \dots, tp_n$  all share join variable  $?j$ . Then we define a **clustered-semi-join** over them as follows.

**DEFINITION 3.1.** A *clustered-semi-join*( $?j, \{tp_1, tp_2, \dots, tp_n\}$ ) is performed as follows: Let  $\mathcal{J} = \{\pi_{?j}(tp_1) \cap \dots \cap \pi_{?j}(tp_n)\}$ . For each triple pattern  $tp_i, 1 \leq i \leq n$ ,  $tp_i = \{t \mid t \in tp_i, t.?j \in \mathcal{J}\}$ .

This definition shows that a clustered-semi-join is nothing but many semi-joins performed together for TPs that share a join variable.

**Example-1:** Now let us evaluate the query in Figure 3.2 using semi-joins and clustered-semi-joins. We do a semi-join  $tp_2 \bowtie_{?friend} tp_1$ , because  $tp_1$  does a left-outer-join with  $tp_2$  over  $?friend$ . That keeps only  $:Larry$  and  $:Julia$  bindings of  $?friend$  in  $tp_2$ , and removes any other bindings, and in turn triples generating those bindings from  $tp_2$  (they are not shown in the figure for conciseness). Followed by it, we do a *clustered-semi-join*( $?sitcom, \{tp_2, tp_3\}$ ). That removes  $:CurbYourEnthu$ ,  $:Veep$ , and  $:NewAdvOldChristine$  bindings of  $?sitcom$ , and the respective triples from  $tp_2$ . Notice that this clustered-semi-join also removes the  $:Larry$  binding of  $?friend$  from  $tp_2$  as a *ripple effect* of the removal of  $:CurbYourEnthu$  binding of  $?sitcom$ . In the end,  $tp_1$  and  $tp_3$  has the same set of triples, but  $tp_2$  now has only one triple ( $:Julia :actedIn :Seinfeld$ ). Now if we evaluate the original join ( $tp_1 \bowtie (tp_2 \bowtie tp_3)$ ) or a reordered one ( $(tp_1 \bowtie tp_2) \bowtie tp_3$ ) on these reduced set of triples, we do not need nullification to ensure consistent bindings of  $?sitcom$ , and there are no subsumed results, because each TP has a *minimal* set of triples. **Minimality** of triples is defined as follows.

Let  $\mathcal{R}$  be the final results of a query  $Q$ , and  $tp$  be a TP in  $Q$ . Let  $tp.s, tp.p, tp.o$  be the respective subject, predicate, and object positions in  $tp$ . Let  $Q$  be such that it SELECTs all the variables as well as fixed positions in all the TPs in  $Q$ . E.g., for the query in Figure 3.2, “SELECT  $:Jerry :hasFriend ?friend :actedIn ?sitcom :location :NewYorkCity WHERE \dots$ ” selects *everything* in the query, and the final results will be  $\{(:Jerry, :hasFriend, :Larry, :actedIn, NULL, :location, :NewYorkCity), (:Jerry, :hasFriend, :Julia, :actedIn, :Seinfeld, :location, :NewYorkCity)\}$ . Note that this assumption of SELECTION is just for the ease of definition of minimality, and not a required condition. Let  $\Delta_{tp}$  be the triples associated with a  $tp$  after a semi-join or clustered-semi-join. Then minimality of  $\Delta_{tp}$  is defined as follows.

**DEFINITION 3.2.** Let  $\mathcal{R}_{tp} = (\text{non-NULL})\pi_{tp.s, tp.p, tp.o}(\mathcal{R})$ , i.e.,  $\mathcal{R}_{tp}$  is a projection of the respective distinct bindings of  $tp.s, tp.p, tp.o$  from  $\mathcal{R}$  without any NULLs. Then  $\Delta_{tp}$  is said to be *minimal*, if  $\Delta_{tp} = \mathcal{R}_{tp}$  ( $\Delta_{tp}$  and  $\mathcal{R}_{tp}$  can be empty as well). In short, in a BGP-OPT query, the set of triples associated with a triple pattern is *minimal*, if every triple creates one or more variable bindings in the final results. There does not exist any triple which may get eliminated as a result of an inner or left-outer-join.

Next we see why minimality of triples is important in the context of an OPT pattern query.

**LEMMA 3.1.** If every triple pattern in an OPT pattern has a minimal set of triples associated with it, nullification and best-match operations are not required if an original query  $tp_1 \bowtie (tp_2 \bowtie tp_3)$  is reordered as  $(tp_1 \bowtie tp_2) \bowtie tp_3$ .  $\square$

The proof of Lemma 3.1 is given in Appendix A.1.

Example-1 and Lemma 3.1 together interest us to find if the set of triples associated with each TP in an OPT pattern can be reduced to minimal through semi-joins and clustered-semi-joins alone, because then nullification and best-match can be avoided even if the joins are reordered.

**Acyclicity:** Bernstein et al [16, 17] and Ullman [43] have proved previously that if a “graph of tables” (GoT) of an inner-join query is a “tree” (i.e., it is acyclic), a bottom-up followed by a top-down pass with semi-joins at each table in this tree, reduces the set of tuples in each table to a minimal. In the context of a SPARQL query, GoT is a graph of TPs, where each TP is treated as a unique table, and two TPs are connected with an undirected edge if they share a join variable. Any redundant cycles in GoT are removed as suggested in [16, 17]<sup>4</sup>. We use and extend the acyclicity property of GoT for our optimization strategies. For this, we construct a “graph of join-variables” (GoJ) as follows. Each unique join variable (jvar) in a query is a node (jvar-node). There is an undirected edge between two jvar-nodes, if they appear together in a TP. Figure 3.3 shows GoT and GoJ for the query given in Figure 3.2. For clarity and simplicity, we treat both GoT and GoJ separate from GoSN.

**LEMMA 3.2.** For a join query, if the GoT is acyclic, then the GoJ is acyclic too.  $\square$

Acyclicity of GoJ follows from its construction. We have given the proof of Lemma 3.2 in Appendix A.2. From Lemma 3.2, we also observe the following property.

<sup>4</sup>Redundant cycles may occur if multiple TPs join over same variable.

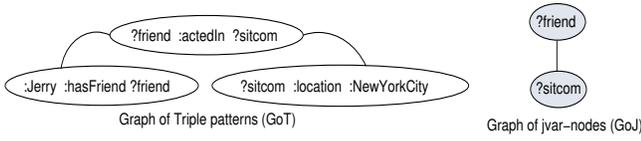


Figure 3.3: GoT and GoJ

PROPERTY 3.1. *For a BGP query without OPT patterns (inner-joins only), if the GoJ is acyclic (is a tree), a bottom-up followed by a top-down pass on the GoJ with “clustered-semi-joins” performed at each jvar-node, reduces the set of triples associated with each TP in the query to a minimal.*

**Note:** We use the query and GoSN in Figure 2.1a, its respective data in Figure 3.2, and GoJ in Figure 3.3 together as a running example in the rest of the text in this paper without mentioning it explicitly every time.

### 3.2 Acyclic well-designed OPT patterns

In Section 3.1, through Example-1 and Lemma 3.1, we showed that if each TP in an OPT query has minimal triples, nullification and best-match are not required. We showed how minimality of triples can be achieved for an acyclic OPT-free BGP through Lemma 3.2 and Property 3.1. *But can we combine all these observations to use the acyclicity of GoJ of a nested OPT query using its graph of supernodes (GoSN)?* We explore it next.

Let the GoJ of a nested OPT pattern without Cartesian products be acyclic, i.e., the GoJ is *connected* and is a *tree*. Cyclic GoJ and Cartesian products are discussed separately in Sections 3.3 and 5.2 respectively. We choose the *least selective* jvar node that appears in an *absolute master* supernode, and fix it as the root of the GoJ tree. Jvar-nodes can be ranked for selectivity as follows: A jvar-node  $?j_1$  is considered more selective than  $?j_2$ , if the most selective TP having  $?j_1$  has fewer triples than the most selective TP having  $?j_2$ , and so on.

With the root fixed as given before, this GoJ tree has all the jvars in absolute masters towards the *top* of the tree (root and internal nodes), and jvars in the slaves towards the *bottom*. This observation follows from the facts that there are no Cartesian products, GoJ is a tree, and we chose the *root* from an absolute master. We can traverse this tree bottom-up with semi-joins and clustered-semi-joins performed at each jvar-node  $?j$  as follows:

- We perform a clustered-semi-join among the TPs that contain  $?j$ , and which appear either in the same supernode, or their supernodes are *peers* of each other.
- We perform a semi-join between pairs of TPs that are in a *master-slave* relationship.

We can do a top-down pass following the same rules. E.g., we choose *?friend* as the root of GoJ tree in our example, because it appears in absolute master  $SN_1$ . In the bottom-up pass, we perform a clustered-semi-join over *?sitcom* among the peers  $tp_2$  and  $tp_3$ , no semi-join for *?sitcom*, because there are no *master-slave* TPs with *?sitcom*. We perform a semi-join  $tp_2 \bowtie_{?friend} tp_1$ , no clustered-semi-join for *?friend* because there are no peer TPs with *?friend*. Next, in the top-down pass, we process *?friend* first followed by *?sitcom*. This process leaves each TP with a *minimal* set of triples.

*But does this give us an optimal order of processing jvars?* No, because a bottom-up pass on the GoJ tree is same as processing OPT patterns per the order imposed in the orig-

inal query – recall that all the jvars in slaves appear towards the *bottom* of the tree, and all the jvars in masters towards the *top* of the tree. So this hardly fetches us any benefits of the selectivity of the master TPs. *We want to find an optimal order of processing jvars, which exploits the selectivity of the masters to aggressively prune the triples.*

For that we use `get_jvar_order` (Alg. 3.1). From the GoJ tree of an OPT pattern, we consider an *induced subtree* consisting only of jvars that appear in the absolute master supernodes (ln 4). Since there are no Cartesian products, this induced subtree is connected. We choose a jvar with the *least selectivity* as the root of this induced subtree, do a bottom-up pass on it, and store the order in *order<sub>bu</sub>* (ln 5–7). Choosing a jvar with least selectivity as the *root* ensures that it is processed *last*.

