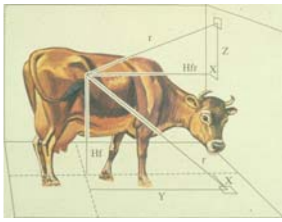
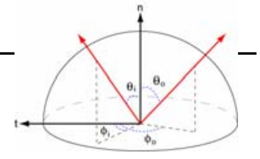
The Rendering Equation & Radiosity II

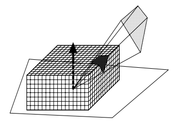


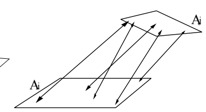
An early application of radiative heat transfer in stables.

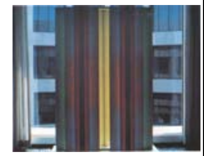
Last Time?

- Local Illumination
  - BRDF
  - Ideal Diffuse Reflectance
  - Ideal Specular Reflectance
  - The Phong Model
- Radiosity Equation/Matrix
- Calculating the Form Factors





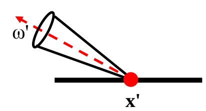




Today

- The Rendering Equation
- Radiosity Equation/Matrix
- Advanced Radiosity
  - Progressive Radiosity
  - Adaptive Subdivision
  - Discontinuity Meshing
  - Hierarchical Radiosity

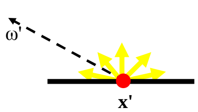
The Rendering Equation



$$L(x',\omega') = E(x',\omega') + \int \rho_{x'}(\omega,\omega')L(x,\omega)G(x,x')V(x,x')\,dA$$

L (x',ω') is the radiance from a point on a surface in a given direction ω'

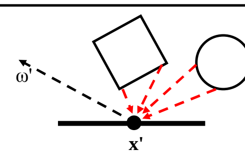
The Rendering Equation



$$L(x',\omega') = E(x',\omega') + \int \rho_{x'}(\omega,\omega')L(x,\omega)G(x,x')V(x,x')\,dA$$

E(x',ω') is the emitted radiance from a point: E is non-zero only if x' is emissive (a light *source*)

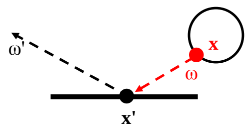
The Rendering Equation



$$L(x',\omega') = E(x',\omega') + \int \rho_{x'}(\omega,\omega')L(x,\omega)G(x,x')V(x,x')\,dA$$

Sum the contribution from all of the other surfaces in the scene

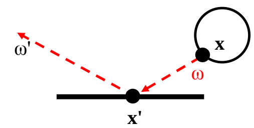
The Rendering Equation



$$L(x',\omega') = E(x',\omega') + \int \rho_{x'}(\omega,\omega')L(x,\omega)G(x,x')V(x,x')\,dA$$

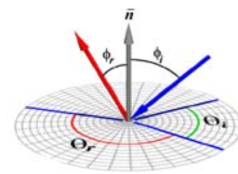
For each x, compute $L(x, \omega)$, the radiance at point x in the direction ω (from x to x')

The Rendering Equation

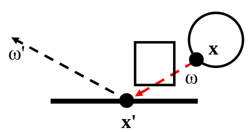


$$L(x',\omega') = E(x',\omega') + \int \rho_{x'}(\omega,\omega')L(x,\omega)G(x,x')V(x,x')\,dA$$

scale the contribution by $\rho_{x'}(\omega,\omega')$, the reflectivity (BRDF) of the surface at x'



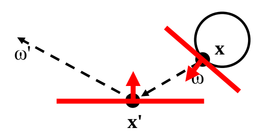
The Rendering Equation



$$L(x',\omega') = E(x',\omega') + \int \rho_{x'}(\omega,\omega')L(x,\omega)G(x,x')V(x,x')\,dA$$

For each x, compute $V(x,x')$, the visibility between x and x': 1 when the surfaces are unobstructed along the direction ω, 0 otherwise

The Rendering Equation

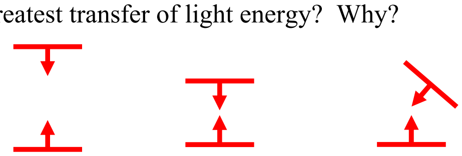


$$L(x',\omega') = E(x',\omega') + \int \rho_{x'}(\omega,\omega')L(x,\omega)G(x,x')V(x,x')\,dA$$

For each x, compute $G(x, x')$, which describes the on the geometric relationship between the two surfaces at x and x'

Intuition about G(x,x')?

