Local vs. Global Illumination & Radiosity

An early application of radiative heat transfer in stables.

Last Time?

• Ray Casting & Ray-Object Intersection
• Recursive Ray Tracing
• Distribution Ray Tracing

Today

• Local Illumination
  – BRDF
  – Ideal Diffuse Reflectance
  – Ideal Specular Reflectance
  – The Phong Model
• Why is Global Illumination Important?
• Radiosity Equation/Matrix
• Calculating the Form Factors

BRDF

• Ratio of light coming from one direction that gets reflected in another direction
• Bidirectional Reflectance Distribution Function
  – 4D
  – \( R(\theta_i, \phi_i; \theta_o, \phi_o) \)

Incoming Radiance

• The amount of light received by a surface depends on incoming angle
  – Bigger at normal incidence (Winter/Summer difference)
• By how much?
  – \( dB = dA \cos \theta \)
  – Same as: \( l \cdot n \)
    (dot product with normal)

Ideal Diffuse Reflectance

• Assume surface reflects equally in all directions (a.k.a. Lambertian)
• An ideal diffuse surface is, at the microscopic level, a very rough surface
• Examples: chalk, clay, some paints

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Ideal Specular Reflectance
- Assume surface reflects only in mirror direction
  - View dependent
- Microscopic surface elements are oriented in the same direction as the surface
- Examples: mirrors, highly polished metals

Non-Ideal Reflectors
- Real materials tend to be neither ideal diffuse nor ideal reflective
- Highlight is blurry, looks glossy

Non-Ideal Reflectors
- Most light reflects in the ideal reflected direction
- Microscopic surface variations will reflect light just slightly offset
- How much light is reflected?

The Phong Model
- How much light is reflected “specularly”? Depends on the angle between the ideal reflection direction and the viewer direction $\alpha$.

Surface $\theta_l$ $\theta_r$ $\theta_v$ $\theta_n$ $\theta_i$

The Phong Model
- Sum of three components:
diffuse reflection + specular reflection + “ambient”.

Ambient Illumination
- In a typical room, everything receives at least a little bit of light
- Ambient illumination represents the reflection of all indirect illumination
  $L(\omega_i) = k_a$
- This is a total hack!
Anisotropic BRDFs

- Surfaces with strongly oriented microgeometry
- Examples:
  - brushed metals, hair, fur, cloth, velvet

Source: Westin et.al 92

Questions?

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  - The Cornell Box
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Why Global Illumination?

- Simulate all light inter-reflections (indirect lighting)
  - in a room, a lot of the light is indirect: it is reflected by walls.
- How have we dealt with this so far?
  - Ambient term to fake some uniform indirect light

Why Radiosity?

- Sculpture by John Ferren
- Diffuse panels

Radiosity vs. Ray Tracing

Original sculpture by John Ferren lit by daylight from behind. Ray traced image. A standard ray tracer cannot simulate the interreflection of light between diffuse surfaces. Image rendered with radiosity, note color bleeding effects.
The Cornell Box

- Careful calibration and measurement allows for comparison between physical scene & simulation

photograph simulation
Light Measurement Laboratory
Cornell University, Program for Computer Graphics

Two approaches for global illumination
- Radiosity
  - View-independent
  - Diffuse materials only
- Monte-Carlo Ray-tracing
  - Send tons of indirect rays

Radiosity vs. Ray Tracing
- Ray tracing is an *image-space* algorithm
  - If the camera is moved, we have to start over
- Radiosity is computed in *object-space*
  - View-independent (just don't move the light)
  - Can pre-compute complex lighting to allow interactive walkthroughs

Questions?

Lightscape http://www.lightscape.com
Today

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Radiosity Overview

- Surfaces are assumed to be perfectly Lambertian (diffuse)
  - reflect incident light in all directions with equal intensity
- The scene is divided into a set of small areas, or patches.
- The radiosity, \( B_i \), of patch \( i \) is the total rate of energy leaving a surface. The radiosity over a patch is constant.
- Units for radiosity: Watts / steradian * meter\(^2\)

Radiosity Equation

\[
L(x',\omega') = E(x',\omega') + \int \rho_x(\omega,\omega')L(x,\omega)G(x,x')V(x,x') \, dA
\]

Radiosity assumption:
perfectly diffuse surfaces (not directional)

\[
B_x = E_x + \rho_x \int G(x,x')V(x,x') \, B_x \, dA
\]

Continuous Radiosity Equation

reflectivity
\[
B_x = E_x + \rho_x \int G(x,x')V(x,x') \, B_x \, dA
\]

G: geometry term
V: visibility term

No analytical solution, even for simple configurations

Discrete Radiosity Equation

Discretize the scene into \( n \) patches, over which the radiosity is constant

\[
B_i = E_i + \rho_i \sum_{j=1}^{n} F_{ij} B_j
\]

- discrete representation
- iterative solution
- costly geometric/visibility calculations

The Radiosity Matrix

\[
\begin{bmatrix}
B_1 \\
B_2 \\
\vdots \\
B_n
\end{bmatrix} =
\begin{bmatrix}
1-ho_{F_{11}} & -\rho_{F_{12}} & \cdots & -\rho_{F_{1n}} \\
-\rho_{F_{21}} & 1-ho_{F_{22}} & \cdots & -\rho_{F_{2n}} \\
\vdots & \vdots & \ddots & \vdots \\
-\rho_{F_{n1}} & -\rho_{F_{n2}} & \cdots & 1-ho_{F_{nn}}
\end{bmatrix}
\begin{bmatrix}
E_1 \\
E_2 \\
\vdots \\
E_n
\end{bmatrix}
\]

\( n \) simultaneous equations with \( n \) unknown \( B_i \) values can be written in matrix form:

A solution yields a single radiosity value \( B_i \) for each patch in the environment, a view-independent solution.
Solving the Radiosity Matrix

The radiosity of a single patch \( i \) is updated for each iteration by gathering radiosities from all other patches:

\[
\begin{bmatrix}
B_i \\
B_j \\
\vdots \\
B_n
\end{bmatrix} =
\begin{bmatrix}
E_i \\
E_j \\
\vdots \\
E_n
\end{bmatrix} + \begin{bmatrix}
\rho F_{i1} & \rho F_{i2} & \cdots & \rho F_{in} \\
\beta F_{j1} & \beta F_{j2} & \cdots & \beta F_{jn} \\
\vdots & \vdots & \ddots & \vdots \\
\beta F_{ni} & \beta F_{nj} & \cdots & \beta F_{nn}
\end{bmatrix}
\]

This method is fundamentally a Gauss-Seidel relaxation.

Computing Vertex Radiosities

- \( B_i \) radiosity values are constant over the extent of a patch.
- How are they mapped to the vertex radiosities (intensities) needed by the renderer?
  - Average the radiosities of patches that contribute to the vertex.
  - Vertices on the edge of a surface are assigned values extrapolation.

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Calculating the Form Factor \( F_{ij} \)

- \( F_{ij} = \frac{\text{fraction of light energy leaving patch } j \text{ that arrives at patch } i}{\text{takes account of both:}} \)
  - geometry (size, orientation & position)
  - visibility (are there any occluders?)

\[
F_{ij} = \frac{1}{A_i} \int \int \frac{\cos \theta_i \cos \theta_j}{\pi r^2} V_{ij} dA_j dA_i
\]
Form Factor Determination

The Nusselt analog: the form factor of a patch is equivalent to the fraction of the unit circle that is formed by taking the projection of the patch onto the hemisphere surface and projecting it down onto the circle.

Hemicube Algorithm

- A hemicube is constructed around the center of each patch
- Faces of the hemicube are divided into "pixels"
- Each patch is projected (rasterized) onto the faces of the hemicube
- Each pixel stores its pre-computed form factor
  The form factor for a particular patch is just the sum of the pixels it overlaps
- Patch occlusions are handled similar to z-buffer rasterization

Form Factor from Ray Casting

- Cast $n$ rays between the two patches
  - $n$ is typically between 4 and 32
  - Compute visibility
  - Integrate the point-to-point form factor
- Permits the computation of the patch-to-patch form factor, as opposed to point-to-patch

Questions?

Reading for Friday 3/16:

Chuang, Goldman, Curless, Salesin, & Szeliski
Shadow Matting and Compositing
SIGGRAPH 2003

Lightscape  http://www.lightscape.com