---

#### Algorithm 3.1: get\_jvar\_order

---

```

input : GoSN, GoJ
output: orderbu, ordertd
1 if GoJ is cyclic then
2   | ordergreedy = greedy-jvar-order(GoSN, GoJ);
3   | return ordergreedy, ordergreedy
4  $\mathcal{J}_m =$  jvars in absolute master supernodes;
5 root =  $?j \in \mathcal{J}_m$  with least selectivity;
6  $\mathcal{T}_m =$  get-tree( $\mathcal{J}_m$ , root);
7 orderbu = bottom-up( $\mathcal{T}_m$ );
8  $SN_{ss} =$  order remaining slaves with masters first;
9 for each slave supernode  $SN_i$  in  $SN_{ss}$  do
10  |  $\mathcal{J}_s =$  jvars in  $SN_i$ ;
11  | root =  $?j \in$  masters( $SN_i$ );
12  |  $\mathcal{T}_s =$  get-tree( $\mathcal{J}_s$ , root);
13  | orderbu = orderbu.append(bottom-up( $\mathcal{T}_s$ ));
14 ordertd = top-down( $\mathcal{T}_m$ );
15 for each slave supernode  $SN_i$  in  $SN_{ss}$  do
16  |  $\mathcal{J}_s =$  jvars in  $SN_i$ ;
17  | root =  $?j \in$  masters( $SN_i$ );
18  |  $\mathcal{T}_s =$  get-tree( $\mathcal{J}_s$ , root);
19  | ordertd = ordertd.append(top-down( $\mathcal{T}_s$ ));
20 return orderbu, ordertd;

```

---



---

#### Algorithm 3.2: prune\_triples

---

```

input: orderbu, ordertd, GoSN
1 for each  $?j$  in orderbu do
2   for each  $tp_i$  do
3     for each  $tp_j$  do
4       | if slave-of( $tp_j$ ,  $tp_i$ ) then
5       | | semi-join( $?j$ ,  $tp_j$ ,  $tp_i$ ); //  $tp_j \bowtie_{?j} tp_i$ 
6   for each  $SN_i$  with  $?j$  do
7     |  $S_{tp} = \{tp \mid ?j \in tp, tp \in (SN_i \cup \text{peers-of}(SN_i))\}$ ;
8     | clustered-semi-join( $?j$ ,  $S_{tp}$ );
9 for each  $?j$  in ordertd do
10  for each  $tp_i$  do
11    for each  $tp_j$  do
12      | if slave-of( $tp_j$ ,  $tp_i$ ) then
13      | | semi-join( $?j$ ,  $tp_j$ ,  $tp_i$ ); //  $tp_j \bowtie_{?j} tp_i$ 
14  for each  $SN_i$  with  $?j$  do
15    |  $S_{tp} = \{tp \mid ?j \in tp, tp \in (SN_i \cup \text{peers-of}(SN_i))\}$ ;
16    | clustered-semi-join( $?j$ ,  $S_{tp}$ );

```

---

We order the remaining supernodes as – masters before their respective slaves, and among any two peer supernodes, a supernode with a more selective triple pattern is ordered first. This order is  $SN_{ss}$  (ln 8). Note that such an ordering of supernodes favours *selective masters* to be processed before their non-selective peers and slaves, thus benefiting

the pruning process. For each supernode  $SN_i$  in  $SN_{ss}$ , we consider an induced subtree of GoJ consisting only of jvars in  $SN_i$ , and choose a *root* jvar such that it also appears in a master of  $SN_i$  (ln 11). Note that since GoJ is a *connected tree*, a slave supernode shares *at least* one jvar with a master. We make a bottom-up pass on this induced subtree of  $SN_i$ , and append it to  $order_{bu}$  (ln 9–13). For a top-down pass, we reverse the above procedure. Starting with the induced subtree of absolute masters, we do a top-down pass, and store it in  $order_{td}$  (ln 14). Using the same order of supernodes,  $SN_{ss}$ , for an induced subtree of each  $SN_i$  in  $SN_{ss}$ , we do a top-down pass, and append it to  $order_{td}$  (ln 15–19).

With  $order_{bu}$  and  $order_{td}$  of jvar-nodes, we prune the triples associated with TPs in a query using `prune_triples` (Alg. 3.2). For each jvar  $?j$  in  $order_{bu}$ , first we do a **semi-join** between TPs that are in a master-slave relationship (ln 2–5). Then we do a **clustered-semi-join** among TPs having  $?j$ , and which appear in the same supernode, or are peers of each other (ln 6–8). Recall that the master-slave or peer relationship among TPs is determined by GoSN. We repeat the same process, but now by following  $order_{td}$  of jvars (ln 9–16). Notice that in this process, **semi-joins** transfer the restrictions on variable bindings from master TPs to the slaves *without* actually performing left-outer-joins, and **clustered-semi-joins** transfer restrictions on variable bindings among all the peers without actually doing inner-joins. Through  $order_{bu}$ ,  $order_{td}$  of pruning we ensure that jvars in masters always get pruned before those in slaves.

**Example-2:** Recalling our running example, which has an acyclic GoJ, we get  $order_{bu} = [(?friend), (?sitcom, ?friend)]$ , and  $order_{td} = [(?friend), (?friend, ?sitcom)]$  from `get_jvar_order`, and with these we use `prune_triples` to do **semi-joins** and **clustered-semi-joins** among the TPs in the master-slave and peer relationships respectively.

LEMMA 3.3. *For a well-designed OPT query with an acyclic GoJ, Algorithm 3.1 followed by Algorithm 3.2 leaves a minimal set of triples for each triple pattern.* □

The proof of Lemma 3.3 is given in Appendix A.3.

### 3.3 Cyclic well-designed OPT patterns

For OPT-free BGPs, i.e., pure inner-joins, with cyclic GoJ, minimality of triples cannot be guaranteed using clustered-semi-joins [16, 17, 43]. This result carries over immediately to cyclic OPT patterns too. For a cyclic OPT pattern, minimality of triples cannot be guaranteed, so we simply return  $order_{greedy}$  from `get_jvar_order` (ln 3), which is a *greedy* order of jvars, i.e., all the jvars are ranked in the descending order of their selectivity. Recall from Section 3.2, that the relative selectivity between two jvars can be determined from the selectivity of the TPs which have those jvars. In `prune_triples`, we use  $order_{greedy}$  in place of  $order_{bu}$  and  $order_{td}$ , and follow the rest of the procedure as is. Since minimality of triples in each TP is not guaranteed, we need to use the *nullification* and *best-match* operations in a re-ordered query to ensure consistent variable bindings, and to remove any subsumed results.

This observation in general holds for all cyclic OPT queries, but we identify a *subclass* of cyclic OPT queries that can avoid nullification and best-match by just using  $order_{greedy}$  in place of  $order_{bu}$  and  $order_{td}$  in `prune_triples` – *in such cyclic OPT queries, each slave supernode has only one jvar in it* (slaves can have one or more non-join variables).

LEMMA 3.4. *For a well-designed OPT query with a cyclic GoJ, if each slave supernode has only one jvar in it, nullification and best-match can be avoided by using Algorithm 3.1 followed by Algorithm 3.2.* □

We have given the proof in Appendix A.4.

From Alg. 3.1 and 3.2, it may seem that `prune_triples` is a “heavy-weight” procedure, than simply doing pairwise joins between the TPs. But as shown by our evaluation in Section 6, our pruning procedure is in fact quite “light-weight”, especially for low-selectivity complex OPT patterns (please note the  $T_{prune}$  values compared to  $T_{total}$  in the query times, and “#initial triples” and “#triples aft pruning” columns in Tables 6.2, 6.3, 6.4). We achieve this through the usage of compressed bitvector indexes on RDF graphs, and procedures that directly work on these compressed indexes without decompressing them. These are described in Section 4. Together they give a competitive query performance.

## 4. INDEXING THE RDF GRAPH

In the recent few years, there have been a lot of advances in the efficient storage and indexing of RDF graphs, with many research systems, e.g., RDF-3X [33], TripleBit [45], BitMat [15], as well as large scale open-source and commercial engines, e.g., Virtuoso [12], MonetDB [7, 42]. Among these options, we have chosen BitMats as the base index structure for implementing our technique. Our reasons of choice are given after we give a brief overview of BitMat, our enhancements in it, and the `fold`, `unfold` operations for the completeness of the text.

If  $V_s$ ,  $V_p$ , and  $V_o$  are the sets of unique subject, predicate, and object values in an RDF dataset, then a 3D bitcube of RDF data has  $V_s \times V_p \times V_o$  dimensions. Each cell in this bitcube represents a unique RDF triple formed by the coordinate values (S P O). If this (S P O) triple is present in the given RDF dataset, that bit is set to 1 in the bitcube. To facilitate joins on S-O dimensions, same S and O values are mapped to the same coordinates of the respective dimensions<sup>5</sup>. Due to space constraints, the exact details of this procedure are given in Appendix D for reader’s convenience. They are borrowed from the details given in Section 3 of [15]. The 3D bitcube of the data given in Figure 3.2 is shown in Figure 4.1.

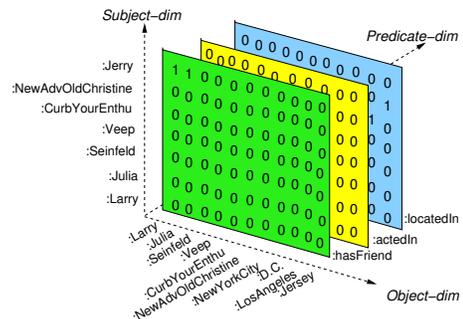


Figure 4.1: 3D Bitcube of RDF data in Figure 3.2

This bitcube is conceptually sliced along each dimension, and the 2D BitMats are created. In general, four types of 2D BitMats are created: (1) S-O and O-S BitMats by slicing the

<sup>5</sup>For the scope of this paper, we do not consider joins on S-P or O-P dimensions.

P-dimension (O-S BitMats are nothing but *transpose* of the respective S-O BitMats), (2) P-O BitMats by slicing the S-dimension, and (3) P-S BitMats by slicing the O-dimension. Altogether we store  $2 * |V_p| + |V_s| + |V_o|$  BitMats for any RDF data. Figure 4.1 shows 2D S-O BitMats that we can get by slicing the predicate dimension (others are not shown for conciseness). Intuitively, a 2D S-O or O-S BitMat of predicate *hasFriend* represents all the triples matching a triple pattern of kind (*?a :hasFriend ?b*), a 2D P-S BitMat of O-value *:Seinfeld* represents all triples matching triple pattern (*?c ?d :Seinfeld*), and so on.

Each row of these BitMats is compressed using run-length-encoding. A bit-row like “1110011110” is represented as “[1] 3 2 4 1”, and “0010010000” is represented as “[0] 2 1 2 1 4”. Notably, in the second case, the bit-row has only two set bits, but it has to use five integers in the compressed representation. So we use a *hybrid* representation in our implementation that works as follows – if the number of set bits in a bit-row are less than the number of integers used to represent it, then we simply store the set bit positions. So “0010010000” will be compressed as “3 6” (3 and 6 being the positions of the set bits). This hybrid compression fetches us as much as 40% reduction in the index space compared to using only run-length-encoding as done in [1].

