Voxels & Collisions

Last Time?

- Spring-Mass Systems
- Numerical Integration (Euler, Midpoint, Runge-Kutta)
- Modeling string, hair, & cloth

Today

- More on Cloth!
  - Taylor Series Analysis
  - Stiffness
  - Implicit Integration
- Implicit Surfaces
- Voxels
- Collisions
- Untangling Cloth

Analysis using Taylor Series

- Expand exact solution \( X(t) \)
  \[ X(t+h) = X(t) + h(\frac{d}{dt}X(t)) + \frac{h^2}{2!} \frac{d^2}{dt^2}X(t) + \frac{h^3}{3!} \frac{d^3}{dt^3}X(t) + \cdots \]

- Euler’s method:
  \[ X(t+h) = X(t) + h f(t, X(t)) \]
  \[ h \rightarrow h/2 \Rightarrow \text{error} \rightarrow \text{error}/4 \text{ per step} \times \text{twice as many steps} \rightarrow \text{error}/2 \]

- First-order method: Accuracy varies with \( h \)
  - To get 100x better accuracy need 100x more steps

The Stiffness Issue

- What relative stiffness do we want for the different springs in the network?
- Cloth is barely elastic, shouldn’t stretch so much!
- Inverse relationship between stiffness & \( \Delta t \)
- We really want a constraints (not springs)
- Many numerical solutions
  - reduce \( \Delta t \)
  - use constraints
  - implicit integration
  - …

How would you simulate a string?

- Springs link the particles. Problems?
  - Stretch, actual length will be greater than rest length
  - Numerical oscillation

- Rigid, fixed-length bars link the particles
  - Dynamics & Constraints
    - (must be solved simultaneously)
The Discretization Problem

- What happens if we discretize our cloth more finely, or with a different mesh structure?

- Do we get the same behavior?
  - Usually not! It takes a lot of effort to design a scheme that does not depend on the discretization.
- Using (explicit) Euler, how many timesteps before a force propagates across the mesh?

Explicit vs. Implicit Integration

- With an explicit/forward integration scheme:
  \[ y_{k+1} = y_k + h \cdot g(y_k) \]
  we must use a very small timestep to simulate stable, stiff cloth.
- Alternatively we can use an implicit/backwards scheme:
  \[ y_{k+1} = y_k + h \cdot g(y_{k+1}) \]
  \[ y_k = y_{k+1} - h \cdot g(y_{k+1}) \]
  Solving one step is much more expensive (Newton’s Method, Conjugate Gradients, …) but overall faster than the thousands of explicit timesteps required for very stiff springs.

Questions?

- Dynamic motion driven by animation

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Implicit Surfaces

- For a sphere:
  \[ H(x,y,z) = x^2 + y^2 + z^2 - r^2 \]
  - If \( H(x,y,z) = 0 \), on surface
  - If \( H(x,y,z) > 0 \), outside surface
  - If \( H(x,y,z) < 0 \), inside surface

Level Sets

- Efficient method for computing signed distance field

  \[ \text{Level Set Methods and Fast Marching Methods, Sethian, 1999} \]

  naive approach
  using level sets
Marching Cubes

- Polygonization: extract triangle mesh from signed distance field


Marching Tetrahedra

"Marching Tetrahedra"

"When the Blobs Go Marching Two by Two", Jeff Lander, Gamasutra

"Interval volume tetrahedrization"

Visualization '97

Nielson & Sung

Marching Tetrahedra

Similarly, we can create volumetric models:

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Collisions

- Detection
- Response
- Overshooting problem (when we enter the solid)

Detecting Collisions

- Easy with implicit equations of surfaces
- $H(x,y,z)=0$ at surface
- $H(x,y,z)<0$ inside surface
- So just compute $H$ and you know that you’re inside if it’s negative

- More complex with other surface definitions
Collision Response

- tangential velocity $v_t$ unchanged
- normal velocity $v_n$ reflects:
  $$v = v_t + v_n$$
  $$v \leftarrow v_t - \varepsilon v_n$$
- coefficient of restitution
  (1 for elastic, 0 for plastic)
- change of velocity $= - (1 + \varepsilon) v$
- change of momentum $\text{Impulse} = - m (1 + \varepsilon) v$

Collisions - Overshooting

- Usually, we detect collision when it’s too late: we’re already inside
- Solutions: back up
  - Compute intersection point
  - Compute response there
  - Advance for remaining fractional time step
- Other solution:
  Quick and dirty fixup
  - Just project back to object closest point

Collision Detection for Solids

- How to detect collision between 2 polyhedra?
- Need an inside/outside test
- Test if a vertex is inside the other polyhedron
- But treat also edge-edge intersection

Cost of Detection?

- Test each edge with each face?
  - $O(N^2)$
- How would you detect collision between two bunnies?
  - $O(N^2)$ is too expensive!
  - Use spatial hierarchy

Conservative Bounding Region

- First check for an intersection with a conservative bounding region
- Early reject

Conservative Bounding Regions

- tight → avoid false positives
- fast to intersect
Overlap test

- Overlap between two axis-aligned boxes?
  - Check if the intervals along the 3 dimensions overlap
- Overlap test between two spheres?
  - $D(\text{center}_1, \text{center}_2) < r_1 + r_2$

General Collision Detection

- Put a hierarchy around your objects
- Use the fast overlap test recursively
- Handle exact case at the leaves (when necessary)
- More difficult for self-collision (e.g. cloth)
  - Because there is more overlap

Reduced Deformation

- Collisions are expensive
- Deformation is expensive
- This is a lot of geometry!
- Simplify the simulation model

Cloth Collision

- A cloth has many points of contact
- Stays in contact
- Requires
  - Efficient collision detection
  - Efficient numerical treatment (stability)

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Reading for Today:

- Baraff, Witkin & Kass, Untangling Cloth, SIGGRAPH 2003
**Reading for Friday 2/8:**

- “Realistic Animation of Liquids”, Foster & Metaxas, 1996

- Post a comment or question on the LMS discussion by 10am on Friday 2/8

**Reading for Tuesday 2/12:**


- Post a comment or question on the LMS discussion by 10am on Tuesday 2/12