Rigid Body Dynamics, Fracture, & Deformation

Announcements: Quiz

- On Friday (3/4), in class
- One 8.5x11 sheet of notes allowed
- Sample quiz (from last year) is posted online
- Focus on “reading comprehension” and material for Homeworks 0, 1, & 2

Last Time?

- Keyframing
- Procedural Animation
- Physically-Based Animation
- Forward and Inverse Kinematics
- Motion Capture

Today

- Rigid Body Dynamics
- Finite Element Method
- Deformation
- Fracture

Rigid Body Dynamics

- Could use particles for all points on the object
  – But rigid body does not deform
  – Few degrees of freedom
- Use only one particle at the center of mass
- Compute Net Force & Net Torque

Energy & Rigid Body Collisions

- Total Energy stays constant if there is no damping and no friction

[http://www.myphysicslab.com/collision.html](http://www.myphysicslab.com/collision.html)
Collision Between Two Objects

- Suppose a vertex on body A is colliding into an edge of body B at point P. Define the following variables:
  - \( m_1, m_2 \) = mass of bodies A, B
  - \( \vec{r}_{a} \) = distance vector from center of mass of body A to point P
  - \( \vec{r}_{b} \) = distance vector from center of mass of body B to point P
  - \( \vec{v}_{a1}, \vec{v}_{b1} \) = initial pre-collision angular velocity of bodies A, B
  - \( \vec{v}_{a2}, \vec{v}_{b2} \) = final post-collision angular velocity of bodies A, B
  - \( \vec{x}_{a1}, \vec{x}_{b1} \) = final post-collision velocities of center of mass bodies A, B
  - \( \vec{v}_{a3}, \vec{v}_{b3} \) = initial pre-collision velocity of impact point on body A
  - \( \vec{v}_{b3} \) = initial pre-collision velocity of impact point on body B
  - \( \vec{n} \) = normal (perpendicular) vector to edge of body B
  - \( \epsilon \) = elasticity (0 = inelastic, 1 = perfectly elastic)

http://www.myphysicslab.com/collision.html

Center of Mass & Moment of Inertia

- Center of Mass: mean location of all mass in the system
- Moment of Inertia: a measure of an object's resistance to changes to its rotation
- If a solid cylinder & a hollow tube have the same radius & the same mass, which will reach the bottom of the ramp first?

http://solomon.physics.sc.edu/~tedeschi/demo/demo12.html

Rigid Body Dynamics

- Physics
  - Velocity
  - Acceleration
  - Angular Momentum
- Collisions
- Friction

from: Darren Lewis
http://www-cs-students.stanford.edu/~dalewis/cs448a/rigidbody.html

Collisions

- Victor J. Milenkovic & Harald Schmidl
  Optimization-Based Animation
  SIGGRAPH 2001

- We know how to simulate bouncing really well
- But resting collisions are harder to manage

Guendelman, Bridson & Fedkiw
Nonconvex Rigid Bodies with Stacking
SIGGRAPH 2003

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Simulation of Non-Rigid Objects

- We modeled string & cloth using mass-spring systems. Can we do the same?
- Yes…
- But a more physically accurate model uses volumetric elements:

Image from O'Brien et al. 1999
Finite Element Method

- To solve the continuous problem (deformation of all points of the object)
  - Discretize the problem
  - Express the interrelationship
  - Solve a big linear system
- More principled than Mass-Spring

Diagram from Debunne et al. 2001

Strain & Stress

- Stress
  - the internal distribution of forces within a body that balance and react to the loads applied to it
  - normal stress & shear stress
- Strain
  - material deformation caused by stress.
  - measured by the change in length of a line or by the change in angle between two lines

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Level of Detail

- Interactive shape deformation
- Use high-resolution model only in areas of extreme deformation

Multi-Resolution Deformation

- Use Voronoi diagrams to match parent & child vertices.
- Interpolate values for inactive interface vertices from active parent/child vertices
- Need to avoid interference of vibrations between simulations at different resolutions


Haptic Device

- “3D mouse” + force feedback
- 6 DOF (position & orientation)
- requires 1000 Hz refresh (visual only requires ~30 Hz)

Sensable’s Phantom

http://www.sensible.com/
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Some Definitions

- *Isotropic*: is a property which does not depend on the direction.
- *Anisotropic*: is a property which is directionally dependent.
Some Definitions

• **Elastic Deformation**: Once the forces are no longer applied, the object returns to its original shape.

• **Plastic Deformation**: An object in the plastic deformation range will first have undergone elastic deformation, which is reversible, so the object will return part way to its original shape.

![Image: Stress-strain1.png](http://en.wikipedia.org/wiki/Image:Stress-strain1.png)

Some Definitions

• **Tension**: The direction of the force of tension is parallel to the string, away from the object exerting the stretching force.

• **Compression**: resulting in reduction of volume

![Image: GraphicalModeling.jpg](http://www.aero.polimi.it/~merlini/SolidMechanics-FluidsElastcity/CompressionBlock.jpg)

Some Definitions

• **Degenerate/Ill-conditioned Element**: a.k.a. how “equilateral” are the elements?
  - Ratio of volume to surface area
  - Smallest solid angle
  - Ratio of volume to volume of smallest circumscribed sphere


Reading for Today:


  ![Fracture block](http://fig.cox.miami.edu/~cmallery/255/255chem/macrosett/sticks.jpg)

  - Fracture threshold
  - Remeshing
    - need connectivity info!
  - Material properties
  - Parameter tuning

Fracture Opening Modes

Figure 6: Three loading modes that can be experienced by a crack. Mode I: Opening, Mode II: In-Plane Shear, and Mode III: Out-of-Plane Shear. Adapted from Anderson [1].

![Fracture block](http://www.aero.polimi.it/~merlini/SolidMechanics-FluidsElastcity/CompressionBlock.jpg)

Local Mesh Refinement

Managing Fracture Adjacency

Fracture Propagation Difficulties

• Need to track direction of fracture propagation?

• Need to track crack tip?

Controlling Speed of Propagation

Questions?

Readings for Tuesday 3/1: (read both)