**Fold** operation is represented as ‘**fold**(BitMat, RetainDimension) returns bitArray’. It takes a 2D BitMat and *folds* it by retaining the *RetainDimension*. More succinctly, a fold operation is nothing but *projection* of distinct values of the particular BitMat dimension, by doing a bitwise OR on the other dimension. It can be represented as:

$$\text{fold}(BM_{tp}, \text{dim}_{?j}) \equiv \pi_{?j}(BM_{tp})$$

$BM_{tp}$  is a 2D BitMat holding the triples matching *tp*, and  $\text{dim}_{?j}$  is the dimension of BitMat that represents variable *?j* in *tp*. E.g., for a triple pattern (*?friend :actedIn ?sitcom*), if we consider the O-S BitMat of predicate *:actedIn*, *?friend* values are in the “column” dimension, and *?sitcom* values are in the “row” dimension of the BitMat.

**Unfold** is represented as ‘**unfold**(BitMat, MaskBitArray, RetainDimension)’. For every bit set to 0 in the *MaskBitArray*, *unfold* clears all the bits corresponding to that position of the *RetainDimension* of the BitMat. *Unfold* can be simply represented as:

$$\text{unfold}(BM_{tp}, \beta_{?j}, \text{dim}_{?j}) \equiv \{t \mid t \in BM_{tp}, t.?j \in \beta_{?j}\}$$

*t* is a triple in  $BM_{tp}$  that matches *tp*.  $\beta_{?j}$  is the *MaskBitArray* containing bindings of *?j* to be retained.  $\text{dim}_{?j}$  is the dimension of  $BM_{tp}$  that represents *?j*, and *t.?j* is a binding of *?j* in triple *t*. In short, **unfold** keeps only those triples whose respective bindings of *?j* are set to 1 in  $\beta_{?j}$ , and removes all other.

**Reasons for choice of BitMats:** RDF stores that achieve high compression ratio using variable length or *delta* encoding, have to decode/decompress the *variable length* IDs, and get them in 4-byte integers for performing joins. This ends up being an overhead for queries that have to access a large amount of data. In BitMats, IDs and in turn triples are represented by *bits* compressed with 4-byte run-lengths, which are manipulated through the **fold-unfold** procedures without decompressing them. Hence we have chosen BitMats as the base index structure in our implementation.

## 5. QUERY PROCESSING

Up to this point, we saw how to construct a GoSN and capture the nesting of OPT patterns in a query in Section

2.1. In Sections 3.2 and 3.3, we presented our optimization strategies to “prune” the triples associated with TPs in acyclic and cyclic well-designed queries, without considering the underlying index structure for RDF data. Then in Section 4, we described BitMat – our index structure.

Now, in this section, we “connect the dots”, i.e., we present how triples associated with TPs in a query are pruned using BitMats, how **semi-join** and **clustered-semi-join** in **prune\_triples** (Alg 3.2) are achieved through BitMats and **fold-unfold** by consulting GoSN, and finally how output results are generated using a *multi-way pipelined join* for any *acyclic or cyclic well-designed* query. We put all these concepts together in Algorithm 5.1.

---

### Algorithm 5.1: Query processing

---

```

input : Original BGP-OPT query
output: Final results

1 GoSN = get-graph-supernodes(Orig BGP-OPT query);
2 GoJ = get-graph-jvars(Orig BGP-OPT query);
3 for each  $tp_i$  in GoSN do
4   |  $\{tp_i \Rightarrow BM_{tp_i}\} = \text{init}()$ ;
5 bool NB-reqd = decide-best-match-reqd(GoSN, GoJ);
   // Alg 3.1
6 ( $order_{bu}, order_{td}$ ) = get_jvar_order(GoSN, GoJ)
7 prune_triples( $order_{bu}, order_{td}, GoSN$ ); // Alg 3.2
8 sorted-tps = sort TPs in master-slave hierarchy;
   // Final result generation - Alg 5.4
9  $allres = \text{multi-way-join}(vmap, sorted-tps, visited, NB-reqd)$ ;
10 if NB-reqd then
11   |  $finalres = \text{best-match}(allres)$ ;
12 else
13   |  $finalres = allres$ ;
14 return  $finalres$ ;

```

---



---

### Algorithm 5.2: semi-join

---

```

input:  $?j, tp_j, tp_i$ 
1  $\beta_{?j} = \text{fold}(BM_{tp_i}, \text{dim}_{?j}) \text{ AND } \text{fold}(BM_{tp_j}, \text{dim}_{?j})$ ;
2 unfold( $BM_{tp_j}, \beta_{?j}, \text{dim}_{?j}$ );

```

---



---

### Algorithm 5.3: clustered-semi-join

---

```

input:  $?j, \{tp_1, \dots, tp_k\}$ 
1  $\beta_{?j} = \text{bitarray}$  with all bits set to 1;
2 for each  $tp_i$  in  $\{tp_1, \dots, tp_k\}$  do
3   |  $\beta_{?j} = \beta_{?j} \text{ AND } \text{fold}(BM_{tp_i}, \text{dim}_{?j})$ ;
4 for each  $tp_i$  in  $\{tp_1, \dots, tp_k\}$  do
5   | unfold( $BM_{tp_i}, \beta_{?j}, \text{dim}_{?j}$ )

```

---

In Algorithm 5.1, we first construct the GoSN and GoJ (ln 1-2). Then with **init**, we load a BitMat for each TP in the query that contains the triples matching that TP (ln 4). Recall from Section 4, that we have, in all, four types of BitMats. We choose an appropriate BitMat for each TP as follows. If the TP in the query is of type (*?var :fx1 :fx2*), i.e., with two fixed positions, we load only one row corresponding to *:fx1* from the P-S BitMat for *:fx2*. Similarly for a TP of type (*:fx1 :fx2 ?var*), we load only one row corresponding to *:fx2* from the P-O BitMat for *:fx1*. E.g., for (*?sitcom :location :NewYorkCity*) we load only one row corresponding to *:location* from the P-S BitMat of *:NewYorkCity*. If the TP is of type (*?var1 :fx1 ?var2*), we load either the S-O or O-S BitMat of *:fx1*. If *?var1* is a join variable and *?var2* is not, we load the S-O BitMat and vice versa. If both,

?var1 and ?var2, are join variables, then we check which of ?var1 and ?var2 appears in  $order_{b_u}$  before the other. If ?var1 comes before ?var2, we load the S-O BitMat and vice versa. Recalling Example-2 from Section 3.2, for (*?friend :actedIn ?sitcom*) we load the S-O BitMat of *:actedIn* because *?friend* comes before *?sitcom* in  $order_{b_u}$ .

While loading the BitMats with `init`, we do active pruning using the TPs that may have been initialized previously. E.g., if we first load BitMat  $BM_{tp_1}$  containing triples matching (*:Jerry :hasFriend ?friend*), then while loading  $BM_{tp_2}$ , we use the bindings of *?friend* in  $BM_{tp_1}$  to actively prune the triples in  $BM_{tp_2}$  while loading it. Then while loading  $BM_{tp_3}$ , we use the bindings of *?sitcom* in  $BM_{tp_2}$  to actively prune the triples in  $BM_{tp_3}$ . We check whether two TPs are joining with each other over an inner or left-outer join using GoSN with the *master-slave* or *peer* relationship, and then decide whether to use other BitMat’s variable bindings.

Next we decide if nullification and best-match are required – they are required for a cyclic query where slaves have more than one jvars (ln 5). Then using `get_jvar_order` (ln 6) we get an optimal order of jvars for the bottom-up and top-down passes on GoJ. Recall that for a *cyclic* GoJ, `get_jvar_order` simply returns ( $order_{greedy}, order_{greedy}$ ). Next, we prune the triples in BitMats using `prune_triples` (ln 7 in Alg 5.1). This procedure has already been explained in Section 3.2, hence we refer the reader to it. Two important operations in `prune_triples` are `semi-join` and `clustered-semi-join` – to remove the triples from the BitMats. These operations make use of the `fold` and `unfold` primitives. We have shown how `fold` and `unfold` are used in `semi-join` and `clustered-semi-join` with Algorithms 5.2 and 5.3 respectively.

Recall from `prune_triples` (Alg 3.2) that we do a `semi-join` (Alg 5.2) between two TPs, if they are in a master-slave relationship over a shared a jvar. The slave TP takes the restrictions on the variable bindings from the master. In `fold` we project out the bindings of *?j* from  $tp_i$  and  $tp_j$  in the form of bitarrays. Through a bitwise AND we take an intersection of these bindings and store the intersection result in  $\beta_{?j}$ . Then we use  $\beta_{?j}$  as the *MaskBitArray* in `unfold` to remove any triples whose respective bindings for *?j* were dropped as a result of the intersection. `Clustered-semi-join` (Alg 5.3) is same as `semi-join`, except that we transfer the restrictions on the variable bindings across all the TPs that share a join variable and are *peers* of each other. Recall Example-2 from Section 3.2, which shows how `semi-join` and `clustered-semi-join` are used.

If an OPT query is *acyclic*, after `prune_triples`, each TP BitMat has a *minimal* set of triples (Lemma 3.3). In case of a *cyclic* OPT query, `prune_triples` only reduces the triples in the BitMats, but they may not be minimal. Note that using `prune_triples`, we prune the triples in BitMats, but we need to actually “join” them to produce the final results. For that we use `multi-way-join` (ln 9 in Alg 5.1). This procedure is described separately in Section 5.1. After `multi-way-join`, we use `best-match` to remove any subsumed results only if the query is cyclic and its slaves have more than one jvars (ln 11) – recall Lemmas 3.3 and 3.4. In `best-match`, we externally sort all the results generated by `multi-way-join`, and then remove the subsumed results with a single pass over them.

In the `init` and `prune_triples` processes, we do a “simple optimization” – if at any point, a TP in an absolute master

supernode has zero triples, we take that as a hint of an empty result, and abandon any further query processing.

Currently Algorithm 5.1 is a main-memory process, i.e., all the TP BitMats are kept in memory during the query processing, and there is no disk spooling. This may pose some limitations on the total size of BitMats in a query. But as seen in our evaluation, LBR could handle low-selectivity queries with up to 13 TPs on a dataset with more than a billion triples on a machine with 8 GB memory, exhibiting the scalability of our technique. Presently LBR does not handle TPs with all variable positions (*?a ?b ?c*), and supporting them is currently under development.

## 5.1 Multi-way Pipelined Join

Before calling `multi-way-join`, we first sort all the TPs in the query as follows. Considering the TPs in absolute master supernodes, we sort them in an ascending order of the number of triples left in each TP’s BitMat. Then we sort remaining TPs in the descending order of master-slave hierarchy and selectivity. That is, among two supernodes connected as  $SN_1 \rightarrow SN_2$ , TPs in  $SN_1$  and any *peers* of  $SN_1$  are sorted before those in  $SN_2$ . Among the peer TPs, they are sorted in the ascending order of the number of triples left in their BitMats (ln 8 in Alg 5.1). This order is `stps`. In `multi-way-join` we use at most  $\sum_{tp_i \in Q} vars(tp_i)$  additional memory buffer, where  $vars(tp_i)$  are the variables in every  $tp_i$  in the query  $Q$ . This is `vmap` in Alg 5.4. Thus we use negligible additional memory in `multi-way-join`.

