Inverse Kinematics

Last Time?
- Navier-Stokes Equations
- Conservation of Momentum & Mass
- Incompressible Flow

Today
- How do we animate?
  - Keyframing
  - Procedural Animation
  - Physically-Based Animation
  - Forward and Inverse Kinematics
  - Motion Capture
- Rigid Body Dynamics
- Finite Element Method

Keyframing
- Use spline curves to automate the in betweening
  - Good control
  - Less tedious than drawing every frame
- Creating a good animation still requires considerable skill and talent

Procedural Animation
- Describes the motion algorithmically, as a function of small number of parameters
- Example: a clock with second, minute and hour hands
  - express the clock motions in terms of a "seconds" variable
  - the clock is animated by varying the seconds parameter
- Example: A bouncing ball
  - $\text{Abs}(\sin(\omega t + \theta_0))e^{it}$

Physically-Based Animation
- Assign physical properties to objects (masses, forces, inertial properties)
- Simulate physics by solving equations
- Realistic but difficult to control

ACM © 1987 "Principles of traditional animation applied to 3D computer animation"
Articulated Models

- Articulated models:
  - rigid parts
  - connected by joints
- They can be animated by specifying the joint angles as functions of time.

Forward Kinematics

- Given skeleton parameters \( p \), and the position of the effector in local coordinates \( V_s \), what is the position of the effector in the world coordinates \( V_w \)?

\[
V_w = T(x_h, y_h, z_h)R(q_h, f_h, s_h)T(q, f, s)T(q, f, f)V_s
\]

\[
V_w = S(p)V_s
\]

Inverse Kinematics (IK)

- Given the position of the effector in local coordinates \( V_s \) and the desired position \( V_w \) in world coordinates, what are the skeleton parameters \( p \)?
- Much harder requires solving the inverse of the non-linear function:

\[
\text{find } p \text{ such that } S(p)V_s = V_w
\]

Under-/Over-Constrained IK

Application: Robot Motion Planning

Searching Configuration Space

- Use gradient descent to walk from starting configuration to target
- Angle restrictions & collisions can introduce local minima

“The good-looking textured light-sourced bouncy fun smart and stretchy page” Hugo Elias, http://freespace.virgin.net/hugo.elias/models/m_ik.htm

“The good-looking textured light-sourced bouncy fun smart and stretchy page” Hugo Elias, http://freespace.virgin.net/hugo.elias/models/m_ik2.htm
IK Challenge

- Find a “natural” skeleton configuration for a given collection of pose constraints
- A vector constraint function $C(p) = 0$ collects all pose constraints
- A scalar objective function $g(p)$ measures the quality of a pose, $g(p)$ is minimum for most natural poses. Example $g(p)$:
  - deviation from natural pose
  - joint stiffness
  - power consumption

Motion Capture

- Optical markers, high-speed cameras, triangulation → 3D position
- Captures style, subtle nuances and realism
- You must observe someone do something

How do they Animate Movies/Games?

- Keyframing mostly
- Articulated figures, inverse kinematics, motion capture
- Skinning
  - Complex deformable skin, muscle, skin motion
- Hierarchical controls
  - Smile control, eye blinking, etc.
  - Keyframes for these higher-level controls
- A huge time is spent building the 3D models, its skeleton and its controls
- Physical simulation for secondary motion
  - Hair, cloth, water

Questions?

Reading for Today:

- Rapid prototyping of realistic character motion from rough low-quality animations
- Obey the laws of physics & stay within space of naturally-occurring movements

What’s a Natural Pose?

- Training database of ~50 “natural poses”
- For each, compute center of mass of:
  - Upper body
  - Arms
  - Lower body
- The relative COM of each generated pose is matched to most the most similar database example
Linear and Angular Momentum

- In unconstrained animation (no contacts), both linear & angular momentum should be conserved
- The center of mass should follow a parabolic trajectory according to gravity
- The joints should move such that the angular momentum of the whole body remains constant

During Constrained Motion

- During constrained motion (when in contact with the ground), the angular momentum follows a spline curve modeled after biomechanics data

System Features

- Automatically detect point/line/plane constraints
- Divide animation into constrained portions (e.g., feet in contact with ground) and unconstrained portions (e.g., free flight)
- Linear and angular momentum constraints without having to compute muscle forces
- Minimize:
  - Mass displacement
  - Velocity of the degrees of freedom (DOF)
  - “Unbalance” (distance the COM projected to ground is outside of constraints)

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- Finite Element Method

Rigid Body Dynamics

- Could use particles for all points on the object
  - But rigid body does not deform
  - Few degrees of freedom
- Use only one particle at the center of mass
- Compute Net Force & Net Torque

Rigid Body Dynamics

- Physics
  - Velocity
  - Acceleration
  - Angular Momentum
- Collisions
- Friction

Simulation of Non-Rigid Objects

- We modeled string & cloth using mass-spring systems. Can we do the same for deformable solids?
- Yes… But a more physically accurate model uses volumetric elements:

Finite Element Method

- To solve the continuous problem (deformation of all points of the object)
  - Discretize the problem
  - Express the interrelationship
  - Solve a big linear system
- More principled than Mass-Spring

Strain & Stress

- Stress
  - the internal distribution of forces within a body that balance and react to the loads applied to it
  - normal stress & shear stress
- Strain
  - material deformation caused by stress.
  - measured by the change in length of a line or by the change in angle between two lines

Reading for Friday 2/15:


Post a comment or question on the LMS discussion by 10am on Friday 2/15