Ray Tracing

Last Time?
- Keyframing
- Procedural Animation
- Physically-Based Animation
- Forward and Inverse Kinematics
- Motion Capture

Today
- Ray Casting
  - Ray-Plane Intersection
  - Ray-Sphere Intersection
  - Point in Polygon
- Ray Tracing
- Recursive Ray Tracing
- Distribution Ray Tracing

Reading for Today

Durer’s Ray Casting Machine
- Albrecht Durer, 16th century

Ray Casting
For every pixel
Construct a ray from the eye
For every object in the scene
Find intersection with the ray
Keep if closest
Shade depending on light and normal vector
A Note on Local Shading

- Surface/Scene Characteristics:
  - surface normal
  - direction to light
  - viewpoint
- Material Properties
  - color/texture
  - diffuse (matte)
  - specular (shiny)
- More later!

Diffuse sphere  Specular spheres

Ray Representation?

- Two vectors:
  - Origin
  - Direction (normalized is better)
- Parametric line (explicit representation)
  - \( P(t) = \text{origin} + t \times \text{direction} \)

3D Plane Representation?

- Plane defined by
  - \( P_0 = (x,y,z) \)
  - \( n = (A,B,C) \)
- Implicit plane equation
  - \( H(P) = Ax + By + Cz + D = 0 \)
  - \( n \cdot P + D = 0 \)
- Point-Plane distance?
  - If \( n \) is normalized, distance to plane, \( d = H(P) \)
  - \( d \) is the signed distance!

Explicit vs. Implicit?

- Ray equation is explicit
  - \( P(t) = R_o + t \times R_d \)
  - Parametric
  - Generates points
  - Harder to verify that a point is on the ray
- Plane equation is implicit
  - \( H(P) = n \cdot P + D = 0 \)
  - Solution of an equation
  - Does not generate points
  - Verifies that a point is on the plane

Ray-Plane Intersection

- Intersection means both are satisfied
- So, insert explicit equation of ray into implicit equation of plane & solve for \( t \)
  \[
  P(t) = \text{R}_o + t \times \text{R}_d \\
  H(P) = n \cdot P + D = 0 \\
  n \cdot (\text{R}_o + t \times \text{R}_d) + D = 0 \\
  t = -\frac{D + n \cdot \text{R}_o}{n \cdot \text{R}_d}
  \]

Additional Housekeeping

- Verify that intersection is closer than previous
  \( P(t) < t_{\text{current}} \)
- Verify that it is not out of range (behind eye)
  \( P(t) > t_{\text{min}} \)
Normal

- Needed for shading
  - diffuse: dot product between light and normal
- Normal of a plane is constant!

Ray-Triangle Intersection

- Use barycentric coordinates:
  - \( P(\alpha, \beta, \gamma) = \alpha \mathbf{a} + \beta \mathbf{b} + \gamma \mathbf{c} \)
  - with \( \alpha + \beta + \gamma = 1 \)
  - If \( 0 < \alpha < 1 \) & \( 0 < \beta < 1 \) & \( 0 < \gamma < 1 \)
    then the point is inside the triangle!

How Do We Compute \( \alpha, \beta, \gamma \)?

- Ratio of opposite sub-triangle area to total area
  - \( \alpha = \frac{A_a}{A} \)
  - \( \beta = \frac{A_b}{A} \)
  - \( \gamma = \frac{A_c}{A} \)
- Use signed areas for points outside the triangle

Using Cramer’s Rule...

- Used to solve for one variable at a time in system of equations

\[
\begin{vmatrix}
    a_x - R_{ox} & a_x - c_x & R_{dx} \\
    a_y - R_{oy} & a_y - c_y & R_{dy} \\
    a_z - R_{oz} & a_z - c_z & R_{dz} \\
\end{vmatrix} = 0
\]

\[
\begin{vmatrix}
    a_x - R_{ox} & a_x - c_x & a_x - R_{oz} \\
    a_y - R_{oy} & a_y - c_y & a_y - R_{oz} \\
    a_z - R_{oz} & a_z - c_z & a_z - R_{oz} \\
\end{vmatrix} = 0
\]

\[
t = \frac{- \begin{vmatrix}
    a_x - R_{ox} & a_x - c_x & a_x - R_{oz} \\
    a_y - R_{oy} & a_y - c_y & a_y - R_{oz} \\
    a_z - R_{oz} & a_z - c_z & a_z - R_{oz} \\
\end{vmatrix}}{\begin{vmatrix}
    a_x - c_x & a_x - c_x & a_x - c_x \\
    a_y - c_y & a_y - c_y & a_y - c_y \\
    a_z - c_z & a_z - c_z & a_z - c_z \\
\end{vmatrix}}
\]

Sphere Representation?