- Which arrangement of two surfaces will yield the greatest transfer of light energy? Why?



Questions?

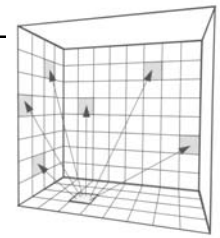
Lightscape http://www.lightscape.com

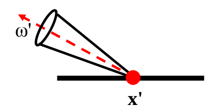
Today

- The Rendering Equation
- Radiosity Equation/Matrix
- Advanced Radiosity
  - Progressive Radiosity
  - Adaptive Subdivision
  - Discontinuity Meshing
  - Hierarchical Radiosity

Radiosity Overview

- Surfaces are assumed to be perfectly Lambertian (diffuse)
  - reflect incident light in all directions with equal intensity
- The scene is divided into a set of small areas, or patches.
- The radiosity, B_i, of patch *i* is the total rate of energy leaving a surface. The radiosity over a patch is constant.
- Units for radiosity: Watts / steradian * meter^2



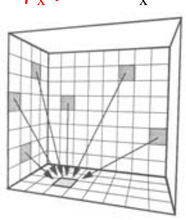


Radiosity Equation

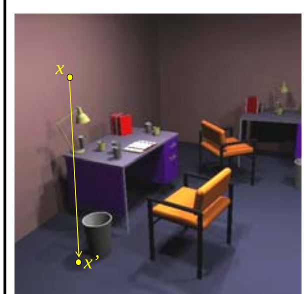
$$L(x',\omega') = E(x',\omega') + \int \rho_{x'}(\omega,\omega') L(x,\omega) G(x,x') V(x,x')\, dA$$

Radiosity assumption:
perfectly diffuse surfaces (not directional)

$$B_{x'} = E_{x'} + \rho_{x'} \int B_x\, G(x,x') V(x,x')$$



Continuous Radiosity Equation



reflectivity

$$B_{x'} = E_{x'} + \rho_{x'} \int G(x,x')\, V(x,x')\, B_x$$

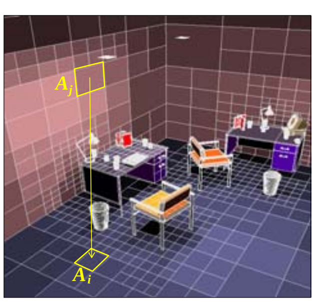
form factor

G: geometry term
V: visibility term

No analytical solution,
even for simple configurations

Discrete Radiosity Equation

Discretize the scene into *n* patches, over which the radiosity is constant



reflectivity

$$B_i = E_i + \rho_i \sum_{j=1}^{n} F_{ij} B_j$$

form factor

- discrete representation
- iterative solution
- costly geometric/visibility calculations

The Radiosity Matrix

$$B_i = E_i + \rho_i \sum_{j=1}^{n} F_{ij} B_j$$

n simultaneous equations with *n* unknown B_i values can be written in matrix form:

$$\begin{bmatrix} 1-\rho_1 F_{11} & -\rho_1 F_{12} & \cdots & -\rho_1 F_{1n} \\ -\rho_2 F_{21} & 1-\rho_2 F_{22} & & \\ \vdots & & \ddots & \\ -\rho_n F_{n1} & \cdots & \cdots & 1-\rho_n F_{nn} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{bmatrix}$$

A solution yields a single radiosity value B_i for each patch in the environment, a view-independent solution.

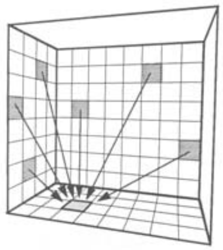
Solving the Radiosity Matrix

The radiosity of a single patch i is updated for each iteration by *gathering* radiosities from all other patches:

$$\begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_i \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_i \\ \vdots \\ E_n \end{bmatrix} + \begin{bmatrix} & & & \\ \rho_i F_{i1} & \rho_i F_{i2} & \cdots & \rho_i F_{in} \\ & & & \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_i \\ \vdots \\ B_n \end{bmatrix}$$

