Sampling, Aliasing, & Mipmaps

Schedule…

- Tonight! Friday 3/23 11:59pm: 2nd progress post for Homework 3 due

- Wednesday 3/28, 11:59pm: Homework 3 due

- Thursday 3/29, 11:59pm: Final Project Proposal & Background Research due

Final Projects

- Teams of 2 highly encouraged
- Individuals or teams > 2 must talk to me first
- Continue to discuss on LMS

- Proposals due next week (Thursday 3/29)
  - Proposed project summary
  - Identify at least 3 related academic papers
  - Description of series of test cases/examples (start very simple for debugging/testing)
  - Timeline & initial task assignment

Last Time?

- Path Tracing vs. Ray Tracing
- Irradiance Caching
- Photon Mapping
- Ray Grammar

Readings for Today:


Truncation    Compression    "Layering"

A      B      C

"Fast Bilateral Filtering for the Display of High-Dynamic Range Images", Durand & Dorsey, SIGGRAPH 2002
"Fast Bilateral Filtering for the Display of High-Dynamic Range Images", Durand & Dorsey, SIGGRAPH 2002

Scene contrast is high
Display contrast is low
Contrast reduced
Details preserved

The Secret:

Bilateral filter

High Dynamic Range Example:

Illuminance & typical Lux values:

- Direct sunlight: > 100,000 lux
- Overcast day/TV studio lighting: ~ 1,000 lux
- Office lighting: ~ 500 lux
- Moonlight: 1 lux

Sunlight checkerboard

Typical office projector: ~ 1:300 contrast ratio
max ~ 500 lux

Tone Mapping

- Convert high dynamic range (HDR) data to low dynamic range (LDR)
  - Linear Scale: loss of contrast & precision
  - Nonlinear Scale: preserve more contrast & precision in important/interesting/prominent ranges
  - Spatially-varying Scaling:

Today

- What is a Pixel?
- Examples of Aliasing
- Sampling & Reconstruction
- Filters in Computer Graphics
- Anti-Aliasing for Texture Maps

What is a Pixel?

- A pixel is not:
  - a box
  - a disk
  - a teeny tiny little light
- A pixel “looks different” on different display devices
- A pixel is a sample
  - it has no dimension
  - it occupies no area
  - it cannot be seen
  - it has a coordinate
  - it has a value
More on Samples

• Most things in the real world are continuous, yet everything in a computer is discrete.
• The process of mapping a continuous function to a discrete one is called sampling.
• The process of mapping a continuous variable to a discrete one is called quantization.
• To represent or render an image using a computer, we must both sample and quantize.

An Image is a 2D Function

• An ideal image is a continuous function \( I(x,y) \) of intensities.
• It can be plotted as a height field.
• In general an image cannot be represented as a continuous, analytic function.
• Instead we represent images as tabulated functions.
• How do we fill this table?

Sampling Grid

• We can generate the table values by multiplying the continuous image function by a sampling grid of Kronecker delta functions.

### The definition of the 2-D Kronecker delta is:

\[
\delta(x,y) = \begin{cases} 
1, & (x,y) = (0,0) \\
0, & \text{otherwise}
\end{cases}
\]

And a 2-D sampling grid:

\[
\sum_{u=-\infty}^{\infty} \sum_{v=-\infty}^{\infty} \delta(u-t,v-v-j)
\]

Sampling an Image

• The result is a set of point samples, or pixels.

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Examples of Aliasing

- Aliasing occurs because of *sampling* and *reconstruction*

Examples of Aliasing

Jagged boundaries

Examples of Aliasing

Improperly rendered detail

Examples of Aliasing

Texture Errors

Questions?

Today

- What is a Pixel?
- Examples of Aliasing
- Sampling & Reconstruction
  - Sampling Density
  - Fourier Analysis & Convolution
- Filters in Computer Graphics
- Anti-Aliasing for Texture Maps
Sampling Density
• How densely must we sample an image in order to capture its essence?
• If we under-sample the signal, we won't be able to accurately reconstruct it...

Sampling Density
• If we insufficiently sample the signal, it may be mistaken for something simpler during reconstruction (that's aliasing!)

Sampling Density
• Aliasing in 2D because of insufficient sampling density

Remember Fourier Analysis?
• All periodic signals can be represented as a summation of sinusoidal waves.

Remember Fourier Analysis?
• Every periodic signal in the spatial domain has a dual in the frequency domain.
• This particular signal is band-limited, meaning it has no frequencies above some threshold

Remember Fourier Analysis?
• We can transform from one domain to the other using the Fourier Transform.
Remember Convolution?