At the beginning, `multi-way-join` gets an empty `vmap` for storing the variable bindings, `stps`, an empty `visited` list, and a flag `nulreqd` indicating if nullification is required (depending on the cyclicity of the query). In `multi-way-join`, we go over each triple in  $BM_{tp_1}$  of the first TP in `stps`, generate bindings for the variables in  $tp_1$ , and store them in `vmap`. We add  $tp_1$  to the `visited` list, and call `multi-way-join` recursively for the rest of the TPs (ln 6–11). Note that in each recursive call, `multi-way-join` gets a partially populated `vmap` and a `visited` list that tells which TP’s variable bindings are already stored in `vmap`. Then we check if any variables in a non-visited  $tp_i$  are already mapped in `vmap` (ln 16–18). Recall that since the query does not have Cartesian products, we always find *at least one*  $tp_i$ , which has one or more of its variables mapped in `vmap`. Also notice that `stps` order ensures that a master TP’s variable bindings are stored in `vmap` before its slaves. If there exist one or more triples  $t$  in  $BM_{tp_i}$  consistent with the variable bindings in `vmap`, then for each such  $t$  we generate bindings for all the variables in  $tp_i$ , store them in `vmap`, and proceed with the recursive call to `multi-way-join` for the rest of the TPs (ln 20–25). Notice that, this way we *pipeline* all the BitMats, and do not do pairwise joins or use any other intermediate storage like hash-tables.

If we do not find any triple in  $BM_{tp_i}$  consistent with the existing variable bindings in `vmap`, then – (1) if  $tp_i$  is an absolute master, we *rollback* from this point, because an absolute master TP cannot have NULL bindings (ln 28), else (2) we map all the variables in  $tp_i$  to NULLs, and proceed with the recursive call to `multi-way-join` (ln 29–32). When all the TPs in the query are in the `visited` list, we check if we require `nullification` to ensure consistent variable bindings in `vmap` across all the slave TPs, and `output` one result (ln 1–4). We continue this recursive procedure till triples in  $BM_{tp_1}$  are exhausted (ln 6–11).

---

**Algorithm 5.4: multi-way-join**

---

```
input : vmap, stps, visited, nulreqd
output: all the results of the query

1 if visited.size == stps.size then
2   if nulreqd then
3     | nullification (vmap);
4     output (vmap); // generate a single result
5     return;
6 if visited is empty then
7   tp1 = first TP from stps;
8   visited.add(tp1);
9   for each triple t ∈ BMtp1 do
10    | generate bindings for vars(tp1) from t, store in vmap;
11    | multi-way-join(vmap, stps, visited, nulreqd);
12 else
13   for each tpi in stps do
14     if tpi ∈ visited then
15       | continue;
16     get bindings for vars(tpi) from vmap;
17     if no bindings found then
18       | continue;
19     atleast-one-triple = false;
20     for each triple t ∈ BMtpi with same bindings do
21       | atleast-one-triple = true;
22       store vars(tpi) bindings from t in vmap;
23       visited.add(tpi);
24       multi-way-join(vmap, stps, visited, nulreqd);
25       visited.remove(tpi);
26     if (atleast-one-triple == false) then
27       if tpi is an absolute master then
28         | return;
29         // This means tpi is a slave
30         set all vars(tpi) to NULL in vmap;
31         visited.add(tpi);
32         multi-way-join(vmap, stps, visited, nulreqd);
33         visited.remove(tpi);
```

---

Intuitively, `multi-way-join` is reminiscent of a relational join plan with reordered left-outer-joins – that is, in `stps` we sort selective masters before their non-selective peers and slaves, and masters generate variable bindings before slaves in `vmap`. *But note that we pipeline all the joins together, and we can skip nullification and best-match for acyclic queries and cyclic queries with only one jvar per slave, because of our optimization techniques, `get_jvar_order` (Alg 3.1), `prune_triples` (Alg 3.2), and Lemmas 3.3 and 3.4.*

Recall Example-2 from Section 3.2. After `prune_triples`, `tp1` has two triples, and `tp2`, `tp3` have one triple each in their BitMats. We sort the TPs as `stps = [tp1, tp2, tp3]`. In `multi-way-join`, we first generate a binding of `tp1.?friend` and store it in `vmap`. In the recursive calls, we locate triples with the same `?friend` binding in `BMtp2`. For each such triple, we generate `tp2.?friend`, `tp2.?sitcom` bindings in `vmap`, and proceed to `tp3`. In a recursive call, if we do not find any triple with the same variable bindings in `tp2` or `tp3`, we set variables in that TP to null in `vmap`. Since this is an acyclic query, we do not need `nullification`. While `outputting` a result, we pick variable bindings generated by masters over their slaves for common variables in `vmap`, e.g., we pick binding of `?friend` from `tp1.?friend` over `tp2.?friend`.

## 5.2 Discussion

For our experiments, we assume that *all* the variables in a query – join as well as non-join – are SELECTed for projection, because analysis of DBpedia query logs shows that over

95% of the queries SELECT all the variables [35]. As per W3C specifications, SPARQL algebra follows “bag semantics” [9], so SELECTION (projection) of particular variables from a query can be readily supported in LBR by just intercepting the output (`vmap`) statement at line-4 in `multi-way-join`, which will output only mappings of the SELECTed variables from `vmap`.

For the scope of this paper, we have focused on the “join” component of SPARQL, i.e., BGP-OPT patterns without UNIONS, FILTERs, or Cartesian products. But for the completeness of the text, here we discuss how our technique can be extended to handle them.

**UNION:** For this discussion, we assume *well-designed* UNIONS (UWD), which are – for every subpattern  $P' = (P_1 \cup P_2)$  in a query, if a variable  $?j$  in  $P'$  appears outside  $P'$ , then it appears in both  $P_1$  and  $P_2$ . UWDs tend to have high occurrence (e.g., 99.97% as shown in [35]). Also UWDs remain unaffected by the difference between SPARQL and SQL over the treatment of NULLs [36], and following equivalences hold on them [34] – (1)  $(P_1 \cup P_2) \bowtie P_3 \equiv (P_1 \bowtie P_3) \cup (P_2 \bowtie P_3)$ , (2)  $(P_1 \cup P_2) \bowtie P_3 \equiv (P_1 \bowtie P_3) \cup (P_2 \bowtie P_3)$ . Additionally we also rewrite – (3)  $P_1 \bowtie (P_2 \cup P_3)$  as  $(P_1 \bowtie P_2) \cup (P_1 \bowtie P_3)$ , and remove any spurious results in the end<sup>6</sup>. Using these rules, we rewrite a *well-designed* BGP-OPT-UNION query in the *UNION normal form* (UNF) [34], i.e., a query of the form  $P_1 \cup P_2 \cup \dots \cup P_n$  where each sub-pattern  $P_i$  ( $1 \leq i \leq n$ ) is UNION-free. We evaluate each UNION-free  $P_i$  using the same LBR technique, remove any spurious results if rule (3) of rewrite is used, and *add* the results from all the  $P_i$ s. Note that we can *add* the output of all the  $P_i$ s without a conventional *set-union*, because SPARQL UNION follows “bag semantics” [9, 41].

**FILTER:** For this discussion, we assume *safe* FILTERs [34, 36], i.e., for  $P_x \mathcal{F}(R)$ , where  $R$  is a filter to be applied on  $P_x$ , all the variables in  $R$  appear in  $P_x$ ,  $\text{vars}(R) \subseteq \text{vars}(P_x)$ . *Unsafe* filters can alter the semantics of OPT patterns (ref [34]). In addition to the previous rewrite rules introduced under UNION, for a *well-designed* BGP-OPT-UNION query with *safe* filters, following equivalences hold [34, 41] – (4)  $(P_1 \bowtie P_2) \mathcal{F}(R) \equiv (P_1 \mathcal{F}(R)) \bowtie P_2$ , (5)  $(P_1 \cup P_2) \mathcal{F}(R) \equiv (P_1 \mathcal{F}(R)) \cup (P_2 \mathcal{F}(R))$ . Using these five rewrite rules, we *push in* filters, *push out* unions, and bring the query in the UNF, i.e.,  $P_1 \cup P_2 \cup \dots \cup P_n$  where each sub-pattern  $P_i$  ( $1 \leq i \leq n$ ) is union-free.

Now we can evaluate each  $P_i$  using an augmented LBR technique as follows. We apply filters either by intercepting `init` (ln 4) and `prune_triples` (ln 7) in Alg 5.1, or we apply them using a new *filter-and-nullification* (FaN) routine, in place of `nullification` (ln 3 in `multi-way-join`). The decision depends on the type of filters, e.g., if a filter has variables from two or more TPs, then we apply it in FaN when variable bindings from all the TPs are generated in `vmap`. Through FaN, we apply filters as well as do nullification for a cyclic  $P_i$ . We apply `best-match` in the end if FaN nullifies *at least* one variable binding in `vmap` or  $P_i$  is cyclic. We remove any spurious results introduced due to rule (3) of UNION rewrite, and obtain the final results by adding the results from each  $P_i$  in the UNF – please recall our note about SPARQL “bag semantics” under UNION. We can use some “cheap” filter optimizations before rewriting a query, e.g., a filter  $P_1 \mathcal{F}(?m = ?n)$  can be eliminated by replacing

<sup>6</sup>Spurious results may get introduced if either  $P_2$  or  $P_3$  is empty, or  $P_2$  or  $P_3$  does not have any matching variable bindings to that of  $P_1$ .

every  $?n$  by  $?m$  in all the TPs in  $P_1$ . Extending LBR technique for OPT patterns with UNIONS and FILTERs is a part of our ongoing work.

**Cartesian products ( $\times$ ):** If a query is in the *Cartesian normal form*, i.e.,  $P_1 \times P_2 \times \dots \times P_n$  where each sub-pattern  $P_i$  ( $1 \leq i \leq n$ ) is  $\times$ -free, then we can evaluate each  $\times$ -free  $P_i$  using the same LBR technique presented in this paper, and generate the final results by taking a cross product of the results from each  $P_i$  sub-pattern. However, a query with an arbitrary nesting of  $\bowtie$  and  $\times$  poses challenges, because  $\bowtie$  is *not distributive* over  $\times$ . A naïve way of handling such a query can be – use the technique presented in this paper only for the sub-patterns that are  $\times$ -free, and then resort to the standard relational technique of performing pairwise joins or  $\times$ -products to get the final results. As a part of the future work, we plan to extend LBR’s technique to handle such nested OPT-Cartesian queries using an *augmented* GoSN construction and modified **multi-way-join** (Alg 5.4). Note that queries with Cartesian products are rare for RDF data. Previously published SPARQL queries in the literature do not have Cartesian products [15, 28, 32, 33, 35, 45, 46].

