The Rendering Equation & Radiosity II

An early application of radiative heat transfer in stables.

Last Time?
- Local Illumination
  - BRDF
  - Ideal Diffuse Reflectance
  - Ideal Specular Reflectance
  - The Phong Model
- Radiosity Equation/Matrix
- Calculating the Form Factors

Today
- The Rendering Equation
- Radiosity Equation/Matrix
- Advanced Radiosity
  - Progressive Radiosity
  - Adaptive Subdivision
  - Discontinuity Meshing
  - Hierarchical Radiosity

The Rendering Equation

\[ L(x', \omega') = E(x', \omega') + \int \rho_{\omega, \omega'}(x, x') L(x, \omega) G(x, x') V(x, x') \, dA \]

- \( E(x', \omega') \) is the emitted radiance from a point: \( E \) is non-zero only if \( x' \) is emissive (a light source)
- Sum the contribution from all of the other surfaces in the scene
### The Rendering Equation

For each \( x \), compute \( L(x, \omega) \), the radiance at point \( x \) in the direction \( \omega \) (from \( x \) to \( x' \))

\[
L(x', \omega') = E(x', \omega') + \int \rho_{x'}(\omega, \omega')L(x, \omega)G(x, x')V(x, x') \, dA
\]

For each \( x \), compute \( L(x, \omega) \), the radiance at point \( x \) in the direction \( \omega \) (from \( x \) to \( x' \))

scale the contribution by \( \rho_{x'}(\omega, \omega') \), the reflectivity (BRDF) of the surface at \( x' \)

### The Rendering Equation

For each \( x \), compute \( V(x, x') \), the visibility between \( x \) and \( x' \): 1 when the surfaces are unobstructed along the direction \( \omega \), 0 otherwise

\[
L(x', \omega') = E(x', \omega') + \int \rho_{x'}(\omega, \omega')L(x, \omega)G(x, x')V(x, x') \, dA
\]

For each \( x \), compute \( V(x, x') \), the visibility between \( x \) and \( x' \): 1 when the surfaces are unobstructed along the direction \( \omega \), 0 otherwise

### Intuition about \( G(x, x') \)?

- Which arrangement of two surfaces will yield the greatest transfer of light energy? Why?

### Questions?

- Lightscape: [http://www.lightscape.com](http://www.lightscape.com)
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Radiosity Equation

\[ L(\mathbf{x}', \omega') = E(\mathbf{x}', \omega') + \int \rho_x(\mathbf{x}, \omega', \omega) L(\mathbf{x}, \omega) G(\mathbf{x}, \mathbf{x}') V(\mathbf{x}, \mathbf{x}') \, dA \]

\[ B_{\mathbf{x}'} = E_{\mathbf{x}'} + \rho_{\mathbf{x}'} \int B_{\mathbf{x}} G(\mathbf{x}, \mathbf{x}') V(\mathbf{x}, \mathbf{x}') \]

Radiosity assumption: perfectly diffuse surfaces (not directional)

Solving the Radiosity Matrix

The radiosity of a single patch \( \mathbf{i} \) is updated for each iteration by gathering radiosities from all other patches:

This method is fundamentally a Gauss-Seidel relaxation

Stages in a Radiosity Solution

Progressive Refinement

- Goal: Provide frequent and timely updates to the user during computation
- Key Idea: Update the entire image at every iteration, rather than a single patch
- How?: Instead of summing the light received by one patch, distribute the radiance of the patch with the most undistributed radiance.
Reordering the Solution for PR

Shooting: the radiosity of all patches is updated for each iteration:

\[
\begin{bmatrix}
B_1 & B_2 & \cdots & \rho F_1 & \cdots \\
B_2 & B_1 & \cdots & \rho F_2 & \cdots \\
\vdots & \vdots & \ddots & \vdots & \ddots \\
B_n & B_1 & \cdots & \rho F_n & \cdots \\
\end{bmatrix}
\]

This method is fundamentally a Southwell relaxation

Progressive Refinement with Ambient Term

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Increasing the Accuracy of the Solution

What’s wrong with this picture?

- Image quality is a function of patch size
- Compute a solution on a uniform initial mesh, then refine the mesh in areas that exceed some error tolerance:
  - shadow boundaries
  - other areas with a high radiosity gradient
Adaptive Subdivision of Patches

Coarse patch solution (145 patches)
Improved solution (1021 subpatches)
Adaptive subdivision (1306 subpatches)

Discontinuity Meshing

- Limits of umbra and penumbra
  - Captures nice shadow boundaries
  - Complex geometric computation to construct mesh

Discontinuity Meshing

“Fast and Accurate Hierarchical Radiosity Using Global Visibility”
Durand, Drettakis, & Puech 1999

Hierarchical Radiosity

- Group elements when the light exchange is not important
  - Breaks the quadratic complexity
  - Control non trivial, memory cost

Practical Problems with Radiosity

- Meshing
  - memory
  - robustness
- Form factors
  - computation
- Diffuse limitation
  - extension to specular takes too much memory

Questions?

Lightscape http://www.lightscape.com
Reading for Today:

- **Optional Reading:** “The Rendering Equation” Kajiya, SIGGRAPH 1986

Reading for Friday 3/4:


Post a comment or question on the LMS discussion by 10am on Friday 3/4