Rigid Body Dynamics, Collision Response, & Deformation

Last Time?
- Navier-Stokes Equations
- Conservation of Momentum & Mass
- Incompressible Flow

Today
- Rigid Body Dynamics
- Collision Response
- Non-Rigid Objects
- Finite Element Method
- Deformation
- Level-of-Detail

Rigid Body Dynamics
- How do we simulate this object’s motion over time?
- We could discretize the object into many particles...
  - But a rigid body does not deform
  - Only a few degrees of freedom
- Instead, we use only one particle at the center of mass
- Compute net force & net torque


Degree of Freedom (DOF)
- Rotations:
  - Translations count too… 6 DOF
Collisions

- Detection
- Response
- Overshooting problem (when we enter the solid)

Collisions - Overshooting

- Usually, we detect collision when it’s too late: we’re already inside
- Solutions: back up
  - Compute intersection point
  - Compute response there
  - Advance for remaining fractional time step
- Other solution: Quick and dirty fixup
  - Just project back to object closest point

Collision Between Two Objects

- Suppose a vertex on body A is colliding into an edge of body B at point P. Define the following variables:

  \[ m_a, m_b = \text{mass of bodies A, B} \]
  \[ \mathbf{r}_{A0} = \text{distance vector from center of mass of body A to point P} \]
  \[ \mathbf{r}_{B0} = \text{distance vector from center of mass of body B to point P} \]
  \[ \mathbf{v}_{A0}, \mathbf{v}_{B0} = \text{initial pre-collision velocities of center of mass bodies A, B} \]
  \[ \mathbf{v}_{A}, \mathbf{v}_{B} = \text{final post-collision velocities of center of mass bodies A, B} \]
  \[ \mathbf{v}_{A0}, \mathbf{v}_{B0} = \text{initial pre-collision velocities of center of mass bodies A, B} \]
  \[ \mathbf{v}_{A}, \mathbf{v}_{B} = \text{final post-collision velocities of center of mass bodies A, B} \]
  \[ \mathbf{v}_{A0} = \text{initial pre-collision velocity of impact point on body A} \]
  \[ \mathbf{v}_{B0} = \text{initial pre-collision velocity of impact point on body B} \]
  \[ \mathbf{n} = \text{normal (perpendicular) vector to edge of body B} \]
  \[ \epsilon = \text{elasticity} \]
  \[ I_{CM} = \text{moment of inertia} \]

Collision Response

- tangential velocity \( v_t \) unchanged
- normal velocity \( v_n \) reflects:
  \[ v = v_t + v_n \]
  \[ v \leftarrow v_t - \epsilon v_n \]
- coefficient of restitution
  - 1 for elastic
  - 0 for plastic
- change of velocity = \(-(1+\epsilon)v\)
- change of momentum \( \text{Impulse} = -m(1+\epsilon)v \)

Energy & Rigid Body Collisions

- Total Energy stays constant if there is no damping and no friction
- Rotational Energy is constant between collisions

Center of Mass & Moment of Inertia

- Center of Mass: mean location of all mass in the system
- Moment of Inertia: a measure of an object’s resistance to changes to its rotation
- If a solid cylinder & a hollow tube have the same radius & the same mass, which will reach the bottom of the ramp first?
### Advanced Collisions
- What about Friction?
- What if the contact between two objects is not a single point?
- What if more than two objects collide simultaneously?

### Rigid Body Dynamics
- Physics
  - Velocity
  - Acceleration
  - Angular Momentum
- Collisions
- Friction

http://www.cs-students.stanford.edu/~dalewis/cs448a/rigidbody.html

### Collisions
- We know how to simulate bouncing really well
- But resting collisions are harder to manage

Victor J. Milenkovic & Harald Schmidl
Optimization-Based Animation
SIGGRAPH 2001

Guendelman, Bridson & Fedkiw
Nonconvex Rigid Bodies with Stacking
SIGGRAPH 2003

### Today
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### Simulation of Non-Rigid Objects
- We modeled string & cloth using mass-spring systems. Can we do the same?
- Yes…
- But a more physically accurate model uses **volumetric elements**

### Strain & Stress
- **Stress**
  - the internal distribution of forces within a body that balance and react to the loads applied to it
  - *normal stress* & *shear stress*
- **Strain**
  - material deformation caused by stress.
  - measured by the change in length of a line or by the change in angle between two lines
  \[
  \varepsilon = \frac{\Delta l}{l_0}
  \]
**Finite Element Method**

- To solve the continuous problem (deformation of all points of the object)
  - Discretize the problem
  - Express the interrelationship
  - Solve a big linear system
- More principled than Mass-Spring

![Diagram from Debunne et al. 2001](image)

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**Level of Detail**

- Interactive shape deformation
- Use high-resolution model only in areas of extreme deformation

![Diagram from Gilles Debunne, Mathieu Desbrun, Marie-Paule Cani, & Alan H. Barr (2001)](image)

**Multi-Resolution Deformation**

- Use Voronoi diagrams to match parent & child vertices.
- Interpolate values for inactive interface vertices from active parent/child vertices

- Need to avoid interference of vibrations between simulations at different resolutions

![Diagram from Debunne et al. “Dynamic Real-Time Deformations using Space & Time Adaptive Sampling”, 2001](image)

**Pre-computation & Simulation**

- FEM matrix pre-computed
- Level of detail coupling pre-computed for rest topology
- What to do if connectivity of elements changes?
  - Cloth is cut or torn
  - Surgery simulation

**Multiple Materials**

- Mueller, Dorsey, McMillan, Jagnow, & Cutler
- Stable Real-Time Deformations Symposium on Computer Animation 2002
Reading for Today:

“Melting and Flowing”
Carlson, Mucha, Van Horn III & Turk
Symposium on Computer Animation 2002

- Object is always a fluid, with time-varying viscosity
- Creating a stable solver, allowing large steps in simulation, for high viscosity fluids
- Modeling temperature-varying viscosity
- 2 resolution (x64) simulation

Reading for Friday:

“Real-time Large-deformation Substructuring”,
Barbic & Zhao, SIGGRAPH 2011
• Mesh for simulation ≠ mesh for rendering
• Mesh hierarchy decomposition
  – not automatic
  – instanced vs duplicated/unique geometry
• Limitations
  – cycles disallowed (“close the loop” problem is common challenge)
  – boundary interfaces assumed to be (nearly) rigid
  – Vibration, lumped mass, & inertia

Components of a well-written research paper?
• Motivation/context/related work
• Contributions of this work
• Clear description of algorithm
  – Sufficiently-detailed to allow work to be reproduced
  – Work is theoretically sound
    (hacks/arbitrary constants discouraged)
• Results
  – well chosen examples
  – clear tables/illustrations/visualizations
• Conclusions
  – limitations of the method are clearly stated

How to read a research paper?
(e specially an advanced paper in a new area)
• Multiple readings are often necessary
• Don’t necessarily read from front to back
• Lookup important terms
• Target application & claimed contributions
• Experimental procedure
• How well results & examples support the claims
• Scalability of the technique (order notation)
• Limitations of technique, places for future research
• Possibilities for hybrid systems with other work

Reading for Tuesday: (pick one)
  • Fracture threshold
  • Remeshing
    – need connectivity info!
  • Material properties
  • Parameter tuning

Guendelman, Bridson, & Fedkiw “Nonconvex Rigid Bodies with Stacking” SIGGRAPH 2003