## 6. EVALUATION

We developed *Left Bit Right* (LBR) query processing technique with C/C++ language, compiled with g++ v4.8.2, -O3 -m64 flags, on a 64 bit Linux 3.13.0-34-generic SMP kernel (Ubuntu 14.04 LTS distribution). For running the experiments, we used a Lenovo T540p laptop with Intel Core i3-4000M 2.40GHz CPU, 8 GB memory, 12 GB swap space, and 1 TB Western Digital 5400RPM SATA hard disk.

### 6.1 Setup for Experiments

**RDF Stores:** To evaluate the competitive performance of LBR’s techniques, we used two columnstores Virtuoso v.7.1.0 [12] and MonetDB v11.17.21 [7], which are popular for RDF data storage. LBR’s core query processing algorithm works with the integer IDs assigned to the subjects, predicates, objects. Hence we loaded RDF triples with integer valued subjects, predicates, objects in Virtuoso and MonetDB’s native columnstore, and translated all the OPT pattern queries to their equivalent SQL counterpart. We setup all the required indexes, configuration, and “vectorized query execution” in Virtuoso as outlined in [13, 23]. For MonetDB, we created separate predicate tables with triples of only that predicate, ordered on S-O columns as outlined in [42], and also created an O-S index on each of these tables. However, due to a large number of predicates in the DBPedia dataset (see Table 6.1), MonetDB failed to create a separate table for each predicate. Hence we loaded the entire DBPedia data in one 3-column table, and created four indexes PSO, POS, SPO, OPS on that table – same as what LBR and Virtuoso use. Thus we ensured that both Virtuoso and MonetDB have an optimized setup. Current published sources of RDF-3X [33] and TripleBit [45] cannot process OPT queries. GH-RDF3X<sup>7</sup>, a third party system built using sources of RDF-3X for OPT pattern support, could not correctly process many OPT queries.

**Datasets and Queries:** We used three popular RDF datasets, (1) Lehigh University Benchmark (LUBM) – a synthetically generated dataset over 10,000 universities using their data generator program, (2) UniProt [10] – a real life

protein network, and (3) DBPedia (English) 2014 [2] – a real life RDF dataset created from Wikipedia. The characteristics of these datasets are given in Table 6.1.

Datasets	#triples	# S	# P	# O
LUBM	1,335,081,176	217,206,845	18	161,413,042
UniProt	845,074,885	147,524,984	95	128,321,926
DBPedia	565,523,796	29,747,387	57,453	153,561,757

Table 6.1: Dataset characteristics

For a competitive evaluation of the three systems, we used a mix of *well-designed* OPT queries with varying degrees of selectivity, complexity, and running times. The LUBM data generator is popular for scalability analysis [15, 28, 44, 45]. However, its queries do not cover OPT patterns well. Therefore, to get a good coverage, we used UniProt queries from [5, 11], and LUBM queries from [6, 15], and introduced OPTIONAL patterns in them by studying the Ontology and graph structure. Similarly, we used DBPedia queries from [32] by removing union and filter clauses wherever necessary, because the main focus of our paper is on the nested BGP-OPT queries. Note that our approach is similar to that of other research systems, which focus on specific SPARQL components [15, 28, 33, 45, 46]. The current RDF benchmarks suffer from limitations as highlighted in [21]. Consequently, several systems use generated queries for specific SPARQL constructs. We evaluated all three systems over the same OPT queries, and they are given in Appendix E.

**Evaluation Metrics:** We used the following metrics for evaluation: (1) Time required for the **init** process in Alg 5.1 ( $T_{init}$ ), i.e., the time to load the BitMats associated with all the triple patterns in a query. (2) Time required for **prune\_triples** (Alg 3.2) ( $T_{prune}$ ). (3) Total query execution time (warm cache) for each system averaged over 5 runs, i.e., end-to-end *clock-time* ( $T_{total}$ ,  $T_{virt}$ ,  $T_{Monet}$  for LBR, Virtuoso, and MonetDB respectively). We ran each query 6 times by discarding the first runtime to warm up the caches. Also, for reporting this time fairly, we redirected output results of all the three systems to `/dev/null`, to eliminate any overhead of I/O latency in writing the results to the disk. Thus this reported time is the core query processing time spent by each system.  $T_{total}$  of LBR is  $T_{init} + T_{prune} + T_{multiway}$ .  $T_{multiway}$  (Alg 5.4) is not explicitly shown in the evaluation for conciseness, as it can be computed by simply subtracting  $T_{init} + T_{prune}$  from  $T_{total}$ . (4) Initial number of triples – the sum of triples matching each triple pattern in the query before the **init** and **prune\_triples** procedures. (5) Sum of triples left in all the BitMats after **prune\_triples**. (6) Number of final results. (7) Number of results, which have one or more NULL valued bindings, i.e., their result rows do not have all the variables bound due to the left-outer-joins in the query. These latter four metrics help us highlight the selectivity properties of the queries. (8) Whether **nullification** and **best-match** operations were required for LBR (see Section 5). Query processing times over all the three datasets are given in Tables 6.2, 6.3, and 6.4.

Note that the LBR system does not run in “server mode” as Virtuoso and MonetDB do, and it does not use any sophisticated cache management techniques of its own. It may only benefit from the underlying operating system kernel’s file-system cache management.

<sup>7</sup><https://github.com/gh-rdf3x/gh-rdf3x>

	$T_{init}$ (LBR)	$T_{prune}$ (LBR)	$T_{total}$ (LBR)	$T_{Virt}$	$T_{Monet}$	#initial triples	#triples aft pruning	#total re- sults	#results with nulls	best-match reqd?
Q1	5.88	3.95	<b>32.69</b>	104.67	>30min	649,375,261	61,662,975	10,448,905	336,455	No
Q2	22.63	7.33	<b>122.75</b>	197.84	>30min	758,743,140	157,571,451	226,641	8449	No
Q3	7.55	6.21	<b>140.02</b>	436.91	>30min	631,261,274	77,041,410	32,828,280	0	No
Q4	0.38	0.86	1.29	<b>0.015</b>	1466.14	352,340,574	1,701,020	11	6	Yes
Q5	0.36	0.85	1.26	<b>0.018</b>	1506.9	352,340,566	1,700,979	10	3	Yes
Q6	0.57	0.54	1.22	<b>0.03</b>	42.71	438,912,504	1,700,913	7	0	No

Table 6.2: Query proc. times (in seconds, warm cache, best times boldfaced) – LUBM 1.33 billion triples

	$T_{init}$ (LBR)	$T_{prune}$ (LBR)	$T_{total}$ (LBR)	$T_{Virt}$	$T_{Monet}$	#initial triples	#triples aft pruning	#total re- sults	#results with nulls	best-match reqd?
Q1	2.53	1.4	<b>8.2</b>	13.1	10.8	546,188,591	13,889,249	917,773	130,161	No
Q2	1.6	0	<b>1.6</b>	16.68	1.79	162,444,409	0	0	0	No
Q3	0.89	0.73	<b>3.25</b>	7.94	6.57	38,089,712	11,196,191	1,009,371	1,001,134	No
Q4	0.83	0.04	<b>3.61</b>	39.7	23.71	16,757,285	6,079,367	6,079,367	6,079,367	No
Q5	0.78	0.47	1.44	<b>0.49</b>	1.88	49,696,660	16,258,093	5625	5625	No
Q6	1.22	0.73	2.8	<b>1.55</b>	1.81	42,463,971	14,503,826	98,842	73,672	No
Q7	2.38	0.48	4.04	3.37	<b>2.92</b>	34,666,636	18,833,950	272,822	1	No

Table 6.3: Query proc. times (in seconds, warm cache, best times boldfaced) – UniProt 845 million triples

	$T_{init}$ (LBR)	$T_{prune}$ (LBR)	$T_{total}$ (LBR)	$T_{Virt}$	$T_{Monet}$	#initial triples	#triples aft pruning	#total re- sults	#results with nulls	best-match reqd?
Q1	1.19	0.19	<b>3.36</b>	6.65	>10min	22,460,545	3,165,560	515,003	446,204	No
Q2	0.025	0	0.025	<b>0.011</b>	>10min	2,231,035	0	0	0	No
Q3	0.036	0	0.036	<b>0.017</b>	>10min	15,799,901	0	0	0	No
Q4	0.63	0.15	0.81	<b>0.15</b>	>10min	13,493,622	501,240	4919	4917	No
Q5	0.37	0.04	0.44	<b>0.023</b>	0.68	8,624,878	94,336	5330	22	No
Q6	0.4	0.06	0.48	<b>0.02</b>	0.83	50,439,668	48	36	36	No

Table 6.4: Query proc. times (in seconds, warm cache, best times boldfaced) – DBPedia 565 million triples

## 6.2 Analysis of the Evaluation

From Tables 6.2, 6.3, and 6.4, we can see that for queries with low selectivity, LBR’s technique clearly excels over Virtuoso and MonetDB.

Queries Q1–Q3 of LUBM need to access more than 50% of the data (see “#initial triples” column for the respective queries). These queries have a large number of triple patterns, have multiple OPT patterns, and generate a large number of results. Notably, these three queries have cyclic graphs of join-variables (GoJ), but only one join variable in each slave supernode. So LBR does not need to use nullification and best-match operations (see Section 3.2 and Lemma 3.4). For these three queries LBR gives several fold better performance than Virtuoso and MonetDB.

Queries Q4–Q6 of LUBM are short running, simpler queries with one OPT pattern. A closer look at these queries shows us that they have one or two highly selective triple patterns, with fixed values in predicate and object positions, and joins on S-S positions. This helps Virtuoso by way of fast merge-joins. Notice the column “#triples aft pruning”, which shows that these queries actually deal with a very small fraction of the entire data – about a million triples out of more than a billion triple dataset, and they produce very small number of results. Notably, for these queries, while Virtuoso performs much better, MonetDB suffers a lot. We conjecture that for such queries with highly selective *master* triple patterns, Virtuoso probably uses the idea of basic reordering of the inner and left-outer joins. Virtuoso’s “explain” tool revealed that it uses a combination of hash and bloom filters to join the selective master with a non-selective slave. Q4 and Q5 have a cycle in their respective GoJ, and since they also have more than one jvars in their slave supernodes, LBR uses nullification and best-match operations to remove any subsumed results (ref Section 3.3).

Among UniProt queries, for Q1–Q4, LBR performs much better. These queries have multiple nestings of BGP-OPT patterns, and do not have highly selective triple patterns in them. Notably, for Q4 a *semi-join* between master and slave removes all the bindings in the slave. Virtuoso however could not exploit this situation (as it did for LUBM Q4–Q6), because this master TP is not very selective and has only predicate position fixed. LBR’s pruning method with semi-joins shows a benefit here. For Q2, LBR’s *init* procedure with active pruning detects empty results of the query much earlier, and abandons further query processing (recall our “simple optimization” from Section 5). Whereas, Virtuoso and MonetDB detect empty results much later in their query processing. All seven UniProt queries are *acyclic*.