Convolution describes how a system with impulse response, \( h(x) \), reacts to a signal, \( f(x) \):

\[
f(x) * h(x) = \int_{-\infty}^{\infty} f(y)h(x-y)dy.
\]

\[ f \] \[ \otimes \] \[ g \] \[ = \] \[ f \otimes g \]

Sampling in the Frequency Domain

Guaranteeing Proper Reconstruction

- Separate by removing high frequencies from the original signal (low pass pre-filtering)

- Separate by increasing the sampling density

- If we can't separate the copies, we will have overlapping frequency spectrum during reconstruction → aliasing.

Remember Convolution?

- Some operations that are difficult to compute in the spatial domain can be simplified by transforming to its dual representation in the frequency domain.

- For example, convolution in the spatial domain is the same as multiplication in the frequency domain.

\[
f(x) * h(x) \rightarrow F(u)H(u)
\]

- And, convolution in the frequency domain is the same as multiplication in the spatial domain:

\[
F(u) * H(u) \rightarrow f(x)h(x)
\]

Sampling Theorem

- When sampling a signal at discrete intervals, the sampling frequency must be **greater than twice** the highest frequency of the input signal in order to be able to reconstruct the original perfectly from the sampled version (Shannon, Nyquist).

Reconstruction

- If we can extract a copy of the original signal from the frequency domain of the sampled signal, we can reconstruct the original signal!

- But there may be overlap between the copies.

\[
\text{LP}(u)(F(u)\ast S(u))
\]
### Questions?

### Today
- What is a Pixel?
- Examples of Aliasing
- Sampling & Reconstruction
- Filters in Computer Graphics
  - Ideal, Gaussian, Box, Bilinear, Bicubic
- Anti-Aliasing for Texture Maps

### Filters
- Weighting function (convolution kernel)
- Area of influence often bigger than "pixel"
- Sum of weights = 1
  - Each sample contributes the same total to image
  - Constant brightness as object moves across the screen.
- No negative weights/colors (optional)

### The Ideal Filter
- Unfortunately it has \( \text{infinite} \) spatial extent
  - Every sample contributes to every interpolated point
- Expensive/impossible to compute

### Problems with Practical Filters
- Many visible artifacts in re-sampled images are caused by poor reconstruction filters
- Excessive pass-band attenuation results in blurry images
- Excessive high-frequency leakage causes “ringing” and can accentuate the sampling grid (anisotropy)
Gaussian Filter
• This is what a CRT does for free!

Box Filter / Nearest Neighbor
• Pretending pixels are little squares.

Tent Filter / Bi-Linear Interpolation
• Simple to implement
• Reasonably smooth

Bi-Cubic Interpolation
• Begins to approximate the ideal spatial filter, the sinc function

Questions?

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• Anti-Aliasing for Texture Maps
  – Magnification & Minification
  – Mipmaps
  – Anisotropic Mipmaps
Sampling Texture Maps

- When texture mapping it is rare that the screen-space sampling density matches the sampling density of the texture.

Spatial Filtering

- Remove the high frequencies which cause artifacts in texture minification.
- Compute a spatial integration over the extent of the pixel
- This is equivalent to convolving the texture with a filter kernel centered at the sample (i.e., pixel center)!
- Expensive to do during rasterization, but an approximation it can be precomputed

MIP Mapping

- Construct a pyramid of images that are pre-filtered and re-sampled at 1/2, 1/4, 1/8, etc., of the original image's sampling
- During rasterization we compute the index of the decimated image that is sampled at a rate closest to the density of our desired sampling rate
- MIP stands for *multum in parvo* which means *many in a small place*

MIP Mapping Example

- Thin lines may become disconnected / disappear

Linear Interpolation

- Tell OpenGL to use a tent filter instead of a box filter.
- Magnification looks better, but blurry
  - (texture is under-sampled for this resolution)

MIP Mapping Example

- Small details may "pop" in and out of view
Examples of Aliasing

Texture Errors

- point sampling
- mipmaps & linear interpolation

Storing MIP Maps

- Can be stored compactly
- Illustrates the 1/3 overhead of maintaining the MIP map

Anisotropic MIP-Mapping

- What happens when the surface is tilted?

Nearest Neighbor  MIP Mapped (Bi-Linear)

Anisotropic MIP-Mapping

- Square MIP-map area is a bad approximation

Anisotropic MIP-Mapping

- We can use different mipmaps for the 2 directions
- Additional extensions can handle non axis-aligned views


Questions?
Reading for Friday 4/2: