Subdivision Surfaces

Last Time?
- Curves & Surfaces
- Continuity Definitions
  \(- C^0, G^1, C^1, \ldots C^\infty \)
- Interpolation vs. Approximation Splines
- Cubic Bezier & BSpline

Today
- Spline Surfaces / Patches
  - Tensor Product
  - Bilinear Patches
  - Bezier Patches
  - Trimming Curves
- Subdivision Surface “Zoo”
- Seams In Subdivision
- Misc. Mesh/Surface Vocabulary
- “Piecewise Smooth Surface Reconstruction”

Tensor Product
- Of two vectors:
  \[ \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix} \otimes \begin{bmatrix} a_4 \\ b_4 \end{bmatrix} = \begin{bmatrix} a_1b_1 & a_1b_2 & a_1b_3 & a_1b_4 \\ a_2b_1 & a_2b_2 & a_2b_3 & a_2b_4 \\ a_3b_1 & a_3b_2 & a_3b_3 & a_3b_4 \end{bmatrix} \]
- Similarly, we can define a surface as the tensor product of two curves....

Bilinear Patch
- Smooth version of quadrilateral with non-planar vertices...
  - But will this help us model smooth surfaces?
  - Do we have control of the derivative at the edges?
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Bicubic Bezier Patch

Notation: \( CB(P_1, P_2, P_3, P_4, \alpha) \) is Bezier curve with control points \( P_i \) evaluated at \( \alpha \)

Define “Tensor-product” Bezier surface

\[
Q(s, t) = CB(P_{10}, P_{15}, P_{11}, P_{16}, s, t) \\
CB(P_{12}, P_{13}, P_{13}, P_{14}, s, t) \\
CB(P_{10}, P_{11}, P_{15}, P_{16}, s, t) \\
CB(P_{12}, P_{13}, P_{13}, P_{14}, s, t)
\]

Editing Bicubic Bezier Patches

Curve Basis Functions

Surface Basis Functions

Bicubic Bezier Patch Tessellation

- Given 16 control points and a tessellation resolution, we can create a triangle mesh

Modeling with Bicubic Bezier Patches

- Original Teapot specified with Bezier Patches

- But it’s not “watertight”: it has intersecting surfaces at spout & handle, no bottom, a hole at the spout tip, a gap between lid & base

Trimming Curves for Patches

Shirley, Fundamentals of Computer Graphics
Spline-Based Modeling Headaches

irregular sampling

“pinched” surfaces

seams & holes

Questions?

• Bezier Patches?
  or
• Triangle Mesh?

Henrik Wann Jensen

Today

• Spline Surfaces / Patches
• Subdivision Surface “Zoo”
  – Doo Sabin (anything!)
  – Loop (triangles only)
  – Catmull Clark (turns everything into quads)
  – … many others!
• Seams In Subdivision
• Misc. Mesh/Surface Vocabulary
• “Piecewise Smooth Surface Reconstruction”

Chaikin's Algorithm

Doo-Sabin Subdivision

May introduce a new vertex for each face
At the midpoint of old vertices, face centroid

Doo-Sabin Subdivision

Original Cube  The 1st subdivision  The 2nd subdivision
The 3rd subdivision  The 5th subdivision

http://www.ke.ics.saitama-u.ac.jp/xuz/pic/doo-sabin.gif
Loop Subdivision

Shirley, Fundamentals of Computer Graphics

Catmull Clark Subdivision

DeRose, Kass & Truong, SIGGRAPH 1998

Questions?

Justin Legakis

Today

- Spline Surfaces / Patches
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Seams don’t Subdivide as Expected
**Genus:** The maximum number of disjoint simple closed curves which can be cut from an orientable surface of genus g without disconnecting it is g.

**Dihedral Angle:**
- the angle between the planes of two triangular faces
- “looking down the edge” between two faces, the angle between the faces.

**Homeomorphic/Topological equivalence:**
- a continuous stretching and bending of the object into a new shape

**Spline Surfaces / Patches**
- Subdivision Surface “Zoo”
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**From input: scanned mesh points**
- Estimate topological type (genus)
- Mesh optimization (a.k.a. simplification)
- Smooth surface optimization
Adding creases to Loop Subdivision

- Vertex & edge masks
- Limit masks
  - Position
  - Tangent

Piecewise Smooth Surface Reconstruction

- Optimization
- Remeshing

Piecewise Smooth Surface Reconstruction

- Crease subdivision masks decouple behavior of surface on either side of crease
- Crease rules cannot model a cone
- Optimization can be done locally
  - subdivision control points have only local influence
- Results
  - Noise?
  - Applicability?
  - Limitations?
  - Running Time

Reading for Tuesday (1/29)

- DeRose, Kass, & Truong, "Subdivision Surfaces in Character Animation", SIGGRAPH 1998
- Post a comment or question on the LMS discussion by 10am on Tuesday 1/29

Reading for Friday (2/1)


Other...

- Assigned readings & discussion
- Anonymous homework discussion

- Homework 1: Simplification & Subdivision Questions/Comments?
- Makefile & OpenGL/glut compiler/platform issues
  - drand48/srand48 vs. rand/srand
  - #include <Assert.h> vs. #include <cassert>