Similar to our previous observations, in case of DBPedia queries, for a query like Q1, which needs to handle larger amount of data, and produces more results, LBR comes out as a clear winner. Q2–Q6 access a much smaller fraction of the data, and generate a small set of results. In case of Q2–Q3, LBR’s initialization with active pruning detects an empty set of results early on and abandons further query processing. MonetDB suffers badly for Q1–Q4. We conjecture that this is because it was unable to create 57,453 distinct predicate tables. We successfully created four types of indexes on MonetDB’s DBPedia table, but its documentation revealed to us that MonetDB does not always honor user created indexes; it relies on its own policies of indexing and managing the data. All six DBPedia queries are *acyclic*.

In general, observe that for low selectivity queries, Q1–Q3 (LUBM), Q1–Q4 (UniProt), and Q1 (DBPedia), LBR prunes a significant portion of the initial triples (see columns “#initial triples” and “#triples aft pruning”), in a relatively small amount of time ( $T_{prune}$  is a very small portion of  $T_{total}$ ). We believe that our novel technique of jvar-tree

traversal (see Section 3.2) along with the compressed index structure and *fold-unfold* operations (see Section 5) make the pruning procedure efficient. Also please notice that LBR processes long running queries *several* seconds or minutes faster than Virtuoso and MonetDB ( $T_{virt} - T_{total}$ ). Virtuoso does better for short running queries, but the difference ( $T_{total} - T_{virt}$ ) is just about *a second*. We feel that for such queries, in the future, LBR can probably use a more sophisticated cache management technique by running in server mode. We trust that for long running queries, the essence of LBR’s optimization technique gets highlighted.

To summarize, our evaluation shows that LBR works much better than the contemporary columnstores for queries with lower selectivity and higher complexity, e.g., queries Q1–Q3 (LUBM), Q1–Q4 (UniProt), and Q1 (DBPedia). The geometric mean of the presented queries for each dataset was as follows – for UniProt, 3.05 sec (LBR), 5.61 sec (Virtuoso), 4.35 (MonetDB), for LUBM, 10.18 sec (LBR) and 2.04 sec (Virtuoso), and for DBPedia 0.28 sec (LBR) and 0.07 sec (Virtuoso). The geometric mean of Virtuoso is lower than LBR for LUBM and DBPedia due to short running queries like Q4–Q6 (LUBM) and Q2–Q6 (DBPedia). We did not compute the geometric mean of MonetDB on LUBM and DBPedia as it took very long time for some of the queries.

**Index Sizes:** The on disk size of LBR indexes, i.e.,  $2|V_p| + |V_s| + |V_o|$  BitMats, with our enhancements of “hybrid compression” technique is 20 GB, 32 GB, and 41 GB for DBPedia, UniProt, and LUBM datasets respectively. But note that we do not need to load all these indexes in memory. For any given query, we just load the BitMats associated with the triple patterns in the query, which are typically a smaller fraction of the total indexes. We ran our experiments on a machine with 8 GB memory, which could fit the BitMats associated with the triple patterns in the queries. For Virtuoso, the on-disk size of the stored data and indexes was 5.5 GB, 9.5 GB, and 11 GB for DBPedia, UniProt, and LUBM datasets respectively, and for MonetDB it was 32 GB, 23 GB, 35 GB for DBPedia, UniProt, and LUBM respectively.

## 7. RELATED WORK

While the SPARQL OPT-free basic graph pattern (BGP) queries and their optimization have enjoyed quite bit of attention from the research community in the past few years, the discussion about OPTIONAL pattern queries has mostly remained theoretical. Previous work, such as [14, 22, 34, 41], has extensively analyzed the semantics of well-designed OPT patterns, as they have high occurrence [27, 35], bear desirable properties for the complexity analysis of the evaluation, and remain unaffected by the disparity between SPARQL and SQL algebra about joins over NULLs. Our query graph of supernodes (GoSN) is reminiscent of well-designed pattern trees (WDPT) [30, 31], but WDPTs are undirected and unordered, whereas GoSN is directed, and establishes an order among the patterns (*master-slave*, *peers*), which is an integral part of our optimization technique. While the work on WDPTs focuses on the analysis of containment and equivalence of well-designed patterns and identifying the tractable components of their evaluation, we use GoSN to focus on the practical aspects of OPT pattern evaluation.

From the practical aspects of the optimization of SQL left-outer-joins, most prominently, Galindo-Legaria, Rosenthal [24, 25, 26] and Rao et al [38, 39] have proposed ways of achieving it through reordering inner and left-outer joins.

Rao et al have proposed *nullification* and *best-match* operators to handle inconsistent variable bindings and subsumed results respectively (see Section 3.1). In their technique, nullification and best-match are required for each reordered query, as the minimality of tuples is not guaranteed (see Lemma 3.1). They do not use methods like semi-joins to prune the candidate tuples. Bernstein et al and Ullman [16, 17, 43] have proved the properties of *minimality* for *acyclic inner-joins* only. Through LBR, we have taken a step forward by extending these semi-join properties in the context of SPARQL OPT patterns, by analyzing the graph of jvar-nodes (GoJ), and finding ways to avoid overheads like *nullification* and *best-match* operations.

For inner-join optimization, RDF engines like TriAD [28], RDF-3X [33], gStore [46] take the approaches like graph summarization, sideways-information-passing etc for an early pruning of triples. Systems like TripleBit [45] use a variable length bitwise encoding of RDF triples, and a query plan generation that favors queries with “star” joins, i.e., many triple patterns joining over a single variable. RDF engines built on top of commercial databases such as DB2RDF [18] propose creation of *entity-oriented* flexible schemas and better data-flow techniques through the query plan to improve the performance of “star” join queries. Along with this, there are distributed RDF processing engines such as H-RDF-3X [29] and SHARD [40]. While many of these engines mainly focus on efficient indexing of RDF graphs, BGP queries, and exploiting “star” patterns in the queries, we have focused on the OPT patterns that may have multiple S-O joins, which cannot exploit the benefits of “star” query optimizers.

Our indexes and pruning procedure resemble the BitMat system [15]. However, BitMat only handles BGP queries. Their query graph cannot capture the structural aspects of a nested BGP-OPT query, and their query processing does not honor any left-outer-joins. Additionally, while using their index structure as a base in our system, we have enhanced it further to reduce as much as 40% of the index size compared to their original method [1].

## 8. CONCLUSION

In this paper, we proposed *Left Bit Right* (LBR), for optimizing SPARQL OPTIONAL pattern queries. We proposed a novel concept of a query graph of *supernodes* to capture the structure of a nested OPT query. We proposed optimization strategies – first of a kind, to the best of our knowledge – that extend the previously known properties of acyclicity, minimality, nullification, and best-match. We presented LBR’s evaluation in comparison with the two mainstream RDF columnstores, Virtuoso and MonetDB, and showed that LBR’s technique works much better for low-selectivity OPT queries with multiple OPT patterns, and for highly selective simpler queries, it gives at par performance with the other systems. In the future, we plan to extend our query processor to handle other SPARQL constructs such as unions, filters, and we intend to investigate methods for better cache management especially for short running queries.

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## APPENDIX

### A. PROOFS

#### A.1 Lemma 3.1

PROOF. Let nullification be required with a minimal set of triples in each TP in a query. This means that there are one or more variable bindings and triples that will be removed as a result of an inner or left-outer-join that caused NULL values for the respective variable bindings. This is a contradiction to our assumption of minimality.

Let us assume that there are subsumed results with a minimal set of triples, and best-match is required to remove them. Let result  $r_2$  be subsumed by  $r_1$  ( $r_2 \sqsubset r_1$ ), and  $non-null(r_1)$  and  $non-null(r_2)$  be the non-NULL variable bindings in  $r_1$  and  $r_2$  respectively. Let  $S_1 = non-null(r_1) \setminus non-null(r_2)$  and  $S_2 = non-null(r_1) \cap non-null(r_2)$ . Per the definition of subsumption,  $non-null(r_2) \subset non-null(r_1)$ . This means that  $S_1$  are the variable bindings contributed by a “slave” TP, which is why they could be set to NULL in  $r_2$  (see the definition of master-slave in Section 2.2). This also means that  $S_2$  variable bindings had corresponding bindings of variables in  $S_1$  in  $r_1$  but not in  $r_2$ , implying that those  $S_1$  bindings and the respective triples were removed in the process of joins. This is a contradiction to our assumption of minimality of triples.  $\square$

#### A.2 Lemma 3.2

PROOF. Consider a query with three join variables, ?a, ?b, ?c, that form a three-cycle in GoJ. Let us assume that the respective GoT of the query is acyclic. An edge between ?a and ?b indicates that there exists a triple pattern  $tp_1$ , which has both ?a and ?b in it. Similarly there must be a  $tp_2$  with join variables ?b and ?c, and  $tp_3$  with join variables ?a and ?c. Now,  $tp_1$  and  $tp_2$  join on ?b,  $tp_1$  and  $tp_3$  join on ?a, and  $tp_2$  and  $tp_3$  join on ?c. This indicates a cycle among the TPs which is a contradiction. The same analysis can be extended to a join-variable cycle of more than three length.

Now consider a GoJ with no cycles, i.e., it is a tree. Consider that there exists a cycle involving TPs  $tp_1 \dots tp_n$  in GoT, such that each  $tp_i$  and  $tp_{i+1}$ ,  $1 \leq i < n$ , join over join variable ? $j_i$ , and  $tp_1$  and  $tp_n$  join over ? $j_n$ . This means that jvar-node graph has an edge between every pair of jvar-nodes ? $j_i$  and ? $j_{i+1}$ ,  $1 \leq i < n$ , and there is an edge between ? $j_n$  and ? $j_1$ . This indicates a cycle in the jvar-node graph, which is a contradiction.  $\square$

#### A.3 Lemma 3.3

PROOF. Let an OPT pattern be  $P_k \bowtie P_l$  with  $P_k$  and  $P_l$  as OPT-free BGPs. Let us temporarily ignore the left-outer-join between  $P_k$  and  $P_l$ , and consider the two BGPs independently.  $P_k$  and  $P_l$  both have an acyclic connected GoJ. This observation follows from the fact that GoJ of  $P_k$  and  $P_l$  together is acyclic, and there are no Cartesian products in the query. Doing a bottom-up followed by top-down pass on the induced subtrees of  $P_k$  and  $P_l$  independently, with only *clustered-semi-joins* for each jvar, ensures that TPs in  $P_k$  and  $P_l$  are left with a minimal set of triples. This follows from Property 3.1 and is proved in [16]. Now consider  $P_k \bowtie P_l$ . Let  $\mathcal{J} = jvars(P_k) \cap jvars(P_l)$ , and  $\mathcal{J} \neq \emptyset$  because GoJ of  $P_k \bowtie P_l$  is connected. In Algorithm 3.1, line 11 chooses one of the jvars in  $\mathcal{J}$  to be the root of  $P_l$ ’s induced subtree. Considering only jvars in  $\mathcal{J}$  and acyclicity,

this means that  $P_l$  has an induced subtree where jvar-nodes shared with its masters ( $\mathcal{J}$ ) appear as the *ancestors* of the jvars that appear only in  $P_l$ , but not in  $P_k$ .