Radiosity values on iteration *t+1*

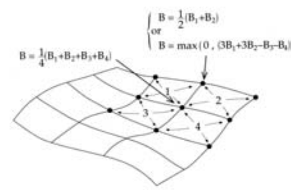
Radiosity values on iteration *t*



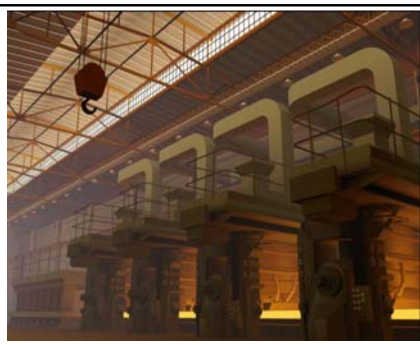
This method is fundamentally a Gauss-Seidel relaxation

Computing Vertex Radiosities

- B_i radiosity values are constant over the extent of a patch.
- How are they mapped to the vertex radiosities (intensities) needed by the renderer?
  - Average the radiosities of patches that contribute to the vertex
  - Vertices on the edge of a surface are assigned values extrapolation



Questions?



Factory simulation. Program of Computer Graphics, Cornell University. 30,000 patches.

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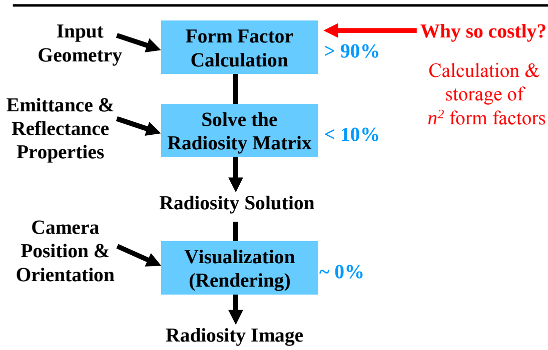
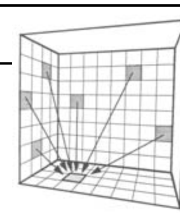
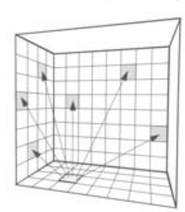
Stages in a Radiosity Solution



Progressive Refinement

- Goal: Provide frequent and timely updates to the user during computation
- Key Idea: Update the entire image at every iteration, rather than a single patch
- How? Instead of summing the light received by one patch, distribute the radiance of the patch with the most *undistributed radiance*.

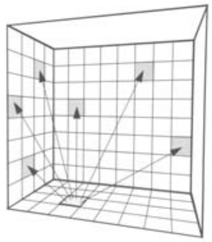




Reordering the Solution for PR

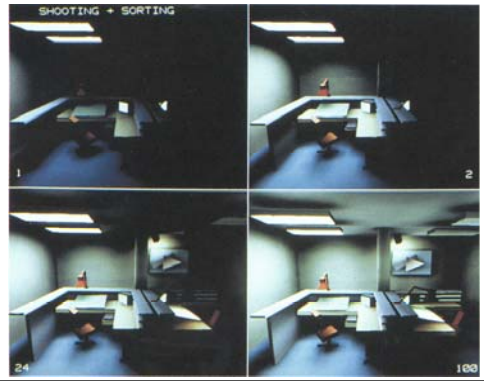
Shooting: the radiosity of all patches is updated for each iteration:

$$\begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ \vdots \\ B_n \end{bmatrix} + \begin{bmatrix} \cdots & \rho_1 F_{1i} & \cdots \\ \cdots & \rho_2 F_{2i} & \\ & & \\ & & \\ \cdots & \rho_n F_{ni} & \cdots \end{bmatrix} \begin{bmatrix} \\ \vdots \\ B_i \\ \vdots \\ \end{bmatrix}$$

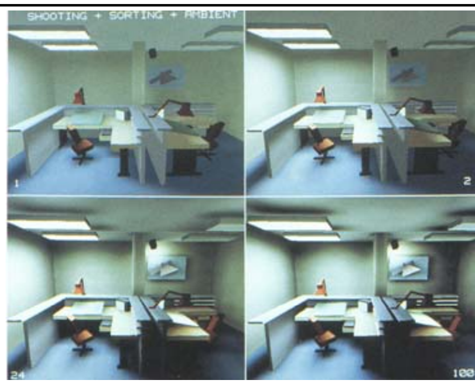


This method is fundamentally a Southwell relaxation

Progressive Refinement w/out Ambient Term



Progressive Refinement with Ambient Term



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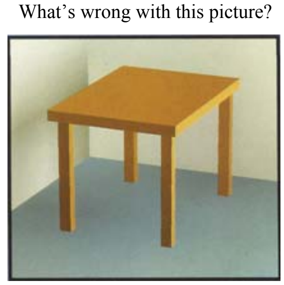
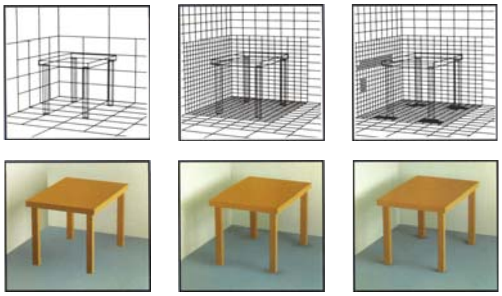
Increasing the Accuracy of the Solution

What's wrong with this picture?



- Image quality is a function of patch size
- Compute a solution on a uniform initial mesh, then refine the mesh in areas that exceed some error tolerance:
  - shadow boundaries
  - other areas with a high radiosity gradient

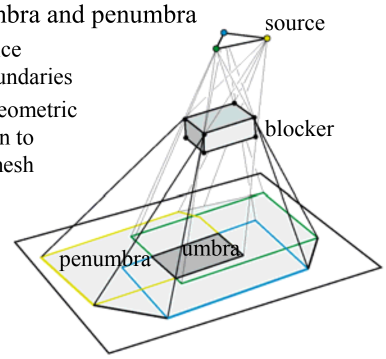
Adaptive Subdivision of Patches



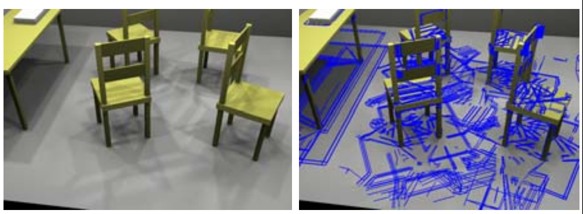
Coarse patch solution (145 patches) | Improved solution (1021 subpatches) | Adaptive subdivision (1306 subpatches)

Discontinuity Meshing

- Limits of umbra and penumbra
  - Captures nice shadow boundaries
  - Complex geometric computation to construct mesh



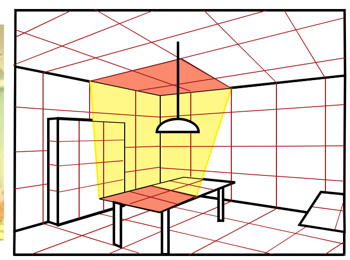
Discontinuity Meshing



"Fast and Accurate Hierarchical Radiosity Using Global Visibility"
Durand, Drettakis, & Puech 1999

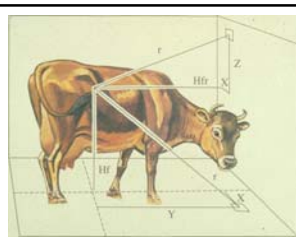
Hierarchical Radiosity

- Group elements when the light exchange is not important
  - Breaks the quadratic complexity
  - Control non trivial, memory cost



Practical Problems with Radiosity

- Meshing
  - memory
  - robustness
- Form factors
  - computation
- Diffuse limitation
  - extension to specular takes too much memory



Cow-cow form factor?

Questions?

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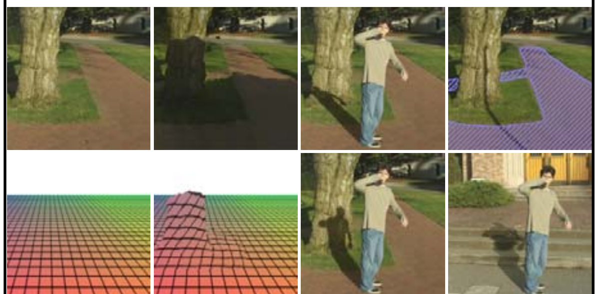
Reading for Today:



Chuang, Goldman, Curless, Salesin, & Szeliski
Shadow Matting and Compositing
SIGGRAPH 2003

Reading for Tuesday 3/20:

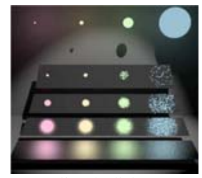
Lokovic and Veach
Deep Shadow Maps
SIGGRAPH 2000

Shadows are important!

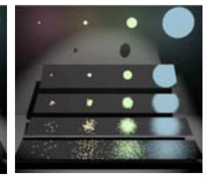
Local illumination only

Reading for Friday 3/23:

Veach & Guibas
"Optimally Combining Sampling Techniques for Monte Carlo Rendering"
SIGGRAPH 95



Sampling the light source



Sampling the BRDF