- Implicit sphere equation
  - Assume centered at origin (easy to translate)
  - \( H(P) = P \cdot P - r^2 = 0 \)

Ray-Sphere Intersection

- Insert explicit equation of ray into implicit equation of sphere & solve for \( t \)

\[
P(t) = R_o + t R_d \quad H(P) = P \cdot P - r^2 = 0
\]

\[
(R_o + t R_d) \cdot (R_o + t R_d) - r^2 = 0
\]

\[
R_d \cdot R_d t^2 + 2 R_d \cdot R_o t + R_o \cdot R_o - r^2 = 0
\]
Ray-Sphere Intersection

- Quadratic: \( at^2 + bt + c = 0 \)
  - \( a = 1 \) (remember, \( ||R_d|| = 1 \))
  - \( b = 2R_d \cdot R_o \)
  - \( c = R_o \cdot R_o - r^2 \)

- with discriminant \( d = \sqrt{b^2 - 4ac} \)
- and solutions \( t_{\pm} = \frac{-b \pm d}{2a} \)

- What does it mean if there are no solutions, 1 solution, or 2 solutions?

Questions?

Today

- Ray Casting
- Ray Tracing
  - Shadows
  - Reflection
  - Refraction
- Recursive Ray Tracing
- Distribution Ray Tracing

How Can We Add Shadows?

Mirror Reflection

- Cast ray symmetric with respect to the normal
- Multiply by reflection coefficient (color)

Reflection

- Reflection angle = view angle
- \( \mathbf{R} = \mathbf{V} - 2(\mathbf{V} \cdot \mathbf{N}) \mathbf{N} \)
Transparency

- Cast ray in refracted direction
- Multiply by transparency coefficient (color)

Snell-Descartes Law:
\[ \eta_i \sin \theta_i = \eta_T \sin \theta_T \]

Total Internal Reflection

- Total internal reflection when the square root is imaginary
- Don't forget to normalize!

Refraction & the Sidedness of Objects

- Make sure you know whether you're entering or leaving the transmissive material:

Questions?
Reading for Today


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- Ray Casting
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Ray Tracing

- Trace ray
  - Intersect all objects
  - For every light
    - Cast shadow ray
    - Color = Color*refl * trace reflected ray
    - If mirror
      - Color += Color*ambient * trace reflected ray
  - If transparent
    - Color += Color*transm * trace transmitted ray

- Does it ever end?

The Ray Tree

- Stopping criteria:
  - Recursion depth
    - Stop after a number of bounces
  - Ray contribution
    - Stop if reflected / transmitted contribution becomes too small

Complexity?

Ray Debugging

- Visualize the ray tree for single image pixel

Today

- Ray Casting
- Ray Tracing
- Recursive Ray Tracing
- Distribution Ray Tracing
  - Soft shadows
  - Antialiasing (getting rid of jaggies)
  - Glossy reflection
  - Motion blur
  - Depth of field (focus)
Shadows
• one shadow ray per intersection per point light source

Soft Shadows
• multiple shadow rays to sample area light source

Reflection
• one reflection ray per intersection

Glossy Reflection
• multiple reflection rays

Antialiasing – Supersampling
• multiple rays per pixel

Shadows & Light Sources
- no shadow rays
- one shadow ray
- clear bulb
- frosted bulb

http://www.pa.uky.edu/~sciworks/light/preview/bulb2.htm
http://www.paulrefer.net/html الرiry/scrresh Público.htm
Motion Blur
- Sample objects temporally

Depth of Field
- multiple rays per pixel

Ray Tracing Algorithm Analysis
- Ray casting
- Lots of primitives
- Recursive
- Distributed Ray Tracing Effects
  - Soft shadows
  - Anti-aliasing
  - Glossy reflection
  - Motion blur
  - Depth of field
  - cost ≈ height * width * num primitives * intersection cost * size of recursive ray tree * num shadow rays * num supersamples * num glossy rays * num temporal samples * num focal samples *
  - can we reduce this?
  - these can serve double duty

Raytracing & Epsilon

Reading for Friday 3/8:
- Goral, Torrance, Greenberg & Battaile “Modeling the Interaction of Light Between Diffuse Surfaces”, SIGGRAPH ’84