A bottom-up pass over jvar subtrees of  $P_k$  and  $P_l$  (ln 1–8 in Alg 3.2), transfers the restrictions on the variable bindings across respective TPs in  $P_k$  and  $P_l$ . Note that *semi-join* transfers the restrictions on variable bindings from  $P_k$  to  $P_l$ , and *clustered-semi-join* transfers the restrictions on bindings among peers. Recall that we build *order<sub>id</sub>* such that induced subtree of  $P_k$  is traversed before  $P_l$ ’s (ln 14–19 in Alg 3.1). A top-down pass on the jvar subtree of  $P_k$  (ln 9–16 in Alg 3.2) leaves a *minimal* set of triples in the TPs in  $P_k$ . Hence a top-down pass on the jvar subtree of  $P_l$  after  $P_k$ ’s, leaves a minimal set of triples in the TPs in  $P_l$  as a *ripple effect*.

This analysis can be inductively applied for any nesting of BGP and OPT patterns, as long as their GoJ is acyclic and connected. Hence with the procedures in Algorithms 3.1 and 3.2, we can get a minimal set of triples in each TP in a nested acyclic BGP-OPT query.  $\square$

#### A.4 Lemma 3.4

PROOF. We need to use *nullification* and *best-match* if a slave supernode has more than one jvars, because removal of bindings of one jvar may change the bindings of another due to the *ripple effect* through GoJ. Let us consider an OPT pattern of kind  $tp_1 \bowtie_{?j} (tp_2 \bowtie_{?j} tp_3)$ . Let  $P_1 = (tp_1)$  and  $P_2 = (tp_2 \bowtie_{?j} tp_3)$ .  $P_2$  has only one jvar ? $j$ , that it shares with the master  $P_1$ . In *prune\_triples* (Alg 3.2), we transfer the restrictions on bindings of ? $j$  from  $tp_1$  to  $tp_2$  and  $tp_3$  through a *semi-join*. Through a *clustered-semi-join* we ensure that restrictions on bindings of ? $j$  are transferred among  $tp_2$  and  $tp_3$ . So even if the original OPT pattern is reordered as  $(tp_1 \bowtie tp_2) \bowtie tp_3$ , there will not be any subsumed results. This is because, after a *clustered-semi-join*,  $tp_3$  does not have any bindings of ? $j$  that do not appear in  $tp_2$  and vice versa.

An alternate way to look at it is: Consider a cyclic OPT pattern where slaves have more than one jvars. Then if a final result  $r_4$  of the query is subsumed by  $r_3$  ( $r_4 \sqsubset r_3$ ),  $non-null(r_3) \setminus non-null(r_4)$  have at least one jvar that appears *only* in the slaves of the query, i.e.,  $non-null(r_3) \setminus non-null(r_4)$  variables do not appear in *absolute masters* – that is why they could be set to NULLs. Let  $S_1 = non-null(r_3) \setminus non-null(r_4)$ ,  $S_2 = non-null(r_3) \cap non-null(r_4)$ . Note that *not all*  $S_1$  can be non-jvars from slaves, because it would mean that while variables in  $S_2$  had the respective bindings for those in  $S_1$  in  $r_3$ , they did not find any bindings for  $S_1$  in  $r_4$ . As per our *prune\_triples* procedure (Alg. 3.2), we only prune the bindings of jvars; bindings of non-jvars get removed as a *side effect* of the pruning of jvar bindings. So if  $S_1$  has all non-jvars, this will be a contradiction. Hence,  $S_1$  would have at least one join variable that *appears only in a slave*, for subsumed result  $r_4$ . But for a cyclic OPT pattern (without Cartesian products) with each slave having only one jvar, each jvar appears in an *absolute master* too. This in turn means that none of the jvars in the query can be set to NULL.  $S_1$  does not have any jvars that appear only in slaves, which means that subsumed results cannot be generated.

Thus we do not need *nullification* and *best-match*, for a cyclic OPT pattern query, where each slave has only one jvar.  $\square$

## B. NON-WELL-DESIGNED PATTERNS

As defined by Pérez et al in [34], a SPARQL nested BGP-OPT pattern query  $P$  is said to be *well-designed* (WD) if for every subpattern  $P' = (P_i \bowtie P_j)$  of  $P$ , if a join variable  $?j$  occurs in  $P_j$  and outside  $P'$ , then  $?j$  occurs in  $P_i$  as well. An OPT pattern that violates this condition is called a non-well-designed (NWD) pattern.

Before we discuss the evaluation of NWD patterns, it is important to take into consideration how NULLs are treated in left-outer-joins or inner-joins. There is a disparity between the SPARQL and SQL algebra over this, which mainly creates a problem for NWD queries. The same NWD query evaluated over a pure SPARQL engine, e.g., Jena, gives counter-intuitive results than a SPARQL-over-SQL engine, e.g. Virtuoso, when there are joins over NULLs. This issue is discussed in detail with an example in Appendix C. However, using the same assumption of “null-intolerant” (or null-rejecting) joins in SQL as done in the previous literature by Rao et al and Galindo-Legaria, Rosenthal [39, 26], we give a way of simplifying NWD patterns. In a “null-intolerant” join evaluation, NULL values are not matched with anything including other NULL values. E.g., in a left-outer-join  $P_a \bowtie_{?j} P_b$ , if  $?j$  has NULL values in  $P_b$  before the join, they get eliminated during the left-outer-join evaluation. NULLs are only introduced as a result of a left-outer-join for missing values in the right hand side pattern.

For any OPT query (WD or NWD), we first serialize it using OPT-free BGPs, join operators  $\bowtie$ ,  $\bowtie$ , and proper parentheses; then build a graph of supernodes (GoSN) as described in Section 2.1. If the given query is NWD, we identify all OPT-free BGPs that *violate* the WD condition, and the corresponding OPT-free BGPs with which they do a violation. E.g., if the query is  $P_x \bowtie (P_y \bowtie P_z)$ , where  $P_z$  has a join variable  $?j$  that appears in  $P_x$  but not in  $P_y$ , then we say, “ $P_z$  violates WD condition with  $P_x$ , and  $(P_z, P_x)$  is a violation pair”. We identify all such violation pairs in the query, and their corresponding violation supernode pairs in the GoSN. For this example  $(SN_z, SN_x)$  will be the violation pair of supernodes. Temporarily ignoring the directionality of edges in GoSN, we identify the undirected path between each such supernode violation pair. Note that according to GoSN’s construction, there is a unique undirected path between any pair of supernodes. Next, considering the original directionality of edges on this path, we convert any unidirectional edges into bidirectional edges.

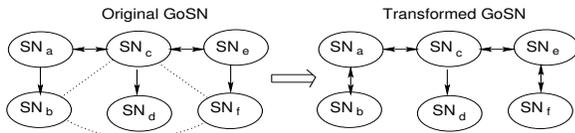


Figure B.1: NWD query GoSN transformation

Conversion of unidirectional edges into bidirectional edges is continued this way until all the violation pairs in GoSN are treated. Note that this is a monotonic process, and always converges – we always convert unidirectional edges into bidirectional edges, and never the other way round. This effectively means we convert one or more left-outer-joins in the original query into inner-joins (recall that a unidirectional edge represents a left-outer-join, and a bidirectional edge represents inner-join). E.g., consider a serialized query  $(P_a \bowtie P_b) \bowtie ((P_c \bowtie P_d) \bowtie (P_e \bowtie P_f))$ , where  $P_b$  violates

WD condition over variable  $?j_1$  with  $P_c$ , and  $P_f$  also violates WD condition over the same variable  $?j_1$  with  $P_c$ . In turn,  $P_b$  and  $P_f$  too violate WD condition with each other over  $?j_1$ . In Figure B.1, we show the original GoSN and the transformed GoSN for this query. The dotted lines in the original GoSN indicate the pairs of respective supernodes that are in violation of the WD condition.

After this transformation process, we use the same query processing techniques given in Algorithms 3.1, 3.2, and 5.1 on this GoSN with null-intolerant join assumption.

## C. TREATMENT OF NULLS

Unlike relational tables, RDF instance data does not contain NULLs<sup>8</sup>. Hence the issue of treatment of NULLs predominantly arises in case of non-well-designed OPT queries. This is because of the difference between the semantics of left-outer-joins between the SPARQL specifications [9] and SQL algebra. Borrowing the definitions from Pérez et al in [34], if we have a pattern  $P_i \bowtie P_j$  with  $\Omega_i$  and  $\Omega_j$  as the set of *mappings* (variable bindings) of  $P_i$  and  $P_j$  respectively, then a SPARQL OPT pattern is defined as:

$$\Omega_i \bowtie \Omega_j = (\Omega_i \bowtie \Omega_j) \cup (\Omega_i \setminus \Omega_j)$$

This definition allows  $P_i \bowtie P_j$  to have results which may have different *arity*. For example, considering the query in Figure 3.2, per above definition, the results will be  $\{(:\text{Larry}), (: \text{Julia}, : \text{Seinfeld})\}$ . Note that the first result has arity 1, whereas the second one has arity 2.

This causes a problem if we do an inner-join of these results (mappings) with another pattern over  $?sitcom$  (as may happen in case of a non-well-designed query). Per SPARQL semantics of inner-join ( $\bowtie$ ), two mappings are considered *join-compatible*, if they match on the variables that are *bound* in the respective mappings. For any unbound variables, the mappings are still considered compatible. E.g., if we join the above mappings with  $\{(:\text{Friends})\}$  over the  $?sitcom$  variable, the result will be counter-intuitive –  $\{(:\text{Larry}, : \text{Friends})\}$ . Note that mapping  $(:\text{Julia}, : \text{Seinfeld})$  got removed, but  $(:\text{Larry})$  was preserved, because it does not have an explicit NULL value for  $?sitcom$ .

In relational databases, evaluation of joins over NULL values has had different interpretations in different contexts (see [34, 38] for a discussion). However, for all practical purposes, most mainstream relational database systems assume a “null-intolerant” join evaluation. Also relational algebra assumes that the two sets of mappings (tuples) that are unioned, are *union compatible*, i.e., they have the same arity and same attributes [37]. Due to these differences in the join and union semantics of SQL and SPARQL, RDF stores that are built on top of relational databases, e.g., Virtuoso, give different results for a non-well-designed query, compared to a native SPARQL processing engine, when joining over NULLs.

We observed that pure SPARQL processing engines, such as ARQ/Jena [3], follow the join semantics which allow union of mappings with different arity, and a *null-tolerant* join, whereas relational RDF stores such as Virtuoso or MonetDB follow the SQL semantics of union-compatibility and null-intolerant joins.

<sup>8</sup>The difference between NULLs and “blank nodes” is elaborated in Section 2.2.

## D. CONCEPTUAL BITCUBE

Borrowing the description of conceptual bitcube construction from [15], here we elaborate on the process of mapping the unique S, P, O values in the RDF data to the each bitcube dimension. As described in Section 4, the dimensions of the 3D bitcube of an RDF dataset are  $V_s \times V_p \times V_o$ . The unique values of S, P, O in the original RDF data are first mapped to integer IDs, which in turn are mapped to the respective bitcube dimensions. Let  $V_{so} = V_s \cap V_o$ . Set  $V_{so}$  is mapped to a sequence of integers 1 to  $|V_{so}|$ . Set  $V_s - V_{so}$  is mapped to a sequence of integers  $|V_{so}| + 1$  to  $|V_s|$ . Set  $V_o - V_{so}$  is mapped to a sequence of integers  $|V_{so}| + 1$  to  $|V_o|$ , and set  $V_p$  is mapped to a sequence of integers 1 to  $|V_p|$ . The common S-O identifier assignment ( $V_{so}$ ) is for the sake of S-O joins.

The 2D BitMats are created by slicing this bitcube along each dimension, and they are compressed using our “hybrid compression” scheme as described in Section 4. Other meta-information such as, the number of triples, and condensed representation of all the non-empty rows and columns in each BitMat, is also stored along with them. This information helps us in quickly determining the number of triples in each BitMat and its selectivity without counting each triple in it, while processing the queries.

## E. QUERIES

### E.1 LUBM Queries

PREFIX ub: <http://www.lehigh.edu/~zhp2/2004/0401/univ-bench.owl#>

**Q1:** SELECT \* WHERE { { ?st ub:teachingAssistantOf ?course . **OPTIONAL** { ?st ub:takesCourse ?course2 . ?pub1 ub:publicationAuthor ?st . } } { ?prof ub:teacherOf ?course . ?st ub:advisor ?prof . **OPTIONAL** { ?prof ub:researchInterest ?resint . ?pub2 ub:publicationAuthor ?prof . } } }

**Q2:** SELECT \* WHERE { { ?pub rdf:type :Publication . ?pub ub:publicationAuthor ?st . ?pub ub:publicationAuthor ?prof . **OPTIONAL** { ?st ub:emailAddress ?ste . ?st ub:telephone ?sttel . } } { ?st ub:undergraduateDegreeFrom ?univ . ?dept ub:subOrganizationOf ?univ . **OPTIONAL** { ?head ub:headOf ?dept . ?others ub:worksFor ?dept . } } { ?st ub:memberOf ?dept . ?prof ub:worksFor ?dept . **OPTIONAL** { ?prof ub:doctoralDegreeFrom ?univ1 . ?prof ub:researchInterest ?resint1 . } } }

**Q3:** SELECT \* WHERE { { ?pub ub:publicationAuthor ?st . ?pub ub:publicationAuthor ?prof . ?st rdf:type :GraduateStudent . **OPTIONAL** { ?st ub:undergraduateDegreeFrom ?univ1 . ?st ub:telephone ?sttel . } } { ?st ub:advisor ?prof . **OPTIONAL** { ?prof ub:doctoralDegreeFrom ?univ . ?prof ub:researchInterest ?resint . } } { ?st ub:memberOf ?dept . ?prof ub:worksFor ?dept . ?prof a ub:FullProfessor . **OPTIONAL** { ?head ub:headOf ?dept . ?others ub:worksFor ?dept . } } }

**Q4:** SELECT \* WHERE { ?x ub:worksFor <http://www.Department9.University9999.edu> . ?x a ub:FullProfessor . **OPTIONAL** { ?y ub:advisor ?x . ?x ub:teacherOf ?z . ?y ub:takesCourse ?z . } }

**Q5:** SELECT \* WHERE { { ?x ub:worksFor <http://www.Department0.University12.edu> . ?x a ub:FullProfessor . **OPTIONAL** { ?y ub:advisor ?x . ?x ub:teacherOf ?z . ?y ub:takesCourse ?z . } } }

**Q6:** SELECT \* WHERE { ?x ub:worksFor <http://www.Department0.University12.edu> . ?x a ub:FullProfessor> . **OPTIONAL** { ?x ub:emailAddress ?y1 . ?x ub:telephone ?y2 . ?x ub:name ?y3 . } }

### E.2 UniProt queries

PREFIX uni: <http://purl.uniprot.org/core/>

PREFIX schema: <http://www.w3.org/2000/01/rdf-schema#>

**Q1:** SELECT \* WHERE { { ?protein rdf:type uni:Protein . ?protein uni:recommendedName ?rn . **OPTIONAL** { ?rn uni:fullName ?name . ?rn rdf:type ?rntype . } } { ?protein uni:encodedBy ?gene . **OPTIONAL** { ?gene uni:name ?gn . ?gene rdf:type ?gtype . } } { ?protein uni:sequence ?seq . ?seq a ?stype . } }

**Q2:** SELECT \* WHERE { { ?a rdf:subject ?b . ?a uni:encodedBy ?vo . **OPTIONAL** { ?a schema:seeAlso ?x } } { ?b a :Protein . ?b uni:sequence ?z . **OPTIONAL** { ?b uni:replaces ?c . } } { ?z a uni:SimpleSequence . **OPTIONAL** { ?z uni:version ?v . } } }

**Q3:** SELECT \* WHERE { { ?protein rdf:type uni:Protein . ?protein uni:organism <http://purl.uniprot.org/taxonomy/9606> . **OPTIONAL** { ?protein uni:encodedBy ?gene . ?gene uni:name ?name . } } { ?protein uni:annotation ?an . **OPTIONAL** { ?an rdf:type uni:Disease\_Annotation . ?an schema:comment ?text . } } }

**Q4:** SELECT \* WHERE { ?s uni:encodedBy ?seq . **OPTIONAL** { ?seq uni:context ?m . ?m schema:label ?b . } }

**Q5:** SELECT \* WHERE { { ?a uni:replaces ?b . **OPTIONAL** { ?a uni:encodedBy ?gene . ?gene uni:name ?name . ?gene rdf:type uni:Gene . } } { ?b rdf:type uni:Protein . ?b uni:modified “2008-01-15” . **OPTIONAL** { ?b uni:sequence ?seq . ?seq uni:memberOf ?m . } } }

**Q6:** SELECT \* WHERE { { ?protein a uni:Protein . ?protein uni:organism <http://purl.uniprot.org/taxonomy/9606> . **OPTIONAL** { ?protein uni:annotation ?an . ?an a uni:Natural\_Variant\_Annotation . ?an schema:comment ?text . } } { ?protein uni:sequence ?seq . ?seq rdf:value ?val . } }

**Q7:** SELECT \* WHERE { ?protein a uni:Protein . ?protein uni:annotation ?an . ?an a uni:Transmembrane\_Annotation . **OPTIONAL** { ?an uni:range ?range . ?range uni:begin ?begin . ?range uni:end ?end . } }

### E.3 DBpedia queries

PREFIX dbpedia: <http://dbpedia.org/resource/>

PREFIX dbpowl: <http://dbpedia.org/ontology/>

PREFIX dbpprop: <http://dbpedia.org/property/>

PREFIX dbpyago: <http://dbpedia.org/class/yago/>

PREFIX dbpcat: <http://dbpedia.org/resource/Category/>

PREFIX rdf: <http://www.w3.org/1999/02/22-rdf-syntax-ns#>

PREFIX rdfs: <http://www.w3.org/2000/01/rdf-schema#>

PREFIX foaf: <http://xmlns.com/foaf/0.1/>

PREFIX geo: <http://www.w3.org/2003/01/geo/wgs84-pos#>

PREFIX owl: <http://www.w3.org/2002/07/owl#>

PREFIX xsd: <http://www.w3.org/2001/XMLSchema#>

PREFIX skos: <http://www.w3.org/2004/02/skos/core#>

PREFIX georss: <http://www.georss.org/georss/>

**Q1:** SELECT \* WHERE { { ?v6 a dbpowl:PopulatedPlace . ?v6 dbpowl:abstract ?v1 . ?v6 rdfs:label ?v2 . ?v6 geo:lat ?v3 . ?v6 geo:long ?v4 . **OPTIONAL** { ?v6 foaf:depiction ?v8 . } } **OPTIONAL** { ?v6 foaf:homepage ?v10 . } **OPTIONAL** { ?v6 dbpowl:populationTotal ?v12 . } **OPTIONAL** { ?v6 dbpowl:thumbnail ?v14 . } }

**Q2:** SELECT \* WHERE { ?v3 foaf:page ?v0 . ?v3 a dbpowl:SoccerPlayer . ?v3 dbpprop:position ?v6 . ?v3 dbpprop:clubs ?v8 . ?v8 dbpowl:capacity ?v1 . ?v3 dbpowl:birthPlace ?v5 . **OPTIONAL** { ?v3 dbpowl:number ?v9 . } }

**Q3:** SELECT \* WHERE { ?v5 dbpowl:thumbnail ?v4 . ?v5 rdf:type dbpowl:Person . ?v5 rdfs:label ?v . ?v5 foaf:page ?v8 . **OPTIONAL** { ?v5 foaf:homepage ?v10 . } }

**Q4:** SELECT \* WHERE { { ?v2 a dbpowl:Settlement . ?v2 rdfs:label ?v . ?v6 a dbpowl:Airport . ?v6 dbpowl:city ?v2 . ?v6 dbpprop:iata ?v5 . **OPTIONAL** { ?v6 foaf:homepage ?v7 . } } **OPTIONAL** { ?v6 dbpprop:nativename ?v8 . } }

**Q5:** SELECT \* WHERE { ?v4 skos:subject ?v . ?v4 foaf:name ?v6 . **OPTIONAL** { ?v4 rdfs:comment ?v8 . } }

**Q6:** SELECT \* WHERE { ?v0 rdfs:comment ?v1 . ?v0 foaf:page ?v . **OPTIONAL** { ?v0 skos:subject ?v6 . } **OPTIONAL** { ?v0 dbpprop:industry ?v5 . } **OPTIONAL** { ?v0 dbpprop:location ?v2 . } **OPTIONAL** { ?v0 dbpprop:locationCountry ?v3 . } **OPTIONAL** { ?v0 dbpprop:locationCity ?v9 . ?a dbpprop:manufacturer ?v0 . } **OPTIONAL** { ?v0 dbpprop:products ?v11 . ?b dbpprop:model ?v0 . } **OPTIONAL** { ?v0 georss:point ?v10 . } **OPTIONAL** { ?v0 rdf:type ?v7 . } }